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MYERS ARITHMETIC

BOOK III

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MYERS ARITHMETIC

BOOK III

BY

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PREFACE

The present book is an outgrowth of the notion that arithmetic as a science of pure number, and arithmetic as a school science, must be treated from essentially different standpoints. Viewed as a finished mental product, arithmetic is an *abstract* science, taking its bearings solely from the needs of the subject; but viewed as a school subject, arithmetic should be an *abstracted* science, taking its bearings mainly from the needs of the learner. The former calls only for logical treatment, while the latter calls for psychological treatment as well. In other words, to be of high educational value the school science of arithmetic must take into full account the particular stage of the pupil's development.

The abstract stage must be approached by steps which begin with the learner, rise with his unfolding powers, and end leaving him in possession of the outlines of the science of arithmetic. To break vital contact with the learner at any stage of the unfolding process is fatal. A controlling principle in the development of the various topics of this book is that any phase of arithmetical work, to be of value, must make an appeal to the life of the pupil.

But the social and industrial factors in American communities enter largely into the pupil's life. This gives to material drawn from industrial sources and from everyday affairs, high pedagogical value for arithmetic. The recent infusion of new life into the curricula of elementary schools through the wide introduction into them of nature study, manual training, and geometrical drawing furnishes a basis for a closer unifying of the pupil's work in arithmetic with his work in the other school subjects. Wide use has been made of all these sources of arithmetical material.

A rational presentation of the processes and principles of arithmetic can be secured as well through material representing real conditions as through material representing artificial conditions. Not only have most of the problems been drawn from real sources but an earnest effort has also been made to have all data of problems correct and consistent, to the end that inferences from them may be relied upon. With so rich a store as the book contains, however, it is perhaps too much to hope that no errors remain. The authors will deem it a favor to be notified of any errors that may be detected.

The majority of the pupils of the elementary school never reach the high school. Even these pupils, whose circumstances cut them off from advanced mathematical study, have a right to claim some useful knowledge of the more powerful instruments of algebra and geometry. For those who will continue their studies into the high school it is important that the roots of the later mathematical subjects be well covered in the soil of the earlier. The present book meets the needs of both classes of pupils through the organic correlation of the elements of geometry and of general number with the arithmetic proper. Treated thus, the geometry serves to illustrate the work of arithmetic, and the algebra emerges from the arithmetic as generalized number.

In particular, this book aims to accomplish four main purposes, viz.:

(1) *To present a pedagogical development of elementary mathematics, both as a tool for use and as an elementary science;*

(2) *To base this development on subject-matter representing real conditions;*

(3) *To open to the pupil a wide range and variety of uses for elementary mathematics in common affairs—to aid him to get a working hold of his number sense; and*

(4) *To give the pupil some training in ways of attacking*

common problems arithmetically and some power to analyze quantitative problems.

The work makes continual call for estimating magnitudes and for actual measurement by the pupils. Supply the children with measures, foot-rules, yardsticks, meter-sticks, etc., and encourage continual use of them. Make so regular a feature of this work that pupils form the habit of estimating distances, areas, volumes, weights, etc., always correcting their estimates by actual measurement.

All models, scales, or standards of measure made by pupils should be carefully kept and used in the later work.

As in Books I and II of this Arithmetic, so here there are groups of problems that bear on the development of some important idea or law, having an interest on its own account. In these groups each problem is a step in a connected line of thought culminating in an important truth. This plan furnishes numerous problems, miscellaneous as to process, thereby requiring original mathematical thought, and still organically related to a central idea, thereby calling for the constant exercise of judgment. For the stage of maturity of pupils of the later grades this is believed to be an important feature. It avoids the danger, always present with lists of promiscuous problems when classified under the arithmetical processes to be exemplified, of reducing to the mechanical what should never be allowed to become mechanical, viz.: the analysis of relations.

But due regard has been had to the necessity of sufficient drill in pure number to enable the pupil to obtain both a conscious recognition of processes and considerable facility in their automatic use. This is done through the conviction that the fundamental arithmetical operations should be reduced to the automatic stage as early as possible consistently with a clear understanding of them.

The attention of teachers is called to the section on Short Methods and Checking at the close of the book. After pupils have clearly grasped the meaning of the arithmetical

processes, and have acquired some mastery of their uses, a relief from the tedium of long arithmetical calculations becomes a matter of great importance. No one can become a rapid computer without short methods, and, considering that the problems of daily life that call for arithmetical treatment do not have answers with them, no one can be certain of his results without means of checking calculations. Every expert accountant uses them, and the more expert he is the more he uses them. In fact, expertness consists very largely in the ability to shorten calculations and to apply rapid checks. Constant use of these sections should be made through the seventh and eighth grades.

Definitions, processes, rules, and even special subjects are worked out under the guidance of the principle: "First its informal, though rational, use, and afterwards conscious recognition of its formal use." Definitions of merely technical terms are given when called for by the development and where the need for them arises.

Book III is a complete arithmetic. All the essentials are given a topical treatment here. The earlier topics, that were carefully taught in Books I and II, are here rounded out sufficiently for elementary school needs. The later topics are carefully taught and applied in numerous directions. The unities of the standard arithmetical subjects are carefully preserved.

Part I reviews thoroughly all the fundamental operations with whole numbers; provides ample material for drill-exercises at suitable places; teaches carefully the operations on fractions, common and decimal, the laws of percentage and interest and their simpler and more common applications.

Part II reviews, from a higher point of view, the topics previously taught, brings out a little of the science, adds the more difficult applications of percentage and interest, and while completing the mensuration of the common

geometrical forms makes use of a little rudimentary practical geometry and general number. The general number brings out and emphasizes the unities of arithmetical topics and laws, and the geometry illuminates the arithmetic and general number and arouses an interest in and an appreciation for the constructive work of the builder.

For the convenience of teachers in reviews, and for aid in estimating the rate of progress from time to time and the stage of advancement, summaries of the work of both parts are given at the end of this book, as was done in Books I and II.

The author's acknowledgments for suggestions and ideas are due to numerous arithmetical writers and teachers. He desires to express his obligations to Miss Katherine M. Stilwell, of the University of Chicago School of Education; to Miss Ada Van Stone Harris, Supervisor of Primary Schools, Rochester, N. Y., both of whom read much of the proof. He desires most heartily to thank Mr. F. W. Buchholz, of the Chicago Normal School, whose unceasing diligence and pains with the manuscripts and the proofs have wrought distinct improvements on nearly every page of the book.

THE AUTHOR.

Chicago, June, 1908.

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PART I

COMMON USES OF NUMBERS

§1. Wind Pressure.

1. If you hold a board 1′ wide and 10′ long flatwise against the wind, how hard does the board push against you if the wind pushes against each square foot of the board with a force of 2 pounds? Of $5\frac{3}{4}$ pounds?

The wind is a stream of air moving along with a certain speed, or *velocity*. The greater the speed, or *velocity*, the stronger it pushes against objects it strikes.

The velocity of wind in miles per hour and the pressure per square foot are given below:

	Velocity in Miles per Hour	Pressure per Square Foot
		lb. oz.
Light breeze	$3\frac{1}{4}$	.. .75
Moderate breeze	$6\frac{1}{2}$	.. 3.33
Fresh breeze	$16\frac{1}{4}$	1, 5
Stiff breeze	$32\frac{1}{2}$	5, 3
Strong gale	$56\frac{1}{4}$	15, 9
Hurricane	$79\frac{1}{2}$	31, 4
Violent hurricane	$97\frac{1}{2}$	46, 12

Note.—Use measures from your schoolhouse or from other buildings in your neighborhood in preference to the numbers in the problems.

2. What is the total pressure in pounds on the west side of a house 30 ft. by 25 ft., due to a light breeze from the west? To a stiff breeze? To a strong gale?

3. What is the pressure on the side of a tall building 265 ft. by 80 ft., due to a hurricane blowing squarely against it?

4. Find the wind pressure against the side of a load of hay 25 ft. long and 12 ft. high, due to a strong gale blowing squarely against it.

5. Find the number of pounds pressure against a sign-board 100 ft. by 14 ft., due to a strong gale blowing squarely against it.

6. The side of a passenger coach against which the wind blows is 70 ft. long and 15 ft. high. (a) Find the wind pressure due to a stiff breeze blowing squarely against the rectangle $15' \times 70'$.

(b) Find the wind pressure due to a strong gale blowing squarely against it.

7. Find the wind pressure on the side of a train of 10 such cars as that of problem 6 due to a hurricane blowing squarely against it.

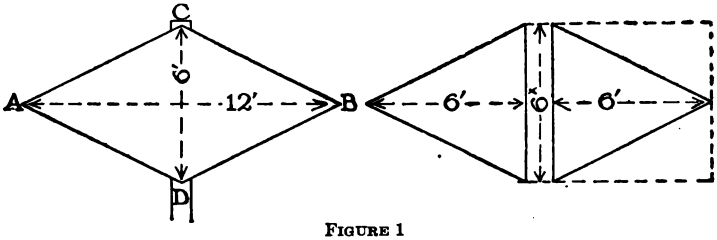


FIGURE 1

8. Find the wind pressure on a diamond-shaped sign-board, due to a violent hurricane blowing squarely against it, if the diamond is 12′ long (AB = 12′) and 6′ broad at the middle (CD = 6′). (See Fig. 1.)

9. Find the wind pressure due to a strong gale blowing squarely against the flat side of a dense tree-top of the shape of a triangle, 30 feet wide and 40 feet high.

*The mark (′) means *foot* or *feet;* (″) means *inch* or *inches.*

§2. Scale Drawing.

One inch in Fig. 1 represents 80 rods.

1. How many acres are in the tract *ABCDE*? What is the farm worth at \$95 per acre?

2. The field is entirely enclosed, except along the river, by a barbed wire fence having 4 lines of wire.

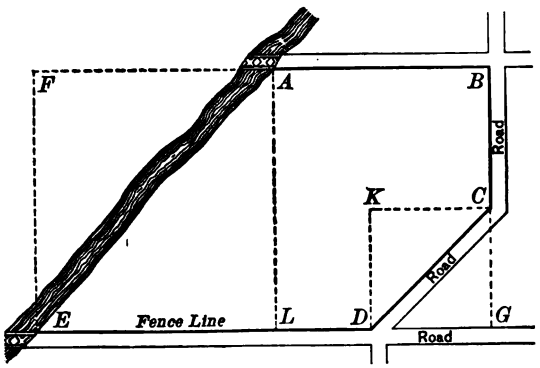


FIGURE 1

Barbed wire weighs about 1 lb. per rod of single wire. Find the cost of the wire at \$3.50 per bale of 100 pounds.

3. If posts cost 35¢ apiece and are set 2 rd. apart, find the cost of the posts.

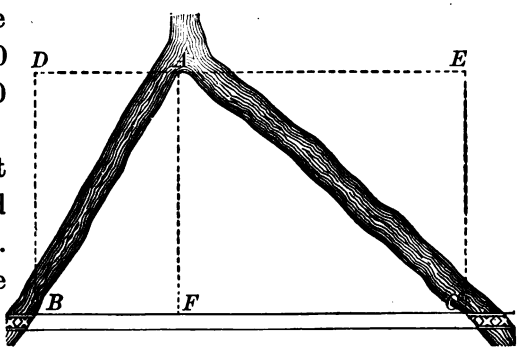


FIGURE 2

4. In Fig. 2, 1 in. represents 80 rd., or *the scale is 1" to 80 rd.* How many acres are in the field ABF? What is the field worth at \$75 per acre?

5. Find how many acres there are in the field AFC. What is its value at \$87½ per acre?

6. Find the value of the field ABF at \$80 per acre.

7. How many rods of fence would be needed to enclose the field ABF? AFC? The entire field DBCE (deduct 65 rd. for river)?

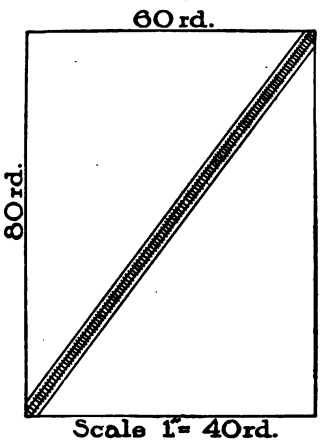


FIGURE 1

8. A railroad runs straight across a rectangular field, dividing it into two triangles as shown in Fig. 1. How much is each triangular part worth at \$120 per acre, supposing the road takes off equally from each part a strip containing $2\frac{1}{2}$ acres?

9. The railroad company fences both sides of the strip with 4-wire fences, each 96 rd. long. Posts are set a rod apart, and cost 25¢ each. The wire used costs \$3.25 per bale of 100 rd., and 2 men at \$1.50 per day work 3 days to build the fence. What did it cost the company to fence the road?

10. If 1 sq. in. of the rectangle of Fig. 2 represents $\frac{5}{8}$ of an acre, how many acres are represented by the whole rectangle?

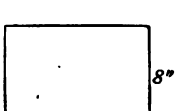


FIGURE 2

11. If one sq. ft. of a sheet of zinc weighs $2\frac{1}{2}$ oz., how much will pieces of the same sheet of the sizes shown in Fig. 3 weigh?

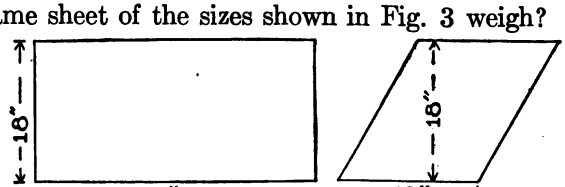


FIGURE 3

NOTATION AND NUMERATION

§3. The Digits.

The symbols, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits, or figures. The symbol, 0, is called zero.

In the number 62,437, what does the 7 stand for? The 3? The 4? The 2? The 6?

What is the value that each 6, in 66,666, stands for?

Besides the value which a digit has from its name, it is seen that it has also a value, that depends on the *place where it stands* in a number.

The zero, 0, is used to occupy places for which there are no units. Thus, the number, 5006, means 5 thousands, no hundreds, no tens, and 6 units.

Four-digit numbers, like 6528, may be read:

(1) 6 thousand 5 hundred twenty-eight, or,

(2) Sixty-five hundred twenty-eight.

The second way is quite commonly used in daily life.

§4. Periods.

The first five *periods* and their names are shown here:

TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			UNITS		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
5	7	1	3	2	4	9	1	8	6	4	5	2	8	4

The names of the higher periods, which are seldom used in practice are: quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions. Notice the similarity of the first parts of the last four names to the names of the last four months of the year.

In Great Britain a notation different from that explained above is used. The method just explained is called the French notation and is used altogether in the United States, France, and Germany.

The English notation groups the digits into periods of six figures each:

Billions Millions Units

865324,786593,279368

In the English notation a million millions make a billion, a million billions a trillion, etc. In the French notation a thousand millions make a billion, a thousand billions a trillion, etc.

§5. The Decimal Point.

In a number like 165.4, read one hundred sixty-five *and* four tenths, the dot, or period (.), is used to point out the digit that stands for units. For this reason the dot is called the *point*. Because it indicates also that the part of the number on its right is a decimal fraction, it is also sometimes called the *decimal* point.

It is well to note that, with a whole number (as 648) in which *no point is written, a point is understood to stand just to the right of the units' digit* (8) as though 648. were the written form.

§6. Notation and Numeration.

Notation is a way of writing or denoting numbers.

The way of writing numbers with the digits and zero, is called the *Arabic* notation, because Europeans first learned it from the Arabs.

Numeration is a way of speaking or reading numbers.

1. Write the following numbers:

Six hundred seven; ninety and nine tenths; seventy-five hundred and seventy-five hundredths; sixty-eight thousand and sixty-eight thousandths; twenty-five million twenty-five thousand twenty-five; twenty-five thousand and twenty-five thousandths.

2. Read aloud the following. Write in words also:

Radius of the earth	=	6,963,455 yards
Distance to moon	=	76,429,120 rods
Radius of sun	=	138,560,000 rods
Distance to sun	=	92,897,500 miles
Light travels in one minute	=	11,199,000 miles

3. Read the following numbers:

.6	27.27	85,000.085
24.6	600.06	3,208,609.34
35.62	9000.009	12,000.012
450.45	9090.09	32,604,321,206.

§7. Roman Notation.

The Roman notation is the way of writing numbers by using capital letters.

The Romans invented this way and it is now often used for dates on foundation stones of buildings, on monuments, for numbering volumes and chapters of books, and in works of art.

In this notation, the letters used and their values are:

I denotes 1	X denotes 10	D denotes 500
V " 5	L " 50	M " 1000
	C " 100	

I. When a letter of less value, as I, is placed before a letter of greater value, as V, the value of the less is to be *taken from* that of the greater; thus, IV means 4 and XC means 90.

II. When a letter of less value follows one of greater value, its value is *added* to that of the greater; as, VI for 6, XV for 15, and CX for 110.

III. Repeating a letter repeats its value; as, XX for 20, CC for 200.

IV. A horizontal line over a letter increases its value a thousandfold; as, $\overline{C}$ meaning 100,000, $\overline{D}$ meaning 500,000.

In interpreting numbers written in the Roman notation, always begin on the right.

§8. Change of Notation.

1. Change from Roman to Arabic notation:

IX	MD	MCC	MCMIX
XV	DC	LXVI	$\overline{MD}$
XL	CD	XLIV	$\overline{DC}$
XIX	CC	LXXXIX	XCIX
CX	CCCC	MDM	MDCCXVII
XC	DD	MCM	MDCCCLXXXIV
CXC	DM	DX	MMDCCIX

2. Change from Arabic to Roman notation:

19	143	2000	10000
90	197	2500	50000
60	508	2409	100000

ADDITION AND SUBTRACTION

§9. Addition. *Answer all you can orally.*

1. Give two numbers, that taken together, make 17; 18; 19; 23; 29.

2. Give three numbers that, taken together, make 13; 22; 37; 45; 63.

3. Give two equal numbers that, taken together, make 14; 18; 24; 32; 50.

4. What is $\frac{1}{2}$ of 14? $\frac{1}{2}$ of 18? Of 24? Of 32? Of 50?

5. Answer the following questions:

(1)	$8+2.5=?$	(6)	$\frac{1}{2}+\frac{1}{4}=?$	(11)	$6.5+2\frac{1}{2}=?$
(2)	$2.5+1.5=?$	(7)	$.5+.25=?$	(12)	$16\frac{2}{3}+3\frac{1}{3}=?$
(3)	$7.8+2.2=?$	(8)	$\frac{1}{2}+\frac{1}{3}=?$	(13)	$15\frac{7}{8}+6\frac{2}{3}=?$
(4)	$1.8+2.9=?$	(9)	$\frac{2}{3}+\frac{1}{2}=?$	(14)	$37\frac{1}{2}+12.5=?$
(5)	$12.5+12.5=?$	(10)	$\frac{4}{5}+\frac{1}{2}=?$	(15)	$.66\frac{2}{3}+.33\frac{1}{3}=?$

Addition is the way of combining numbers into a single number. The numbers to be combined are the addends, and the result is the sum.

6. Find the sums of the following:

(1)	(2)	(3)	(4)
16.25	3.875	2068.121	63.2471
206.09	40.161	110.011	2.0604
8.26	325.654	1216.714	1000.
$.75\frac{1}{2}$	.079	618.	2.0095
9.28	14.687	.928	316.
$10.06\frac{1}{2}$	90.896	3.2	.78

When decimal numbers are to be added write the numbers in a column, so that the decimal points stand in a vertical line and add as with whole numbers.

In integers, the decimal point is supposed to stand to the right of the unit's digit.

Form the habit of checking addition of whole and decimal numbers by casting out the 9's.

CHECKING ADDITION BY CASTING OUT THE NINES

To cast out the nines from a number add its digits and whenever the sum equals or exceeds 9, drop 9 and continue adding the next digits to what remains, dropping 9 whenever the sum equals or exceeds 9. The last remainder is called the *excess*.

ILLUSTRATION.—Cast the 9's out of 647,255.

Beginning on the left, $6+4=10$; drop 9, giving the remainder 1. $1+7+2=10$, drop 9. $1+5+5=11$, drop 9. The last remainder is 2 and this is the excess of 647,255.

To check addition by casting out nines, cast the nines out of the addends and the sum. Then cast out the nines from the excesses of the addends. If this last excess of the excesses equals the excess of the sum, the work is probably correct.

ILLUSTRATION	EXCESS
8465	5
3282	6
4497	6
2957	5
7642	1
26,843	5=5

To obtain the excess of the excesses of the addends:

$5+6=11$ drop 9, leaving 2. $2+6+5=13$, drop 9, leaving 4. $4+1=5$, the excess of the excesses of the addends. Since this equals the excess of the sum, the work is checked.

7. Add the following and check by casting out the nines:

(1)	(2)	(3)	(4)	(5)
328	3264	6835	832.06	8.012
605	7264	876	15.75	700.07
564	5004	9863	5.4	17.38
876	7385	9	1268.	1236.5
333	9816	257	86.37	.9
603	3563	24	200.02	3.875
818	9090	8364	1683.1	7.9
925	1827	418	11.07	9605.09

§10. Subtraction.

1. What number added to 8 gives 14?

Think thus: 2 added to 8 gives 10; and 4 more makes 14. 2 and 4 (or 6) added to 8 gives 14. Or, one may say directly, 14 minus 8, or 8 from 14, 6.

2. What number added to 18 gives 24? 34? 44? 54?

3. Answer the following:

(1) $16 - 9 = ?$	(5) $\frac{3}{4} - \frac{3}{7} = ?$	(9) $16\frac{2}{3} - 12\frac{1}{2} = ?$
(2) $28 - 15 = ?$	(6) $\frac{5}{9} - \frac{2}{11} = ?$	(10) $37 - 15\frac{2}{5} = ?$
(3) $12\frac{1}{2} - \frac{1}{4} = ?$	(7) $\frac{5}{6} - \frac{2}{7} = ?$	(11) $50 - 37\frac{1}{2} = ?$
(4) $19\frac{3}{4} - \frac{1}{2} = ?$	(8) $\frac{8}{9} - \frac{4}{5} = ?$	(12) $25 - 18\frac{7}{9} = ?$

Subtraction is the way of finding the difference between two numbers, or of taking from a number a part of itself.

The number to be subtracted is the subtrahend.

The number from which to subtract is the minuend.

The result is the difference, or remainder.

4. Find the differences of the following pairs of numbers:

(1)		(2)
$18.32\frac{1}{4}$	$= 18.31\frac{25}{20}$	$16.287\frac{1}{2}$
$6.98\frac{3}{5}$	$= 6.98\frac{12}{20}$	$.986\frac{1}{4}$
	$11.33\frac{13}{20}$	

(3)	(4)	(5)
$201.30\frac{1}{4}$	100.00	6581.01
$67.42\frac{1}{3}$	75.25	136.78

Write the numbers to be subtracted in a column, so that the decimal points are in a vertical line. Write zeros enough in the minuend to make the same number of decimal places as there are in the subtrahend. Then subtract as with whole numbers.

CHECKING SUBTRACTION

Check subtraction by adding the difference to the subtrahend and noticing whether the sum equals the minuend.

Subtraction may be checked also by casting out the nines.

All that is necessary is to cast out the nines from the minuend, the subtrahend and the remainder. If the difference between the excesses of the minuend and the remainder equals the excess of the remainder, the work is probably correct.

ILLUSTRATIONS

Check by casting out nines. Check by addition.

		Excesses		
Minuend,	87659.....	8	Or, 36984,	subtrahend.
Subtrahend,	36984.....	3	50675,	remainder.
Remainder,	50675..	5=5	87659,	minuend.

5. Subtract the following and check in both ways:

(1)	(2)	(3)	(4)	(5)
8700	58.00	1062.700	9036.0900	681.7060
7916	19.64	898.807	972.0168	309.0098

Solution for (5):

	Excess
681.7060.....	1+9=10
309.0098.....	2..... 2
372.6962.....	8 - 8

§11. Problems in Addition and Subtraction.

1. The published daily circulation of a city newspaper from week to week is here given:

First Week		Second Week		Third Week		Fourth Week		Fifth Week	
Day	Copies	Day	Copies	Day	Copies	Day	Copies	Day	Copies
1.	255,572	8.	305,725	15.	304,636	22.	304,836	29.	303,383
2.	303,062	9.	303,489	16.	302,173	23.	304,698	30.	302,005
3.	323,513	10.	304,991	17.	302,650	24.	310,870		
4.	312,724	11.	304,746	18.	304,255	25.	299,780		
5.	306,009	12.	305,515	19.	302,942	26.	301,455		
6.	297,918	13.	299,095	20.	297,684	27.	298,446	Total	
Total		Total		Total		Total		Total for mo.	

Find the weekly totals and the total for the month.

2. In the year 1891 the United States imported 94,628,119 lb. of coffee, which was 54,262,757 lb. less than was imported during the two previous years. How many pounds were imported during the two previous years?

3. The United States imported 79,192,253 lb. of tea in 1889; 83,494,956 lb. in 1890, and 82,395,924 lb. in 1891. How many pounds of tea were imported in the three years?

4. In 1889, $27,024,551 worth of molasses was imported into the United States; in 1890, $31,497,243 worth, and in 1891, $32,659,172 worth. How many dollars' worth was imported during the three years?

§12. Areas of States and Territories.

Such problems may be selected from the following list as have a special geographical interest to each school.

From the facts on page 13, solve the following:

1. Find the total area of the North Atlantic division of states; of the South Atlantic division. Check the work.

2. Which is the larger, and by how much, the North or South Atlantic division? Check the work.

3. Find the total area of the North Central division; of the South Central division. Check the work.

4. Find the total area of the Western division. Check the work.

5. Which is the larger, and by how much, the Western or the North Atlantic division? The Western or the South Atlantic division? The Western or the South Central division? Check the work.

6. Find the total area of the United States without the outlying territory. Check the work.

7. Find the total area of the outlying territory of the United States. Check the work.

8. Find the total area of the United States including the outlying territory. Check the work.

9. Compare the area of the *section* you live in with that of the other *section*, or sections of the same division.

10. Solve other problems on the table of page 13.

AREAS IN SQUARE MILES OF STATES AND TERRITORIES GEOGRAPHICALLY GROUPED

SECTION	NORTH ATLANTIC DIVISION	AREA IN SQ. MILES
EASTERN	Me.	33,040
	N. H.	9,305
	Vt.	9,565
	Mass.	8,315
	R. I.	1,250
WESTERN	Conn	4,990
	N. Y.	49,170
	N. J.	7,815
	Penn	45,215
	TOTAL	

SECTION	SOUTH CENTRAL DIVISION	AREA IN SQ. MILES
EASTERN	Ky.	40,400
	Tenn.	42,050
	Ala.	52,250
	Miss.	46,810
WESTERN	La.	48,720
	Tex	265,780
	Okl.	70,430
	Ark	53,850
	TOTAL	

SECTION	SOUTH ATLANTIC DIVISION	AREA IN SQ. MILES
NORTHERN	Del.	2,050
	Md.	12,210
	D. C.	70
	Va.	42,450
	W. Va.	24,780
SOUTHERN	N. C.	52,250
	S. C.	30,570
	Ga.	59,475
	Fla.	58,680
	TOTAL	

SECTION	WESTERN DIVISION	AREA IN SQ. MILES
EASTERN	Mont.	146,080
	Wy.	97,890
	Col.	103,925
	N. M.	122,580
	Ariz.	113,020
CENTRAL	Utah	84,970
	Nev.	110,700
	Idaho	84,800
WESTERN	Wash.	69,180
	Ore.	96,030
	Cal.	158,360
	TOTAL	

SECTION	NORTH CENTRAL DIVISION	AREA IN SQ. MILES
EASTERN	Ohio	41,060
	Ind.	36,350
	Ill.	56,650
	Mich.	58,915
	Wis.	56,040
WESTERN	Minn.	83,365
	Iowa	56,025
	Mo	69,415
	N. D.	70,795
	S. D.	77,650
	Neb.	77,510
	Kan.	82,080
	TOTAL	

	OUTLYING TERRITORY	AREA IN SQ. MILES
	Alaska	577,390
	Hawaii	6,740
	Phil. Is.	114,410
	Tutuila	77
	Guam	150
	Porto Rico	3,531
	TOTAL	

11. Of the yearly trade within New York state, $1,650,000,000 worth of the freight passes over the railroads; $150,000,000 over the canals; and $250,000,000 over Long Island Sound and the lakes. What is the total value of this trade?

§13. Exercises for Practice in Addition and Subtraction.

The following table contains the number of 500-pound bales of cotton shipped from the United States and sold to the countries named during the years indicated at the top of the columns:

Country	1880	1890	1900	1905	1906
United Kingdom	2,433,254	2,905,152	2,304,445	3,969,165	3,181,143
Germany	308,045	837,641	1,659,659	2,056,917	1,871,442
France	359,693	484,758	738.027	821,074	817,583
Italy	59,126	129,751	444,638	535,445	486,607
Spain	133,873	175,340	246,675	295,821	241,747
Belgium	17,896	93,588	151,413	156,912	114,673
Russia	204,500	193,162	54,950	129,060	112,480
Other Countries.	88,121	37,665	186,390	171,932	119,351
Total to Europe					
Japan	.....	1684	323,202	336,575	147,269
Canada	19,619	58,473	111,373	117,225	141,908
Mexico	19,763	26,095	18,523	79,092	29,285
China	.....	.....	11,215	4241	699
All others	232	201	1941	5185	3003
Total to all					

1. Find from the table the total shipments in 1880 (1) to Europe; (2) to all countries.

2. Find similar totals for each year, and tell what the totals mean.

3. Read some of the larger numbers of this table.

4. How much did the total shipments of cotton to all countries increase from 1880 to 1906?

5. What is the difference between the shipments to the United Kingdom in 1880 and in 1906? Is it an increase or a decrease?

6. Find this difference for other countries, as, for Japan, China, Russia, etc., and mark the difference + or − according as it is an *increase* or a *decrease*. Tell what the differences mean.

The yearly yields, in bushels, of 4 of the principal farm crops of the United States for 11 years are given here:

Year	Corn	Wheat	Oats	Potatoes
1896	2,283,875,165	427,684,346	707,346,404	252,234,540
1897	1,902,967,933	530,149,168	698,767,809	164,015,964
1898	1,924,184,660	675,148,705	730,906,643	192,306,338
1899	2,078,143,933	547,303,846	796,177,713	228,783,232
1900	2,105,102,516	522,229,505	809,125,989	210,926,897
Totals..				
1901	1,522,519,891	748,460,218	736,808,724	187,598,087
1902	2,523,648,312	670,063,008	987,842,712	284,632,789
1903	2,244,176,925	637,821,835	784,094,199	247,127,880
1904	2,467,480,934	552,399,517	894,595,552	332,830,300
1905	2,707,993,540	692,979,489	953,216,197	260,741,294
1906	2,927,416,091	735,260,970	964,904,522	308,038,382
Totals..				

Check the work of the following by casting out the nines, or otherwise.

7. Find the total yield of oats from 1896 to 1900, and check the work; of potatoes; of wheat; of corn.

8. Find and check similar totals from 1901 to 1906 for each farm product.

9. Find the average yearly yield for each farm product from 1896 to 1900, of problem 7, omitting fractions of a bushel.

10. Find the average yearly yield for each farm product from 1901 to 1906, of problem 8, omitting fractions.

11. Find the difference of the averages for each farm product, and check the work.

12. Make, solve, and check other problems on the yields for single years, or for chosen periods.

§14. General Number.

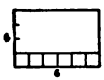


FIGURE 1

1. What is the area of a rectangle 4 units high and 6 units long? 5 units high and 7 units long? 7 units high and 9 units long? 3 units high and 12 units long? 3 units high and 20 units long?

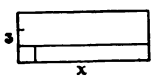


FIGURE 2

The area of a rectangle 3 units high and x units long is $3 \times x$, or $3x$, square units.

2. What is the area of a rectangle:

(1) 3 units high and y units long?
(2) 3 units high and a units long?
(3) 3 units high and z units long?
(4) 5 units high and x units long?
(5) 5 units high and y units long?
(6) 8 units high and c units long?
(7) 8 units high and m units long?
(8) 10 units high and d units long?

3. How many times 7 is 3 times 7 and 5 times 7?

12×7 and 3×7? $6 \times 7 + 5 \times 7$?
10×7 and 6×7? $12 \times 7 + 8 \times 7$?

4. How many times x is 3 times x + 6 times x?

$7 \times x + 2 \times x$? $4x + 3x$? $8x + 12x$?
$8 \times x + 5 \times x$? $6x + 3x$? $25x + 8x$?

5. How many times 10 is 5×10 minus 2×10?

$8 \times 10 - 4 \times 10$? $20 \times 10 - 15 \times 10$?

$12 \times 10 - 7 \times 10$? $32 \times 10 - 12 \times 10$?

$16 \times 10 - 10 \times 10$? $48 \times 10 - 18 \times 10$?

6. How many times a is $4 \times a$ minus $2 \times a$?

$7 \times a - 5 \times a$? $12a - 8a$? $20a - 12a$?

$11 \times a - 7 \times a$? $16a - 10a$? $18a - 9a$?

7. Add the following:

(1)	(2)	(3)	(4)	(5)
$3x$	$10y$	$6.3c$	$180.3d$	$3\frac{1}{2}z$
$5x$	$6y$	$12.6c$	$24.7d$	$4\frac{1}{3}z$
$7x$	$7y$	$.8c$	$6.2d$	$5\frac{2}{3}z$
$12x$	$9y$	$1.9c$	$11.8d$	$6\frac{3}{4}z$

8. Add the following:

(1)	(2)	(3)
$2x + 3y$	$12a + 7b$	$8m + 3c + 2d$
$6x + 2y$	$2a + 5b$	$5m + 6c + 7d$
$3x + 8y$	$7a + 9b$	$9m + 8c + 8d$

9. Subtract the following:

(1)	(2)	(3)
$8x + 5y$	$12c + 7d$	$15c + 12n + 7y$
$2x + 3y$	$8c + 5d$	$8c + 6n + 2y$

(4)	(5)
$18x + 6y + 5z$	$20a + 12b + 15c$
$9x + 3y + 4z$	$11a + 9b + 12c$

(6)	(7)
$17c + 28d + 11e$	$23m + 16n + 13s$
$9c + 19d + 8e$	$18m + 7n + 7s$

MULTIPLICATION AND DIVISION

§15. Multiplication of Integers and Decimals.

A whole number is called an integer.

1. Give two integers that, multiplied together, make 24, 36, 56, 100, 140, 450.

Integers, which multiplied together, give a certain number are called factors of the number.

A number that has no (integral) factors, but 1 and itself, is a prime number, as: 2, 3, 7, 23, 29, 31, etc.

All integers that are not prime are composite numbers, as: 4, 10, 15, 24, 50.

2. Tell which of the following numbers are (1) prime, and (2) composite, and give the reason:

5 11 14 17 21 31 37 42 47 51 100 125

3. Give two factors of each of the following numbers:

36 45 48 54 56 64 63 72 81 88 90 121 132 144

4. Give two equal factors of each of the following:

25 81 64 49 144 100 121 225

The square root of a number is one of its two equal factors.

5. Give the square roots of the following:

4 16 9 49 64 121 144 100

6. Give three equal factors of 8, 64, 27, 125, 216.

The cube root of a number is one of its 3 equal factors.

7. Of what number is 3 the cube root? 4? 2? 6? 10?

8. Give two integral divisors of each of the following:

20 32 35 42 48 45 27 63 72 75

Factors and integral divisors mean the same thing.

$75 \times 2 = ?$	$75 \times 5 \times 10 = ?$	$75 \times 20 = ?$
$80 \times 3 = ?$	$80 \times 3 \times 5 = ?$	$80 \times 15 = ?$
$85 \times 4 = ?$	$85 \times 4 \times 5 = ?$	$85 \times 20 = ?$
$45 \times 10 = ?$	$45 \times 10 \times 3 = ?$	$45 \times 30 = ?$

9. To multiply a number by 5 and that product by 7 is the same as multiplying at once by what single number?

10. Multiply the following numbers mentally by 10:

85, 75, 9, 90, 7.3, 8.4, 16.28, .832.

11. Multiply the same numbers mentally by 100; by 1000; by 10000.

12. Divide the numbers of problem 10 mentally by 10; by 100; by 1000.

13. How may any number be quickly multiplied by 1 with any number of zeros following it, by the use of the decimal point only?

14. How may any number be divided by 10, 100, 1000, 10000, etc., by using the decimal point only?

15. To multiply a number by 100 and then divide the product by 2 is the same as multiplying by what *single* number?

16. Make a rule for quickly multiplying a number by 50; by 25; by 500; by 250; by $33\frac{1}{3}$; by $333\frac{1}{3}$.

17. Find the products of the following pairs of numbers:

	(1)	(2)	(3)	(4)
Multiplicands,	681	681	681	681
Multipliers,	325	32.5	3.25	.325
Products,				

18. How do the multipliers of problem 17 compare? The multiplicands? How, then, must the products compare?

19. Find the products of the following:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
681	68.1	6.81	.681	.681	6.81	68.1
325	325	325	325	32.5	3.25	3.25

The figures on the right of the decimal point in a number are called *decimal places.*

20. How many decimal places are there in the product of (2) problem 19? Of (3)? Of (4)? Of (5)? Of (7)?

21. How many decimal places are there in both the multiplicand and the multiplier of the product of (2) problem 19? Of (3)? Of (4)? Of (5)? Of (7)?

22. What relation is there between the number of decimal places in the product and the number of decimal places in both the multiplicand and the multiplier?

Decimal numbers are multiplied precisely as are whole numbers; then the point must be so placed as to give as many decimal places in the product as there are in both the multiplicand and the multiplier.

§16. Division.

1. Write the quotients of the following from the one that is given *without dividing:*

	Dividend	Divisor	Quotient		Dividend	Divisor	Quotient
(1)	221,325	÷ 681	= 325	(5)	22,132.5	÷ 68.1	= ....
(2)	22,132.5	÷ 681	= ...	(6)	221.325	÷ 6.81	= ....
(3)	2213.25	÷ 681	= ...	(7)	22.1325	÷ 68.1	= ....
(4)	221.325	÷ 681	= ...	(8)	22.1325	÷ .681	= ....

2. In each part of problem 1 compare the number of decimal places in the quotient with the difference between the number in the dividend and the number in the divisor. What seems to be the law for pointing the quotient?

To divide decimal numbers, *divide first as in whole numbers, then so point the quotient that the number of places in the quotient is the number of places in the dividend minus the number of places in the divisor.*

Note: This usual way of pointing quotients in decimals is not convenient when there are more decimal figures in the divisor than in the dividend.

Another way that some prefer is to imagine the decimal point moved over the same number of places in the same direction in both dividend and divisor, until only *one* digit stands to the *left* of the decimal point *in the divisor*. Then according to what the digit is, ask: "About what is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of the modified dividend?" and place the point in the quotient to make it as near the correct answer to the question as possible.

3. Why may the decimal point be moved toward the right over 2, or 3, or 4, places in both dividend and divisor without changing the quotient? Toward the left over the same number of places?

§17. Multiplying When Some Digits of the Multiplier Are Factors of Others.

1. From twice a number, how may you obtain 4 times the number? From 4 times a number, how obtain 8 times the number?

2. Multiply (1) 19,279 by 842; (2) 15,423 × 328.

(1)		(2)	
19279		15423	
842		328	
38558	= 2 × 19279	123384	= 8 × multiplicand
771160	= 20 × 38558	4935360	= 40 × first product
15423200	= 20 × 771160	5058744	= 328 × 15423
16232918	= 842 × 19279		

3. Observe the relations of the digits of each multiplier below, and shorten the work of multiplication, as above.

(1) 4783 × 363 (3) 7853 × 639 (5) 86,729 × 393

(2) 12,875 × 442 (4) 29,847 × 248 (6) 78,431 × 217

§18. Checking Multiplication.

Any of the above ways may be used to test the correctness of products obtained in the usual way.

To check by casting out the nines, cast the nines out of the multiplicand, the multiplier, and the product. Multiply the excesses of the multiplicand and the multiplier. Cast the nines out of this product. If this last excess equals the excess of the original product, the work is probably correct.

ILLUSTRATION.—To check $6848 \times 619 = 4238912$, excess in $6848 = 8$; excess in $619 = 7$; excess in $4238912 = 2$, $7 \times 8 = 56$, excess in $56 = 2$. As excess in 4238912 and in 56 are equal, the work is checked.

A very useful check against blunders is to examine the facts of the problem and to decide about what the answer must be, before beginning to solve it.

§19. Original Problems.

Make and solve multiplication problems from the following facts, and check the work:

1. Silver is worth 56¢ an oz., 12 oz. to the pound.

2. Hay costs \$23 per T.; oats 42¢ per bu. A horse is fed 162 lb. of hay and 10 bu. of oats a month. Bedding costs \$2 a month.

3. Standard silver is $\frac{9}{10}$ pure silver and $\frac{1}{10}$ copper. A silver dollar weighs 412.5 grains. The total number of silver dollars coined on this basis was 378,166,769.

4. The number of grains in an ounce of silver is 480. The amount of silver bought under the Sherman Law by the United States government to coin into money was 168,674,682 ounces.

5. Light travels 186,600 mi. per second. It requires 498 seconds for light to reach the earth from the sun.

6. It reaches the earth from the moon in $1\frac{7}{25}$ seconds.

7. An oz. of pure gold is worth \$20.67. There are 12 oz. in 1 lb. of gold.

8. An Alaskan miner can take away 200 lb. of gold from the mining district.

9. A cu. ft. of granite weighs 170 lb. A granite step measures $\frac{1}{2}' \times 2' \times 8'$.

10. The distance around the driving wheel of a locomotive engine is 22 ft. In going a certain distance the driving wheel turned 5280 times.

11. In the year 1901, 82,305,924 lb. of tea and 511,041,459 lb. of coffee were imported into the United States. Tea was worth 48¢ and coffee 26¢ per pound.

12. A prize-winning steer weighed 15.03 cwt. (1 cwt. = 100 lb.) and sold for $9.00 per hundredweight.

13. The price of oats was $48\frac{1}{8}$¢ a bushel. A farmer raised 38 acres of oats, averaging 48 bushels per acre.

14. Pupils should prepare and solve problems based upon price lists obtained from the grocer and the butcher, or from the market reports of the daily papers.

§20. Division with Larger Numbers.

1. 5,128,672 ÷ 9272 = ?

```
        553
    ------
9272)5128672
     46360
     -----
     49267
     46360
     -----
     29072
     27816
     -----
      1256
```

SOLUTION.—The divisor, 9272, being too large to use readily, we first use a trial divisor. For the same reason, we select a trial dividend. 92, the trial divisor, is contained in 512, 5 times; but as the whole divisor contains four digits the partial dividend must be enlarged. 9272 is contained in 51,286, 5 times. The quotient figure 5 is of the same order as the last figure of the trial dividend, which is hundreds. We write 5 in hundreds place in the quotient. Multiplying the whole divisor by the quotient figure we have the product 46,360. Subtracting this product from the trial dividend, 4926 remains. 4926 hundreds = 49,260 tens; and 49,260 tens + 7 tens = 49,267 tens. Continue in the same manner with each step that follows.

The last subtraction gives a remainder of 1256. This remainder must also be divided by the divisor 9272.

$$5{,}128{,}672 \div 9272 = 553\tfrac{1256}{9272}$$

Check by multiplication for problem 1:

9272
553
27816
46360
46360
5127416
1256
5128672

The 553 is the whole, or integral part of the quotient and the $\frac{1}{9}\frac{2}{2}\frac{5}{7}\frac{6}{2}$ is the fractional part.

Check: Multiply the divisor by the quotient, and to the product add the remainder. The result should equal the dividend.

Solve the following problems and check your work:

2. 131,320 ÷ 536 = ? **4.** 630,861 ÷ 2731 = ?

3. 195,936 ÷ 624 = ? **5.** 1,057,536 ÷ 4352 = ?

§21. Applications to Areas and Bulk.

1. Find the areas of these rectangles:

$6'' \times 8''$; $12'' \times 28''$; $40' \times 64'$;

$375' \times 486'$; $9' \times x'$; 9 yd. $\times$ x yd.;

a mi. $\times$ b mi.; x units $\times$ y units.

How can you find the number of square units in any rectangle?

NOTE.—$x \times y$ is written xy and read "x, y."

2. How many cubic feet in 1 cu. yd.? In 18 cu. yd.?

3. What is the capacity of a square-cornered box

$3'' \times 3'' \times 9''$? $3' \times 3' \times 9'$?

Of a square-cornered room or space

3 yd. $\times$ 3 yd. $\times$ 9 yd.? $3 \times 3 \times 9$?

3 rd. $\times$ 3 rd. $\times$ 9 rd.? $3 \times 3 \times a$?

4. Find the capacity of a square-cornered box

$3'' \times 4'' \times 7''$; $3'' \times 4'' \times 8''$; $3' \times 4' \times 8'$;

$3' \times 4' \times 15'$; 3 yd. $\times$ 3 yd. $\times$ 3 yd.;

3 units $\times$ 4 units $\times$ 15 units;

$3 \times 4 \times x$; $3 \times x \times y$; a units $\times$ b units $\times$ c units.

NOTE.—The product $x \times y \times z$ is written xyz and read "x, y, z."

5. How can you find the number of cubic units in any square-cornered vessel?

6. Find the total weight of a snow load of 25 lb. per sq. ft. on a flat rectangular roof $25' \times 48'$. Find the weight of a load of 15 lb. per square foot.

7. Find the total weight of the shingles and sheathing for both sides of the roof shown in Fig. 1, if shingles weigh 3 lb. per sq. ft., and sheathing 5 lb. per square foot.

NOTE.—The eaves project 2 ft. over the plate.

8. Find the weight of the snow load on both sides of the roof, the weight on each sq. ft. of surface being 12 pounds.

9. Find the cost at 32¢ per sq. yd. of lathing and plastering the four walls and the ceiling, $24' \times 36'$, no allowances being made.

10. What is the area of the square *ABCD* of Fig. 1? What is the area of the triangle *DEC*?

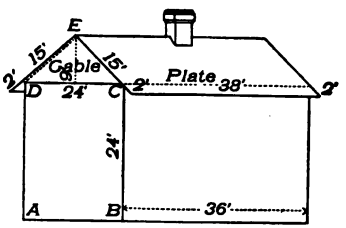


FIGURE 1

11. A strong gale, giving a pressure of 16 lb. per sq. ft., blows squarely against the end of the building. What is the total wind pressure against the end including the gable?

12. How many cu. ft. of space are inclosed by the outside walls to the base, *DC*, of the gables?

§22. Miscellaneous Problems.

1. Porto Rico has an area of 3531 sq. mi., and a population of 953,243. How many inhabitants does it support to the square mile?

2. The greatest ocean depth found is 31,614 ft. near the island of Guam, in the Pacific ocean. The highest

mountain in the world is Mt. Everest in Asia, which rises 29,002 ft. above sea level. Find the difference of level in miles between the greatest ocean depth and the greatest land altitude.

Check the work of the following problems by multiplying the quotient by the divisor, adding the remainder, and comparing the sum with the dividend.

3. The state of New York, with an area of 49,170 sq. mi., supports a population of 7,268,894. How many inhabitants does it average to the square mile?

4. Hawaii has an area of 6449 sq. mi., and a population of 154,001. Find the average population per square mile.

5. In 1891, the total wheat area of North and South Dakota was 4,882,157 A.; the yield was 81,819,000 bu. What was the average yield per acre?

6. In the year 1900, Kentucky had 22,488 A. of rye under cultivation. The total yield was 292,344 bushels. What was the average yield per acre?

7. In the year 1890, there were 210,366 persons employed in manufacturing in Chicago. The total wages paid amounted to $123,955,001. What was the average wage paid to each person?

8. In 1880, 3519 factories in Chicago together yielded $249,022,948 worth of products. In 1890, 9977 factories yielded $577,234,446 worth. During which year was the average product per factory the greater, and by how much was it the greater?

9. A comparison of the density of population (population per square mile) may be made by finding the density of population of these countries:

COUNTRY	AREA (Sq. Mi.)	POPULATION	DENSITY
German Empire	208,830	56,345,014	
Great Britain	120,979	41,454,578	
Austria	115,903	26,107,304	
France	204,092	38,641,333	
Italy	110,646	32,449,754	
Russia	8,660,395	135,000,000	
United States	3,688,110	76,212,168	

NOTE.—Only the whole numbers need be found in the problems.

§23. Checking Division.

To check division by casting out the nines, add the excess in the product of the excesses of divisor and quotient to the excess of the remainder. If the excess of this sum equals the excess of the dividend, the division is probably correct.

ILLUSTRATION.—Check the work of problem 1, p. 23.

Cast the 9's out of the dividend, 5,128,672. The excess is *4*.

Cast the 9's out of the divisor, 9272. The excess is *2*.

Cast the 9's out of the quotient, 553. The excess is *4*.

Cast the 9's out of the remainder, 1256. The excess is *5*.

The product of the excesses of divisor and quotient is *8*.

The excess of 8 is *8* itself. Add this 8 to the excess of the remainder, giving 13.

The excess of this 13 is *4*, and as this equals the excess *4* of the dividend, the division is probably correct.

It is even more important in division than in multiplication to examine a problem carefully before beginning to solve it. Try to foresee about what the answer must be. This often avoids blunders.

ILLUSTRATION.—If it requires 480 slates to cover a square (100 sq. ft.) of roof surface, how many squares are there in a roof which requires 13,680 slates to cover it?

Pupils should at once notice that if it required 500 slates to cover a square, there would be a little more than 13,680 ÷ 500, or 136 ÷ 5 = 27 squares and the guess might be 28 squares. The actual division of 13,680 by 480 gives 28.5 squares.

First form a rough estimate of the answer and then solve these exercises:

1. $238 was paid for flour @ $3.50 per bbl. How many bbl. were bought?

SUGGESTION.—How many bbl. would there have been if the price had been $7.00 a barrel?

2. In still air a hawk flew 375 miles in $2\frac{1}{2}$ hr. What was its speed per hour?

3. In 3.9 hr. a crow flew 97.5 miles. Find the rate of flight per hour.

4. From a certain cow $\frac{1}{32}$ of the milk was butter-fat. The cow gave 12 lb. of milk per day. How many pounds of butter-fat will the milk from this cow yield in 70 days?

§24. Family Expenses.

1. The monthly expenditures of a family for clothing are here tabulated, beginning with Oct. 1900. Find, without rewriting the numbers, the total expenditures for clothing (*a*) for the first six months; (*b*) for the last 6 months; (*c*) for the year:

OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	TOTAL
$25.50	$48.37	$15.78	$5.83	$20.26	$37.90	.....
APR.	MAY	JUNE	JULY	AUG.	SEPT.	
$3.75	$18.64	$25.95	$8.47	$12.35	$20.16	.....

2. Find the daily average per month for each of the 6 months from October to March.

3. Find the monthly average for the 6 months from April to September.

Problems 4 and 5 refer to the table on page 29.

4. Add horizontally and find the total expenditure for the first day of the 6 months; for the second day.

5. Find the average expenditure for the first day of the 6-month period. For the second day.

6. The daily totals for the expenditures of this family are here given for the half year beginning Oct. 1. Find the monthly totals. Note the time you take to add the column for each month.

7. Find the total for the 6 months (1) by adding the monthly totals, and (2) by finding the daily totals and adding them. Compare the two results.

DATE	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.
1....	$51.57	$55.28	$53.15	$ 3.70	$ .35	$ 2.86
2....	8.94	3.65	.13	51.26	58.10	56.55
3....	2.09	7.25	1.68	3.67	1.02	.60
4....	7.35	.25	3.75	.55	5.16	3.16
5....	1.50	1.65	1.85	5.90	1.37	4.60
6....	8.22	4.78	5.75	.12	2.15	6.51
7....	.05	2.83	1.25	1.60	3.17	2.75
8....	4.75	3.86	8.21	6.12	1.16	1.28
9....	1.77	2.28	.68	3.26	4.18	3.82
10....	4.13	6.98	1.86	1.81	.22	.75
11....	9.10	.15	1.17	3.60	6.03	2.29
12....	1.19	3.65	4.76	2.31	2.58	1.69
13....	3.87	1.63	5.16	.05	1.15	9.60
14....	.10	.85	1.11	5.86	5.60	.17
15....	2.17	6.87	2.68	1.03	1.81	1.25
16....	3.98	2.37	.68	4.32	2.81	2.18
17....	18.47	8.86	3.27	9.82	.75	.85
18....	1.80	.12	7.32	2.18	.81	5.63
19....	2.55	4.10	1.28	3.28	15.61	3.05
20....	8.81	1.17	2.18	.75	5.78	1.06
21....	.25	1.68	11.12	6.29	4.78	11.61
22....	.25	3.76	16.25	1.18	1.05	4.28
23....	3.55	9.26	.22	2.28	3.27	2.57
24....	.55	7.81	3.20	7.60	1.60	.60
25....	1.65	.55	1.68	3.91	2.86	3.05
26....	3.70	3.86	.54	2.83	8.07	3.18
27....	5.74	7.52	3.48	.25	3.35	1.75
28....	.76	.28	6.15	1.65	4.15	3.10
29....	2.80	18.26	2.97	5.16	.00	6.21
30....	3.86	1.60	.21	18.21	.00	.76
31....	2.85	.00	2.08	6.38	.00	.15
Monthly Totals						

§25. Fencing a Farm.

Answer all you can orally.

A farm was divided into fields and seeded as shown in the drawing (Fig. 1). The scale of the drawing is 1 in. to 80 rd.

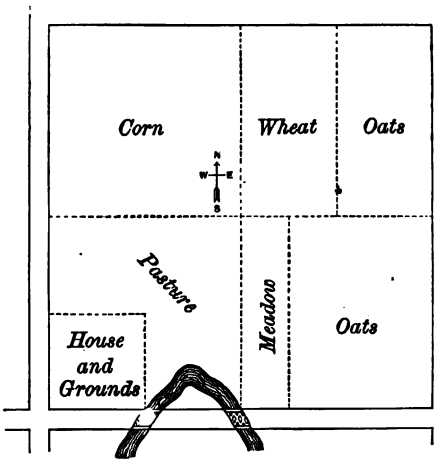


FIGURE 1

1. How long is the cornfield north and south? How wide?

2. How long and how wide is the wheatfield? The north oatfield?

3. How long and how wide is the meadow? The south oatfield?

4. The drawing of the road is $\frac{1}{10}$ in. wide. How wide is the road?

5. How long and how wide is the farm?

6. How long must a wire be to reach entirely round the farm?

7. How many rods of barbed wire will be needed to inclose the farm with a 3-wire fence? How many miles (320 rd. = 1 mile)?

Use pencil only when needed.

8. How many rods long is the partition (division) fence across the farm from east to west? What part of a mile is its length? How many rods of wire will make it a 4-wire fence?

9. How many rods of wire will run a 4-wire fence

across the farm from the middle of the north side to the middle of the south side?

10. How many rods of wire will run a 4-wire fence between the wheatfield and the north oatfield? Between the meadow and the south oatfield?

11. How many bales of wire of 100 rd. each will be needed for these cross fences and division fences?

12. How much will the wire cost for the 3-wire inclosing fence @ $3.50 a bale of 100 rods?

13. The posts for all the barbed wire fences are set 1 rd. apart. How many posts will be needed to fence around the farm?

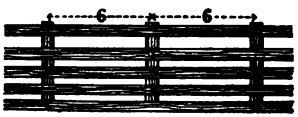
14. How much will these posts cost @ 25¢ apiece?

15. How much will the wire cost @ $2.60 a bale for the two 4-wire cross fences, and the two division fences between the wheat and the north oatfield and between the meadow and south oatfield?

16. How much will the posts for these fences cost @ 15¢ apiece?

17. Find the total cost of wire and posts for these fences.

18. The house and grounds lot is separated from the pasture by a 5-board fence. The boards used are 1 in. thick, 6 in. wide, and 12 ft. long. How many boards will reach once along the north side of this lot? Once along the east side?



TWO FENCE PANELS
FIGURE 1

19. How many boards will be needed for the 5-board fence along both sides?

20. The posts are set 6 ft. between centers, that is, a

post is set every 6 ft. How many posts will be needed for the north side? For the east side? For both?

21. How much will these posts cost @ 23¢ apiece?

Lumber is sold by the *board foot.* A board foot is a board 1 ft. long, 1 ft. wide, and 1 in., or less, thick.

22. How many board feet of lumber in a board 1 in. thick, 1 ft. wide, and 5 ft. long? The same thickness and width and 10 ft. long? 12 ft. long?

23. How many board feet in a board 1 in. thick, 10 ft. long, and 2 ft. wide? The same thickness and length and 6 in. wide? 9 in. wide? 18 in. wide? 1 in. wide? 8 in. wide?

24. How many board feet in a board 12 ft. long, 4 in. wide, and 1 in. thick? 2 in. thick?

25. How many board feet in a "two by four" scantling 10 ft. long ($2'' \times 4'' \times 10'$)? 16 feet long?

26. How many board feet in a "four by four" scantling 10 ft. long? 12 ft. long? 18 ft. long?

27. How many board feet in a fencing board $1'' \times 6'' \times 12'$? In 5 such boards?

28. How much will the fencing lumber of problem 19 cost @ $18 per M board feet?

29. What will be the total cost of the lumber and posts for the fence on the north and east sides of the lot?

30. It cost 8¢ apiece to have the post holes bored, and it took 4 days' work by 3 men @ $1.50 per day to build the fence. Three patent gates costing $12 apiece were put in the fence. Find the total cost of the lot fence, including gates.

31. What was the total cost of all the items mentioned in the fencing done on the farm?

§26. Areas of Fields.

Answer all you can orally.

Problems 1 to 14 refer to Fig. 1, p. 30.

1. The whole farm contains 160 acres. How many acres are there in the cornfield? In the wheatfield? In the meadow? In the south oatfield? In the pasture? In the house and grounds lots?

2. What is the width of each of these fields in rods?

3. If the meadow were divided by a north and south central line, how many rods wide would each half be? How many acres would there be in each half?

4. How many square rods in a strip 1 rd. wide and 80 rd. long?

5. How many square rods in an acre?

6. How wide must a strip of land be to contain an acre, if it is 80 rd. long? 40 rd. long? $\frac{1}{2}$ mile long?

7. Compare the sizes of the wheatfield and the north oatfield; of the cornfield and the wheatfield; of the meadow and the wheatfield; of the meadow and the south oatfield; of the meadow and the house and grounds lots; of the lots and the pasture; of the pasture and the south oatfield.

Use pencil and paper only when needed.

8. What would be the income per acre from the farm planted in corn, if the average yield were 47 bu. per acre, sold at 28¢ per bu.? The gross (total) income from the farm (except the house and grounds lots) for this year?

9. If the farm were rented @ $5 per acre, how much would remain for the tenant* from each acre (as in prob. 8)? From the entire farm?

10. What would be the owner's income from the whole farm, not allowing for expenses?

*The tenant is the farmer who raises the crop on another man's farm.

11. Which is the more profitable way for the owner to rent his farm, @ \$6 per acre cash, or for ½ of all the crop delivered to market, supposing that the farm, except lot around house, is planted to corn and that a yield of 48 bu. per acre and a price of 30¢ per bushel is obtained?

NOTE.—Rent of the first sort is called "cash rent," of the second sort, "grain rent."

12. If this same farm will produce 20 bu. of wheat per acre and a price of 60¢ per bushel can be obtained for it, which is the more profitable crop to the owner, wheat or corn? How much more profitable? (Conditions as in prob. 8.)

13 Compare the owner's profits of a grain-rented farm from an oats crop of 40 bu. per A. sold at 22¢ per bushel with the profits of the wheat crop of problem 12.

14. In problem 12 which would bring the larger income to the owner, and how much, cash rent @ \$5 per A., or grain rent @ ½ delivered?

15. A cornfield of 68 acres produced an average yield of 48 bu. per acre. How many bushels did the field yield?

16. It cost \$5.85 per acre to raise the crop, problem 15. At 46¢ per bu. what was the net value of the crop?

17. If the tenant paid cash rent @ \$5.50 per acre, what was his net profit from the crop? How much did the owner receive?

§27. Farm Account Keeping.

The following forms show how a certain farmer keeps a systematic account of receipts and expenditures with each of the fields shown in Fig. 1, p. 30.

The items for which money was paid out or received were put down with the dates of payment or of receipt in a day-book thus:

Day-Book for Forty-Acre Cornfield

1. Paid for 4 da. work removing stalks @ $1.15, Mar. 18.
2. Paid for 9 da. plowing @ $1.15, Apr. 30.
3. Paid for 2 da. harrowing @ $1.00, May 1.
4. Paid for 3½ da. planting corn @ $1.20, May 18.
5. Paid for 6 bu. seed corn @ 90¢, May 18.
6. Paid for 2 da. replanting corn @ $1.15, May 30.
7. Paid for 5 da. harrowing corn @ $1.15, May 30.
8. Paid for 7 da. cultivating @ $1.15, June 15.
9. Paid for 6 da. cultivating @ $1.15, June 30.
10. Paid for 6 da. cultivating @ $1.15, July 15.
11. Paid for cutting 240 shocks corn @ 8⅓¢, Sept. 15.
12. Paid for husking 1392 bu. corn @ 3¢, Nov. 20.
13. Paid for shelling 1336 bu. corn @ ¾¢, Jan. 10.
14. Sold 1336 bu. corn @ 38¢, Jan. 10.
15. Received pay for 3 mo. pasture @ $1.50, Feb. 28.

1. These items are here arranged in the form of a receipt and expenditure account. Fill out the vacant columns, and find the totals and the net profit for this forty-acre cornfield:

In Account with Forty-Acre Cornfield

	EXPENDITURES				RECEIPTS	
Mar. 18	Removing stalks, 4 da., @ $1.15			Jan. 10	1336 bu. corn @ 38¢	
Apr. 30	Plowing, 9 da., @ $1.15			Feb. 28	3 mo. pasture @ $1.50	
May 1	Harrowing, 2 da., @ $1.00				TOTAL	
" 18	Planting corn, 3½ da., @ $1.20				Expenditures to be deducted	
" 18	Seed corn, 6 bu., @ 90¢				Net profit from 40 acres	
" 30	Replanting, 2 da., @ $1.15				Net profit per acre	
" 30	Harrowing corn, 5 da., @ $1.15				Account closed March 1, 1901	
June 15	Cultivating, 7 da., @ $1.15					
" 30	Cultivating, 6 da., @ $1.15					
July 15	Cultivating, 6 da., @ $1.15					
Sept. 15	Cutting 240 shocks corn @ 8⅓¢					
Nov. 20	Husking 1392 bu. corn @ 3¢					
Jan. 10	Shelling 1336 bu. corn @ ¾¢					
	TOTAL					

2. From the following list of day-book items for the twenty-acre wheatfield the account below is arranged. Fill out the vacant columns and find the totals, the net profit on the whole field, and the net profit per acre.

Day-Book for Twenty-Acre Wheatfield

1. Paid for 4 da. plowing @ $2.50, Oct. 20.
2. Paid for 3 da. harrowing and rolling @ $2.25, Oct. 25.
3. Paid for 25 bu. seed wheat @ $1.25, Oct. 25.
4. Paid for 2½ da. drilling wheat @ $2.50, Oct. 30.
5. Paid for cutting 20 acres wheat @ 75¢, July 15.
6. Paid for 2 da. shocking wheat @ $1.75, July 15.
7. Paid for threshing 480 bu. wheat @ 6¢, Aug. 30.
8. Paid for 3½ da. help in threshing @ $1.50, Aug. 30.
9. Sold 425 bu. wheat @ 68¢, Dec. 22.
10. Received pay for 4 mo. pasture @ $1.50, Mar. 2.

In Account with Twenty-Acre Wheatfield

Expenditures				Receipts			
Oct.	20	Plowing, 4 da., @ $2.50		Dec.	22	Wheat, 425 bu., @ 68¢	
"	25	Harrowing and rolling, 3 da., @ $2.25		Mar.	2	Pasture, 4 mo., @ $1.50	
"	25	Seed wheat, 25 bu., @ $1.25					
"	30	Drilling, 2½ da., @ $2.50				Expenditures to be deducted	
July	15	Cutting 20 acres wheat @ 75¢				Net profit from the field	
"	15	Shocking wheat, 2 da., @ $1.75				Net profit per acre	
Aug.	30	Threshing 480 bu. wheat @ 6¢					
"	30	Help threshing, 3½ da., @ $1.50					
		Total					

3. Draw up these facts into an account, and find the totals and the net profit, and treat in the same way as above:

Day-Book for Thirty-Acre Pasture

1. Bought 120 bu. corn @ 37¢, Apr. 23.
2. Sold 5 yearling steers @ $18.00, Apr. 25.
3. Sold 3 yearling heifers @ $15.00, July 6.
4. Sold 2 milk-cows @ $46, July 8.
5. Bought 4 two-yr. old heifers @ $25, July 10.
6. Sold 8 hogs weighing 2064 lb., @ $5 per 100 lb., July 20.
7. 8 head of hogs, worth $12 each, died July 25.

8. Bought 20 pigs @ $2.50, Aug. 10.
9. Sold 3 two-yr. old colts @ $72, Sept. 30.
10. Sold 2 draught horses @ $195, Sept. 30.
11. Sold 9 four-yr. old steers @ $68, Oct. 25.
12. Bought 10 yearling calves @ $12, Nov. 1.
13. Sold 3 Jersey milk-cows @ $45, Nov. 15.
14. Paid 6 mo. wages for one man @ $30, Dec. 23.
15. Sold 8 head of hogs @ $15, Jan. 10.
16. Bought 12 head of hogs @ $9.50, Feb. 14.

4. Fill out this account of the fifty-acre oatfield.

	EXPENDITURES				RECEIPTS		
Mar. 20	Removing old stalks 5 da. @ $1.25			Feb. 15	1800 bu. oats @ 22½c		
" 20	100 bu. seed oats @ 35¢			" 20	85 loads straw @ $2.75		
" 20	6½ da. work sowing oats @ $1.25				Expenditures to be deducted		
Aug. 18	Harvesting 50 acres oats @ 75c				Net profit from 50 acres		
" 18	4½ da. labor in harvesting @ $1.75				Net profit per acre		
Sept. 15	Threshing 2148 bu. @ 3¢						
" 15	5 da. help threshing @ $1.50						
	TOTAL						

5. Treat the meadow account similarly.

In Account with Ten-Acre Meadow

	EXPENDITURES				RECEIPTS		
Nov. 20	6 bu. timothy seed @ $2.00			Dec. 15	12 tons hay @ $9.50		
Aug. 30	Cutting 10 acres hay @ 35c				Expenses to be deducted		
" 30	Raking and shocking hay 5 da. @ $1.50				Net profit from 10 acres		
" 30	Stacking 15 tons hay @ 55c				Net profit per acre		

6. Draw up the following items into an account like the one above, and find totals and profit or loss to the farmer.

Day-Book for House and Grounds Lot (10 acres).

1. Apr. 15. Bought 6 bu. seed potatoes @ $2
2. " 20. Bought 30 pkg. garden seeds @ 10¢
3. " 20. Bought 4 pkg. garden seeds @ 25¢
4. May 15. Paid 5 da. wages @ $1.25
5. " 25. Sold 100 bunches onions @ 2½¢
6. " 30. Sold 148 bunches celery @ 3¢

7. May 30. Sold 200 bunches lettuce @ 2½¢
8. " 30. Sold 240 bunches asparagus @ 3¢
9. June 30. Sold 14 bu. peas @ 4¢ per qt.
10. " 30. Sold 28 bu. green beans @ 4¢ per qt.
11. " 30. Sold 300 bunches asparagus @ 5¢
12. " 30. Paid 18 da. wages @ $1.25
13. " 31. Sold 10 bu. new potatoes @ $1
14. " 31. Paid 15 da. wages @ $1.50
15. July 31. Sold 50 heads cabbage @ 8¢
16. " 31. Sold 20 doz. ears sweet corn @ 8¢ per dozen
17. Aug. 31. Sold 140 lb. tomatoes @ 8¢
18. " 31. Sold 10 bu. early apples @ 75¢
19. " 31. Sold 25 heads cabbage @ 7¢
20. " 31. Sold 15 bu. peaches @ $1
21. " 31. 10 doz. ears corn @ 7¢
22. " 31. Paid 20 da. wages @ $1.50
23. Sept. 30. Sold 55 bu. potatoes @ 65
24. " 30. Sold 20 bu. lima beans @ $1.25
25. " 30. Paid 20 da. wages @ $1.25
26. Dec. 20. Sold 30 bu. apples @ $1
27. " 30. Sold 10 bbl. (40 gal. each) cider vinegar @ 30¢ per gal.

7. Make a statement from these items of general expense, not included in any of above accounts, and find the total profit or loss:

1. Apr. 20. Bought 3 sets of harness @ $28
2. " 25. Paid tax on 160 acres @ 25¢
3. " 30. Bought 1 wagon @ $60
4. " 30. Bought 2 plows @ $35
5. May 10. Bought 3 cultivators @ $25
6. Aug. 20. Paid 84 rods tile ditching @ 50¢
7. Aug. 30. Built corn crib, cost $112.00
8. Sept. 10. Paid for 120 mo. pasturing stock @ $1.50
9. Feb. 28. Paid for 45 mo. pasturing stock @ $1.25
10. " 28. Paid 30 da. fertilizing @ $1.25
11. " 28. Paid 8 da. mending fence @ $1.25

8. Add all the net receipts and all the net expenditures for the separate fields as given in the records of these fields in problems 1 to 7 above. Find the net earnings for the year of the whole farm.

§28. Exercises for Practice in Division.

1. Divide:

(1) 8450 by 26
(2) 10914 by 34
(3) 15281 by 37
(4) 12958 by 38
(5) 17864 by 44
(6) 24384 by 48
(7) 16686 by 54
(8) 23199 by 57
(9) 26752 by 64
(10) 32708 by 68
(11) 59422 by 73
(12) 79083 by 87

2. Divide:

(1) 9690 by 102
(2) 10769 by 121
(3) 13668 by 134
(4) 17150 by 175
(5) 19474 by 182
(6) 20304 by 216
(7) 66795 by 365
(8) 102910 by 502
(9) 190035 by 615
(10) 361398 by 899
(11) 1083903 by 901
(12) 1078032 by 1214

3. Divide:

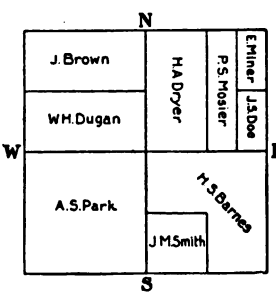
(1) 6015[illegible]6 by 1763
(2) 13036697 by 2683
(3) 13799478 by 3234
(4) 17713971 by 3807
(5) 24534861 by 4007
(6) 21443457 by 4237
(7) 32398582 by 5141
(8) 50537881 by 7109
(9) 133764048 by 13648
(10) 401292144 by 29403

§29. Measuring Land.

A *section* of land is a tract 320 rd., or 1 mi., square. It is usually divided into halves, quarters, eighths and sixteenths, as shown.

1. How many square rods make a section of land?

2. An acre = 160 sq. rd.; how many acres in a section of land? In a quarter section?



SECTION OF LAND—FIGURE 1

3. A section of land is divided up into farms, as shown in Fig. 1, p. 39. How many acres are there in the farm belonging to A. S. Park? To H. S. Barnes? To J. Brown? H. A. Dryer? J. M. Smith? P. S. Mosier? J. S. Doe?

4. Point out the south half of the section; the east half; the N $\frac{1}{2}$; the W $\frac{1}{2}$.

5. Point out the SE $\frac{1}{4}$; the NW $\frac{1}{4}$; the SW $\frac{1}{4}$; the NE $\frac{1}{4}$.

6. Point out the N $\frac{1}{2}$ of the SE $\frac{1}{4}$; the E $\frac{1}{2}$ of the SE $\frac{1}{4}$; the W $\frac{1}{2}$ of the NE $\frac{1}{4}$; the S $\frac{1}{2}$ NW $\frac{1}{4}$; the W $\frac{1}{2}$ E $\frac{1}{2}$.

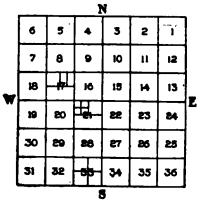
7. Point out the SW $\frac{1}{4}$ of the SE $\frac{1}{4}$; the NE $\frac{1}{4}$ SW $\frac{1}{4}$; the NW $\frac{1}{4}$ NW $\frac{1}{4}$; the NE $\frac{1}{4}$ SW $\frac{1}{4}$.

8. The farm of E. Miner would be described as the N $\frac{1}{2}$ E $\frac{1}{2}$ E$\frac{1}{2}$ NE $\frac{1}{4}$. Describe the farms of the other 8 owners of this section.

9. How many rods of fence would be required to enclose the section and separate it into farms as shown?

10. How many rods long is a ditch starting from the NE corner of H. A. Dryer's farm, thence running due south to H. S. Barnes's N line, thence due W to Barnes's W line, thence S to J. M. Smith's N line, thence due W across Park's farm to his W line?

11. Each small square in Fig. 1 represents a section. The large square represents a *township*. How long is a township? How wide?



TOWNSHIP—FIGURE 1

12. How many square miles in 1 township? How many acres?

13. Point out in Fig. 1 the following:

(1) NW $\frac{1}{4}$ Sec. 33.

(2) S $\frac{1}{2}$ Sec. 17; NE $\frac{1}{4}$ Sec. 17; E $\frac{1}{2}$ NE $\frac{1}{4}$ Sec. 17.

(3) NE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21; SE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21.

14. How many farms of 160 A. each could be made of a township?

15. How long and how wide is a square 160-acre farm?

§30. Plotting Observations and Measurements.

1. The hourly temperatures from 6 A.M. to 6 P.M. December 26, 1902, were:

6 A.M.	7 A.M.	8 A.M	9 A.M.	10 A.M.	11 A.M.
7°	7°	10°	10°	12°	13°

12 M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.
14°	14°	15°	16°	15°	14°	14°

Draw 13 equally spaced vertical parallels, one for each hour. Draw a horizontal line as YX across the parallels. Using $\frac{1}{16}$" to represent 1°, measure off on the first vertical the distance Y1 to represent 7°; on the second vertical the distance 7-2 to represent the second 7°; on the third parallel measure off the distance 10°, and so on. Mark the top of each measured vertical distance with a dot. Draw lines connecting these dots as in Fig. 1. *This is called plotting the readings.*

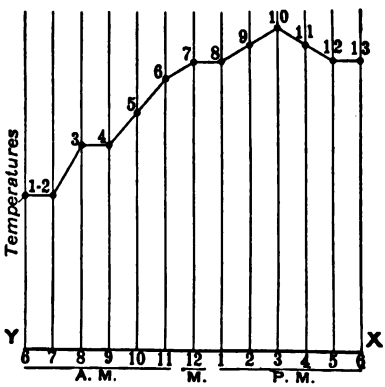


FIGURE 1

This line shows the temperature change during the 12 hours.

This plotting is very much aided by the use of cross-lined paper, ruled into small squares. A horizontal side of one of the small squares might be used to represent 1 hr. and a vertical side to represent 1° of temperature.

NOTE: Pupils in arithmetic should have notebooks containing several pages of squared paper for such work as this.

2. Read the out-door temperatures from hour to hour at your schoolhouse and plot the readings as in prob. 1.

3. The average hourly temperatures from 6 A.M. to 6 P.M., December 26 to January 2, were:

6 A.M.	7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.
21.5°	22.1°	22.2°	22.2°	22.5°	23.2°

12 M.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.
23.9°	24°	24.5°	25.2°	25°	24.7°	24.3°

Plot these readings on squared paper, or by measurement, as in Fig. 1, p. 41. Plot the tenths by estimate.

4. Does the line for the averages for a week agree in a general way with the line for December 26? What does a comparison of the two lines show?

This table gives the heights in feet and weights in pounds of boys and girls of ages given in the first column:

Age	Height in Feet		Weight in Pounds	
	BOYS	GIRLS	BOYS	GIRLS
2	2.6	2.6	25.0	23.5
4	3.0	3.0	31.4	28.7
6	3.4	3.4	38.8	35.3
9	4.0	3.9	50.0	47.1
11	4.4	4.3	59.8	56.6
13	4.7	4.6	75.8	72.7
15	5.1	4.9	96.4	89.0
17	5.4	5.1	116.6	104.4
18	5.4	5.1	127.6	112.6
20	5.5	5.2	132.5	115.3

5. Using squared paper, or making a drawing such as is indicated in Fig. 1, p. 43, plot the numbers in the first column horizontally and those in the second column vertically. Draw free-hand a smooth line through all the plotted points and obtain the curve for growth in stature of boys. At what age do boys cease growing rapidly in height?

FIGURE 1

6. Using the numbers of columns 1 and 3 obtain a similar curve for girls. At what age do girls cease growing rapidly in stature?

7. Compare the two curves and note when boys and when girls grow fastest?

8. Using the numbers of columns 1 and 4 draw the curve for growth of boys in weight.

9. Use numbers of columns 1 and 5 similarly for girls.

10. Compare the curves of problems 8 and 9 and note any likenesses or differences in the two curves.

11. Plot these 12 lengths of sides and areas in square inches of squares, using any convenient horizontal and vertical scales.

Sides...	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
Areas...	1	4	9	16	25	36	49	64	81	100	121	144

Do the plotted points lie on a straight line?

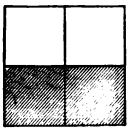
Date of Census	Population of U. S. in Millions	Date of Census	Population of U. S. in Millions
1790	3.9	1850	23.2
1800	5.3	1860	31.4
1810	7.2	1870	38.6
1820	9.6	1880	50.2
1830	12.9	1890	62.6
1840	17.1	1900	76.3

12. Plot the above dates on horizontal, and populations on vertical, lines.

Draw a smooth free-hand curve through the plotted points. By continuing the curve, can you predict (tell) about what the population will be in 1910? Is the line through the plotted points a straight line?

COMMON FRACTIONS

§31. Reducing Fractions.



$\frac{1}{2} = \frac{2}{4}$

FIGURE 1

1. Multiply both terms of $\frac{1}{2}$ by 2; by 3; by 5; by 10.

2. How do $\frac{1}{2}$ and $\frac{2}{4}$ compare in size, or in value? $\frac{1}{2}$ and $\frac{3}{6}$? $\frac{1}{2}$ and $\frac{5}{10}$; $\frac{1}{2}$ and $\frac{10}{20}$.

3. Multiply both terms of $\frac{2}{3}$ by 10; by 12; by 30; by 100.

4. How do $\frac{2}{3}$ and $\frac{20}{30}$ compare in value? $\frac{2}{3}$ and $\frac{24}{36}$? $\frac{2}{3}$ and $\frac{60}{90}$? $\frac{2}{3}$ and $\frac{200}{300}$?

5. Show with a fraction of your own selection that the value of a fraction is *not changed* by multiplying *both* terms by the same number.

6. Choose a fraction and show that if the *numerator alone* is multiplied by any number the fraction itself (its value) is *multiplied by that number.*

7. Choose a fraction and show that if the *denominator alone* is multiplied by any number, the fraction itself (its value) is *divided by that number.*

8. Choose a fraction and show that if the numerator alone is *divided by* any number, the fraction itself is *divided by that number.*

9. Show that if the denominator alone is *divided* by any number, the fraction itself is *multiplied by that number.*

10. Divide both terms of $\frac{20}{30}$ by 10. How does the result compare in value with $\frac{20}{30}$?

11. Divide both terms of $\frac{45}{60}$ by 15. How does the result compare in value with $\frac{45}{60}$?

12. Show what effect on the value of a fraction *dividing both terms* by the same number has.

The *terms* of a fraction may be either *both multiplied*, or *both divided*, by the same number without changing the *value* of the fraction.

When fractions are expressed so that their terms are the *smallest possible whole* numbers, they are said to be in their *lowest terms*.

13. Reduce to their lowest terms:

$\frac{6}{8}$ $\frac{10}{12}$ $\frac{10}{15}$ $\frac{20}{35}$ $\frac{25}{50}$ $\frac{35}{40}$ $\frac{25}{75}$ $\frac{26}{39}$ $\frac{49}{63}$

In the work of reducing fractions to their lowest terms, tests for divisibility should be used to find common factors of numerator and denominator.

14. What is the test for divisibility by 2? By 10? By 5? By 4? By 9? By 3? By 6? By 8? By any composite factor?

15. Reduce to lowest terms, discovering correct divisors by the tests for divisibility:

$\frac{26}{34}$ $\frac{33}{36}$ $\frac{100}{210}$ $\frac{75}{80}$ $\frac{60}{72}$ $\frac{52}{60}$ $\frac{24}{104}$ $\frac{600}{800}$

16. Give the values of these fractions in their lowest terms:

$\frac{5}{10}$ $\frac{6}{9}$ $\frac{21}{28}$ $\frac{8}{16}$ $\frac{35}{49}$ $\frac{32}{56}$

To obtain the value of a fraction in the smallest possible terms, it is often necessary to divide both terms by the largest exact common divisor of both terms.

This divisor is called the greatest common divisor of the terms. It is indicated by the initial letters G. C. D.

Dividing both terms of a fraction by their G. C. D. reduces the fraction to its lowest terms.

17. Why are these fractions in their lowest terms?

$\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{3}{7}$ $\frac{13}{15}$ $\frac{15}{23}$ $\frac{10}{27}$

Among the latter fractions is there a factor common to any numerator and its denominator?

Two numbers that have no common factor, except 1, are said to be prime to each other.

18. Reduce these fractions to their lowest terms:

$\frac{16}{24}$ $\frac{27}{81}$ $\frac{60}{80}$ $\frac{25}{75}$ $\frac{39}{52}$ $\frac{74}{111}$ $\frac{169}{351}$

19. Reduce to their lowest terms:

$\frac{63}{77}$ $\frac{88}{121}$ $\frac{96}{132}$ $\frac{65}{156}$ $\frac{91}{143}$ $\frac{119}{136}$ $\frac{171}{209}$

§32. Factors, Prime and Composite.

NOTE: Use tests for divisibility through this section and the next, when searching for factors.

1. Write down all the factors, or exact divisors, of 36 and of 48.

By factor is here meant exact divisor, or a divisor that is contained without a remainder.

CONVENIENT FORM

2)36, 48
2)18, 24
3)9, 12
2)3, 4
3, 2

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

2. What factors are common to both 36 and 48?

3. What is the *greatest* factor that is common to 36 and 48?

4. Arrange the factors of 42 and 105 as the factors of 36 and 48 are arranged in problem 1, and answer questions like 2 and 3 for the factors of 42 and 105.

5. Give the factors that are common to the 3 numbers, 18, 24, and 36. Give the *greatest* factor common to all three numbers.

6. Answer questions like those of problem 5 for these numbers: (1) 12, 72, and 84 (2) 42, 98, and 168 (3) 36, 84, 96, and 108.

7. Are there factors of 5 other than itself and 1? Of 7? Of 13?

A number that has no factors except itself and 1 is a prime number.
A number that has factors beside itself and 1 is a composite number.

8. Name the prime numbers from 1 to 100; the composite numbers from 1 to 100.

QUERY: Is 2 a prime or a composite number?

A number that can be exactly divided by 2 is an even number.
All numbers that can not be exactly divided by 2 are odd numbers.

9. How can an even number be quickly recognized?

10. Name the even numbers from 0 to 50; the odd numbers.

11. Tell which of the following numbers are (1) even, (2) odd, (3) prime, (4) composite: 5, 8, 9, 2, 21, 15, 19, 26, 27, 38, 41, 42.

12. Mention some numbers that are both odd and composite; even and composite; odd and prime.

13. Factors of 28 are 7 and 4. As 7 is a prime number it is called a *prime factor*. What is a prime factor?

14. Write the prime factors of 21; of 24; of 25; of 27; of 30.

NOTE: Write out the prime factors of the above numbers as they are here written out for 96:

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3.$$

15. How many times does 2 occur as a prime factor in 96? This may be indicated by writing 96 thus: $2^5 \times 3$. The small 5 written to the right and above the 2 is to show how many times 2 is to be used *as a factor*. Express 80 as 96 is expressed above.

§38. Greatest Common Divisor by Prime Factors.

1. What are the prime factors of 36 and of 48? Write 36 in the form given for 96 in problem 15 of the last section. Write 48 also in this form.

2. Will each of the *common* prime factors of 36 and 48 divide the G. C. D. of 36 and 48? Will the product of all the common prime factors divide the G. C. D. exactly?

3. Answer questions like 1 and 2 for the numbers 42 and 105; for 96 and 216; for 75 and 250.

4. When any common prime factor occurs oftener than once in one or both of the numbers, how often does it occur in the G. C. D. of those numbers? Answer by examining the following pairs of numbers:

(1) 12 and 48
(2) 54 and 405
(3) 75 and 250
(4) 98 and 343

5. Make a rule for finding the G. C. D. of two or more numbers from their common prime factors when no common prime factors are repeated in any of the numbers. Test your rule by finding the G. C. D. of 30 and 42; of 105 and 231; of 30 and 70.

6. Find the G. C. D. of 792 and 1080.

CONVENIENT FORM

2)792	1080
2)396	540
2)198	270
3)99	135
3)33	45
11	3)15
	5

$792 = 2^3 \times 3^2 \times 11$

$1080 = 2^3 \times 3^3 \times 5$

G. C. D. $= 2^3 \times 3^2 = 8 \times 9 = 72$

The G. C. D. of two or more numbers equals the product of all the prime factors common to all the numbers, each common prime factor being used the smallest number of times it occurs in any one of the given numbers.

7. Find the G. C. D. of the following pairs of numbers:
(1) 72 and 108 (2) 288 and 648 (3) 675 and 1125.

8. Find the G. C. D. of the following, applying tests for divisibility:

(1) 100 and 120
(2) 54 and 144
(3) 225 and 315
(4) 215 and 1935
(5) 108, 162, and 216
(6) 66, 176, and 286
(7) 315, 630, and 756
(8) 162, 1134, and 1458

9. Reduce these fractions to their lowest terms by dividing both terms by their G. C. D.:

(1) $\frac{16}{18}$ (2) $\frac{36}{49}$ (3) $\frac{40}{75}$ (4) $\frac{180}{210}$ (5) $\frac{108}{168}$ (6) $\frac{240}{264}$ (7) $\frac{360}{416}$ (8) $\frac{63}{147}$ (9) $\frac{102}{136}$ (10) $\frac{90}{315}$ (11) $\frac{68}{748}$ (12) $\frac{96}{672}$

10. A boy spent $\frac{2}{5}$ of his money and gave away $\frac{1}{3}$ of it; what part of it did he have left?

To solve such problems as 10 we must know how to add and to subtract fractions.

§34. Addition and Subtraction.

1. Find the following sums and differences:

(1) $\frac{7}{17}+\frac{8}{17}=?$ (2) $\frac{13}{24}+\frac{8}{24}=?$ (3) $\frac{3}{a}+\frac{5}{a}=?$ (4) $\frac{13}{b}+\frac{8}{b}=?$ (5) $\frac{6}{13}+\frac{5}{13}=?$ (6) $\frac{8}{11}-\frac{3}{11}=?$ (7) $\frac{7}{26}-\frac{2}{26}=?$ (8) $\frac{4}{a}-\frac{3}{a}=?$ (9) $\frac{9}{b}-\frac{7}{b}=?$ (10) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ (11) $\frac{6}{7}-\frac{3}{14}=?$ (12) $\frac{6}{a}-\frac{3}{2a}=\frac{12}{2a}-\frac{3}{2a}$ (13) $\frac{7}{15}+\frac{1}{5}=?$ (14) $\frac{6}{3a}+\frac{1}{a}=?$ (15) $\frac{8}{2x}-\frac{1}{x}=?$

Fractions whose units are not the same, must be expressed in a common unit before adding or subtracting

them. To do this we must find a fractional unit whose denominator can be exactly divided by each of the given denominators. It will be the simplest always to select the fractional unit whose denominator is the *least* number that each of the given numbers will divide.

A denominator which is common to two or more fractions is called a common denominator.

When this common denominator is the least number that can be found which may be used as a common denominator of the fractions, it is called the least common denominator, and is written L. C. D.

Before studying more difficult fractions we must learn how to find least common denominators. We begin by a study of multiples, common multiples, and least common multiples of numbers.

§35. Multiples.

Answer all you can orally.

42 yd. is exactly measured by 7 yd., 3 yd., and 2 yards.

$21 is exactly measured by $7 and $3.

56 pk. is exactly measured, or divisible, by 7 pk. and by 8 pecks.

42 is a multiple of 7, 3, and 2. 21 is a multiple of 7 and 3. 56 is a multiple of 7 and 8. What are some of the multiples of 3 and 5; of 5 and 7; of 2 and 11?

1. What then is a multiple of a number?

A number that can be exactly divided by another number is called a multiple of the latter number, or a multiple of a number is found by multiplying it by some whole number.

Following is a list of multiples of 2, from 2 to 30:

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

Following is a list of all the multiples of 3, to 30:

3 6 9 12 15 18 21 24 27 30

2. Why is each number in the first row a multiple of 2?

3. Reading the two rows underscore the numbers that are the same in both rows.

4. What is a common multiple of two numbers?

A number that can be exactly divided by two or more numbers, is called a common multiple of those numbers.

5. Rewriting the common multiples of 2 and 3 we have:

6 12 18 24 30

Supply a few more common multiples of 2 and 3.

6. What number, besides 2 and 3, will exactly divide all the common multiples of 2 and 3?

7. On the blackboard write out rows of multiples of 3 and of 5, as in prob. 1, underscoring, or writing in colored chalk, the multiples that are common to both rows.

8. Will the least of the common multiples of 3 and 5 divide all the other common multiples? What is the least common multiple of two numbers?

The least common multiple of two or more numbers is the least number that is exactly divisible by each of the numbers. It is usually written L. C. M. for brevity.

§36. Finding the L. C. M.

Use pencil only when necessary.

1. Make lists of the multiples of 2, 5, and 7, and find their L. C. M.

2. In the same manner, find the L. C. M. of 3, 4, and 5.

3. Note that all the given numbers in problems 1 or 2 are prime to each other. In such a case how can the L. C. M. be found from the numbers?

The L. C. M. of two or more numbers, which are all prime to each other, is their product.

4. Illustrate the truth of this statement by two sets

of numbers of your own selection, one to contain two numbers, and the other three, the numbers of each set being prime to each other.

5. 4, 8, 12, 16, 20, 24 are multiples of 4, and 6, 12, 18, 24, 30 are multiples of 6. What is the *least* common multiple of 4 and 6?

6. This number is the product of 4 and 6 divided by what? 2 is the common factor of 4 and 6.

7. Are 4 and 10 prime to each other? What is their greatest common factor or divisor?

8. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 are multiples of 4, and 10, 20, 30, 40 are multiples of 10. What are some common multiples of 4 and 10? What is their least common multiple?

9. Divide the product of 4 and 10 by 2, their G. C. D. What is the result? Compare this result with the last answer to problem 8.

The L. C. M. of two numbers, which are not prime to each other, is their product divided by their greatest common divisor (G. C. D.).

10. What is the G. C. D. of 15 and 20? What is the product of 15 and 20? How can you find the L. C. M.?

11. In the same manner find the L. C. M. of 8 and 10; of 12 and 24; of 15 and 25; of 24 and 60; of 40 and 36.

§37. Shorter Process for Three or More Numbers.

Answer all you can orally.

1. In a certain number there are 8 24's. How many 12's are there in the number? How many 8's? How many 6's?

2. In a certain number, x, there are 15 6's. How many 3's are there in x? How many 2's?

3. If 6 exactly divides a certain number, x, what other numbers also exactly divide x?

4. If any composite number exactly divides a number, y, what other numbers also exactly divide y?

Any multiple of a number is divisible by all the prime factors of that number.

Use pencil only when necessary.

5. Find the L. C. M. of 150, 504, and 540.

SOLUTION: $150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$.

$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7$.

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5$.

The L. C. M. is a multiple of each of these numbers severally. By the above principle it must then contain the prime factors 2, 3, 5, and 7. But, since 25 or 5^2 is a factor of 150, the L. C. M. must contain 5 twice as a factor. Since 8, or 2^3, is a factor of 504, the L. C. M. must contain 2 as a factor three times. How many times must the L. C. M. contain 3 as a factor? How many times must the L. C. M. contain 7 as a factor? The number that contains just these factors, and no others, is $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^3 \times 5^2 \times 7 = 37{,}800$.

37,800 is the least common multiple, because it contains the necessary factors and no others. Any other common multiple must contain all these factors and some other, and it would therefore be larger than 37,800.

What prime factors must be in the L. C. M. of three numbers?

If a prime factor is repeated in one or more of the numbers, how often must it occur in the L. C. M.?

The L. C. M. of two or more numbers is the product of all the different prime factors of all the numbers, each factor occurring the greatest number of times it occurs in any one of the numbers.

6. Find the L. C. M. of these numbers, and show work:

(1) 9, 12, 18

(2) 18, 30, 36

(3) 15, 25, 42, 50

(4) 6, 18, 27, 36

7. The work can be shortened by this arrangement.

SOLUTION of problem 2, (4):

CONVENIENT FORM

3)	6,	18,	27,	36
2)	2,	6,	9,	12
3)	1,	3,	9,	6
	1,	1,	3,	2

L. C. M. $= 3 \times 2 \times 3 \times 3 \times 2 = 108$.

EXPLANATION: Select any prime number, as 3, that will divide two or more of the numbers whose L. C. M. is sought. Divide it into all the numbers that are exactly divisible by it, writing in the line below the quotients and any numbers the chosen divisor does not exactly divide.

Select another prime number and do as before. Continue until the numbers last brought down are all prime. The continued product of the divisors and the numbers remaining in the last horizontal line is the L. C. M.

The least common denominator defined in §34 is the L. C. M. of all the denominators of the fractions.

8. Find the least common denominator, and then add:

(1) $\frac{2}{15}$, $\frac{3}{25}$, $\frac{9}{42}$, and $\frac{13}{50}$

(2) $\frac{5}{7}$, $\frac{1}{28}$, $\frac{1}{24}$, and $\frac{5}{42}$

(3) $\frac{1}{6}$, $\frac{5}{18}$, $\frac{2}{27}$, and $\frac{1}{36}$

(4) $\frac{2}{9}$, $\frac{5}{13}$, $\frac{3}{26}$, and $\frac{7}{39}$

9. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$.

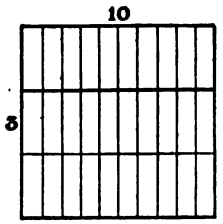


FIGURE 1

$\frac{2}{3} = \frac{20}{30}$

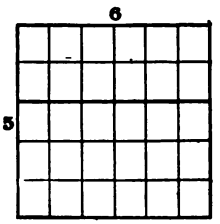


FIGURE 2

$\frac{3}{5} = \frac{18}{30}$

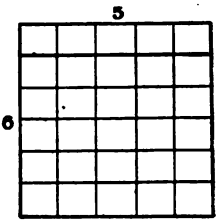


FIGURE 3

$\frac{5}{6} = \frac{25}{30}$

Show from Fig. 1 that $\frac{2}{3} = \frac{20}{30}$; from Fig. 2, that $\frac{3}{5} = \frac{18}{30}$, and from Fig. 3, that $\frac{5}{6} = \frac{25}{30}$.

Then, $\frac{20}{30} + \frac{18}{30} + \frac{25}{30} = \frac{20+18+25}{30} = \frac{63}{30} = 2\frac{1}{10}$.

Or, with numbers, the least common denominator of the fractions $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$, is 30. Then,

$$\frac{2}{3}+\frac{3}{5}+\frac{5}{6}=\frac{2\times10}{3\times10}+\frac{3\times6}{5\times6}+\frac{5\times5}{6\times5}=\frac{20+18+25}{30}=\frac{63}{30}=2\frac{3}{30}=2\frac{1}{10}.$$

The last step merely simplifies the result.

10. $\frac{2}{11}$ of the weight of a quantity of soil was gravel, and $\frac{3}{7}$ was sand; what part of the soil by weight was sand and gravel? Draw a figure and show the solution by a drawing. Give also the arithmetical solution.

11. One side of a triangle is $2\frac{3}{4}$ in.; another is $2\frac{2}{3}$ in., and the third is $3\frac{2}{5}$ in.; how long is the perimeter of (distance around) the triangle? Add first the whole numbers, then the fractions, and finally add the sums.

12. One side of a 4-sided figure is $6\frac{4}{5}$ in. long; a second side is $3\frac{2}{3}$ in.; the third $7\frac{3}{8}$ in., and the fourth $7\frac{7}{12}$ in. What is the perimeter of the figure?

13. A coal dealer bought 4 carloads of coal of the following weights: $21\frac{11}{10}$ T., $27\frac{7}{8}$ T., $29\frac{4}{5}$ T., and $30\frac{3}{4}$ T. Find the combined weight, in tons.

14. How many feet of lumber would there be in a baseboard not over 1″ thick, 1 ft. wide, extending entirely around the inside of the house, deducting 6 board feet for doors, Fig. 1?

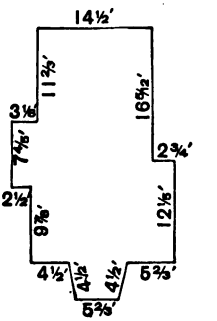


FIGURE 1

§38. Exercises and Problems.

1. Solve the following:

(1) $\frac{1}{5}+\frac{1}{7}=?$ (6) $\frac{2}{5}+\frac{2}{7}=?$ (11) $\frac{6}{7}+\frac{5}{9}=?$

(2) $\frac{1}{8}-\frac{1}{11}=?$ (7) $\frac{3}{7}-\frac{3}{11}=?$ (12) $\frac{7}{11}-\frac{5}{12}=?$

(3) $\frac{1}{7}-\frac{1}{12}=?$ (8) $\frac{8}{11}+\frac{8}{13}=?$ (13) $\frac{4}{17}+\frac{9}{10}=?$

(4) $\frac{1}{a}+\frac{1}{b}=?$ (9) $\frac{a}{c}+\frac{a}{b}=?$ (14) $\frac{a}{b}+\frac{c}{d}=?$

(5) $\frac{1}{a}-\frac{1}{b}=?$ (10) $\frac{m}{x}-\frac{m}{y}=?$ (15) $\frac{1.1}{n}-\frac{x}{y}=?$

2. Make a rule for finding quickly the sum, or the difference, of any two fractions.

The rule may be shown thus,

$$\text{sum} = \frac{\text{1st num.} \times \text{2d den.} + \text{2d num.} \times \text{1st den.}}{\text{1st den.} \times \text{2d den.}}, \text{ and}$$

$$\text{difference} = \frac{\text{1st num.} \times \text{2d den.} - \text{2d num.} \times \text{1st den.}}{\text{1st den.} \times \text{2d den.}}$$

The result must always be reduced to the simplest form.

3. In three successive runs, a train loses $\frac{1}{3}$ hr, $\frac{1}{5}$ hr., and $\frac{1}{2}$ hr. How much time does it lose in all?

4. $\frac{2}{5}$ of a man's salary is spent for clothing; $\frac{1}{3}$ for board and lodging; $\frac{1}{6}$ for books and stationery; $\frac{1}{12}$ for traveling expenses. What part of his salary was spent for these purposes? What part remained?

5. In the six days of one week, a man works: $8\frac{1}{2}$ hr., $9\frac{3}{4}$ hr., $8\frac{2}{3}$ hr., $9\frac{1}{4}$ hr., $8\frac{3}{4}$ hr., $10\frac{3}{4}$ hr. What is the whole number of hours of work for the week? What are his wages at \$.30 an hour?

6. Four children are to do a piece of work. Three of them do $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{2}{9}$ of it. What part is left for the fourth?

7. $\frac{2}{5}$, $\frac{1}{6}$, and $\frac{2}{15}$ of a man's money are invested in three different enterprises. What part of his money is invested? What part is not invested?

8. A merchant sold $\frac{7}{12}$ of a gross of buttons to one customer, and $\frac{1}{8}$ of a gross less to another. What part of a gross did the second customer buy?

9. A tank of oil is $\frac{4}{5}$ full. If $\frac{1}{2}$ of the contents of the tank is drawn off and then $\frac{4}{15}$ of the remainder, what part is left?

10. At \$1 a day how much money does a man earn by working $5\frac{3}{4}$ da., $6\frac{2}{3}$ da., and $8\frac{3}{5}$ days?

11. A boy lives where school is taught 1100 hr. a year. Through sickness and other causes he loses, from month to month, the following numbers of hours:

$115\frac{2}{3}$ $39\frac{3}{4}$ $15\frac{5}{6}$ $6\frac{11}{12}$ $4\frac{4}{5}$ $3\frac{2}{5}$ $5\frac{3}{5}$ $18\frac{1}{3}$ $89\frac{7}{10}$

How many hours does he lose in all? What part of the whole school year does he lose?

NOTE: In adding the fractions of problem 11 group together fractions whose denominators are the same, or are easily made the same.

MULTIPLICATION AND DIVISION

§39. Multiplying a Fraction by a Whole Number.

1. Give the value of the letter in each of the following problems and make a rule for multiplying a fraction quickly by some factor of its denominator. Use cancellation, thus: $\cancel{6} \times \frac{7}{\cancel{24}_{4}} = \frac{7}{4} = 1\frac{3}{4}$.

(1) $3 \times \frac{5}{6} = \frac{x}{6}$ (3) $12 \times \frac{7}{24} = \frac{x}{24}$ (5) $25 \times \frac{19}{125} = \frac{x}{125}$

(2) $5 \times \frac{7}{15} = \frac{x}{3}$ (4) $9 \times \frac{5}{36} = \frac{x}{4}$ (6) $13 \times \frac{16}{91} = \frac{x}{7}$

2. How may a fraction be multiplied by a whole number?

A fraction is multiplied by a whole number by

(1) *Multiplying its numerator by the multiplier;* or,

(2) *Dividing its denominator by the multiplier.*

NOTE: The usefulness of the method of cancellation consists in making these two processes undo (balance) each other. (See problem 1).

QUERY: When will the second method be the more convenient?

A fraction is multiplied by its denominator by dropping its denominator.

3. Give orally the following products:

(1) $12 \times \frac{5}{12} = ?$ (3) $16 \times \frac{7}{16} = ?$ (5) $30 \times \frac{7}{30} = ?$

(2) $10 \times \frac{3}{10} = ?$ (4) $25 \times \frac{11}{25} = ?$ (6) $b \times \frac{a}{b} = ?$

4. Solve the following:

(1) $9 \times \frac{7}{8} = ?$ (6) $8 \times \frac{7}{8} = ?$ (11) $27 \times \frac{7}{9} = ?$

(2) $12 \times \frac{5}{6} = ?$ (7) $12 \times \frac{5}{15} = ?$ (12) $b \times \frac{a}{b} = ?$

(3) $20 \times \frac{4}{5} = ?$ (8) $35 \times \frac{3}{14} = ?$ (13) $16 \times \frac{a}{16} = ?$

(4) $25 \times \frac{2}{13} = ?$ (9) $40 \times \frac{7}{8} = ?$ (14) $18 \times \frac{2}{9} = ?$

(5) $30 \times \frac{21}{60} = ?$ (10) $12 \times \frac{7}{24} = ?$ (15) $c \times \frac{a}{c} = ?$

§40. Multiplying a Whole Number by a Fraction.

Just as $4 \times 7 = 7 \times 4$, or $3 \times 8 = 8 \times 3$, so is $\frac{5}{6} \times 12$ equal to the same number as $12 \times \frac{5}{6}$; and $\frac{8}{3} \times 26 = 26 \times \frac{8}{3}$. Thus, when a problem is stated, "What is $\frac{7}{11} \times 47$?" it may be at once replaced by the question, "What is $47 \times \frac{7}{11}$?" This change need be made only in thought.

1. Solve the following:

(1) $\frac{4}{5} \times 25 = ?$ (7) $\frac{12}{13} \times 26 = ?$ (13) $\frac{3}{7} \times a = ?$

(2) $\frac{4}{7} \times 21 = ?$ (8) $\frac{11}{12} \times 36 = ?$ (14) $\frac{4}{5} \times c = ?$

(3) $\frac{5}{7} \times 35 = ?$ (9) $\frac{5}{8} \times 14 = ?$ (15) $\frac{6}{9} \times x = ?$

(4) $\frac{5}{6} \times 12 = ?$ (10) $\frac{7}{8} \times 38 = ?$ (16) $\frac{c}{d} \times ad = ?$

(5) $\frac{6}{7} \times 13 = ?$ (11) $\frac{7}{11} \times 33 = ?$ (17) $\frac{28}{39} \times 78 = ?$

(6) $\frac{7}{9} \times 15 = ?$ (12) $\frac{6}{7} \times 28 = ?$ (18) $\frac{x}{y} \times my = ?$

2. Solve the following:

(1) $\frac{7}{15} \times 10 = ?$ (4) $\frac{5}{21} \times 14 = ?$ (7) $\frac{9}{25} \times 15 = ?$

(2) $\frac{5}{6} \times 9 = ?$ (5) $\frac{9}{14} \times 42 = ?$ (8) $\frac{b}{4} \times 6 = ?$

(3) $\frac{1}{4} \times 18 = ?$ (6) $\frac{11}{36} \times 81 = ?$ (9) $\frac{4}{2c} \times 3c = ?$

§41. Problems.

1. Make and solve problems based on facts obtained by yourself, or on those given below:

(1) To plow an acre with a plow cutting a furrow 10″ wide (a 10″-plow) a horse must walk $9\frac{9}{10}$ mi.; with a

12″-plow, a horse must walk $8\frac{1}{4}$ mi.; with a 15″-plow, $6\frac{3}{5}$ miles.

(2) For a team drawing a plow, a distance of 16 mi. is a fair day's work and a distance of 18 mi. is a large day's work. The following table shows to what fractional part of an acre a furrow $\frac{1}{2}$ mi. long and of the stated widths is equivalent:

Width of Furrow	Equivalent	Width of Furrow	Equivalent
12″	$\frac{2}{33}$ A.	16″	$\frac{8}{99}$ A.
13″	$\frac{13}{198}$ A.	17″	$\frac{17}{198}$ A.
14″	$\frac{7}{99}$ A.	18″	$\frac{1}{11}$ A.
15″	$\frac{5}{66}$ A.	20″	$\frac{10}{99}$ A.

§42. Multiplying a Mixed Number by a Mixed Number.

1. How many square inches in the area of a rectangle $4\frac{3}{4}'' \times 5\frac{1}{2}''$?

Solution: (1) How long is *a* (Fig. 1)? How wide? What is its area?

(2) Answer similar questions for *b*, *c* and *d*.

(3) How long is the entire rectangle? How wide? What is its area?

Since the area of the entire rectangle equals the sum of the areas of its parts we may write:

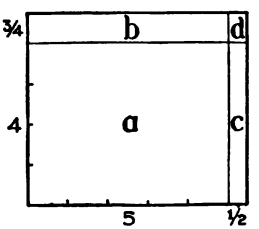


Figure 1

$4\frac{3}{4} \times 5\frac{1}{2} = 4 \times 5 + \frac{3}{4} \times 5 + 4 \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2} = 20 + 3\frac{3}{4} + 2 + \frac{3}{8} = 26\frac{1}{8}$. *Ans.* $26\frac{1}{8}$ sq. in.

Point out on Figure 1, the areas that represent all the parts of the product.

2. Find the areas of rectangles of the following dimensions:

(1) $3\frac{2}{3}'' \times 12\frac{1}{2}''$ (3) $8\frac{2}{3}'' \times 30\frac{3}{4}''$ (5) $20\frac{3}{5}'' \times 15\frac{1}{4}''$

(2) $5\frac{3}{4}'' \times 16\frac{2}{5}''$ (4) $16\frac{1}{2}'' \times 32\frac{3}{4}''$ (6) $16\frac{2}{3}'' \times 18\frac{3}{4}''$

3. Find the cost of $9\frac{1}{2}$ T. hard coal at $\$7\frac{3}{4}$.

SOLUTIONS: If 1 T. costs $\$7\frac{3}{4}$, $9\frac{1}{2}$ T. will cost $9\frac{1}{2} \times \$7\frac{3}{4}$.

CONVENIENT FORMS

(1)
$$\begin{array}{l} 7\frac{3}{4} \\ 9\frac{1}{2} \\ \hline \frac{3}{8} = \frac{1}{2} \times \frac{3}{4} \\ 3\frac{1}{2} = \frac{1}{2} \times 7 \\ 6\frac{3}{4} = 9 \times \frac{3}{4} \\ 63 \ = 9 \times 7 \\ \hline 72\frac{1}{8}\frac{3}{} = 73\frac{5}{8} \end{array}$$

(2)
$$7\tfrac{3}{4} = \tfrac{31}{4}$$
$$9\tfrac{1}{2} = \tfrac{19}{2}$$
$$\tfrac{31}{4} \times \tfrac{19}{2} = \tfrac{589}{8} = 73\tfrac{5}{8}$$

Ans. $\$73\frac{5}{8}$.

Find areas of surfaces having the following dimensions:

4. $44\frac{3}{7}$ ft. $\times$ $28\frac{3}{4}$ feet.

5. $36\frac{5}{8}$ ft. $\times$ $27\frac{2}{7}$ feet.

6. $24\frac{3}{4}$ ft. $\times$ $18\frac{2}{3}$ feet.

7. $45\frac{5}{6}$ ft. $\times$ $30\frac{1}{3}$ feet.

Find the cost of the following items:

8. $24\frac{3}{4}$ doz. eggs @ $\$.16\frac{2}{3}$.

9. $46\frac{3}{4}$ gal. of oil @ $\$.12\frac{1}{2}$.

10. $15\frac{1}{3}$ yd. of cloth @ $\$.66\frac{2}{3}$.

11. $52\frac{3}{4}$ lb. of sugar @ $\$.05\frac{5}{9}$.

12. $265\frac{3}{8}$ M. of pine flooring @ $\$35\frac{1}{4}$.

13. Make original problems from the following items:

(1) A cubic inch of water weighs $252\frac{2}{2}\frac{4}{5}$ grains.

(2) A water tower is $2\frac{4}{6}\frac{7}{3}$ ft. higher than the mound upon which it is built. The mound is $81\frac{1}{2}$ ft. high.

(3) $69\frac{1}{5}$ statute miles = 1 degree of longitude at the equator.

(4) A pine tree 87 ft. high has no branches for $\frac{1}{2}\frac{3}{9}$ of its height.

(5) $\frac{3}{8}\frac{1}{9}$ of a library of 55,447 volumes was burned.

(6) Flaxseed cost $\$\frac{2}{5}$ per pound when a certain linseed oil factory bought supplies.

(7) A spring furnishes $28\frac{7}{16}$ bbl. of water daily.

(8) A steam-boat makes $11\frac{3}{4}$ miles per hour, on an average.

(9) A room is 32 ft. long and $24\frac{5}{8}$ ft. wide. Painting costs $\$\frac{1}{50}$ per square foot.

(10) A cubic foot of water weighs 62.5 pounds.

(11) Gold is $19\frac{1}{4}$ times as heavy as water.

(12) $30\frac{1}{4}$ sq. yd. = 1 square rod.

§43. Dividing a Fraction by a Whole Number.

1. $\frac{7}{8} \div 5 = \frac{7}{x}$. What number does x stand for?

SOLUTION:

$\frac{7}{8} \div 5 = ?$

$\frac{7}{8} \div 5 = \frac{1}{5}$ of $\frac{7}{8}$, or

$\frac{7}{8}$ of $\frac{1}{5} = \frac{7}{40}$. x stands for 40.

2. What number should stand in place of x in the following?

(1) $\frac{16}{17} \div 4 = \frac{x}{17}$

(2) $\frac{80}{37} \div 40 = \frac{x}{37}$

(3) $\frac{40}{47} \div 8 = \frac{x}{47}$

(4) $\frac{144}{167} \div 24 = \frac{x}{167}$

(5) $\frac{7}{9} \div 4 = \frac{7}{x}$

(6) $\frac{9}{10} \div 7 = \frac{9}{x}$

(7) $\frac{15}{3} \div 14 = \frac{15}{x}$

(8) $\frac{240}{173} \div 3 = \frac{240}{x}$

(9) $\frac{40}{45} \div 5 = \frac{40}{x}$

(10) $\frac{40}{45} \div 5 = \frac{x}{45}$

(11) $\frac{180}{211} \div 10 = \frac{x}{211}$

(12) $\frac{180}{211} \div 10 = \frac{180}{x}$

3. In what two ways may a fraction be divided by a whole number? When would you use the one way? When the other way?

A fraction is divided by a whole number (1) *by dividing its numerator, or* (2) *by multiplying its denominator by the whole number.*

4. A strip of tin-foil $\frac{8}{9}$ ft. long is to be cut across into 4 equal pieces. What is the length of one strip?

5. 4 children have a garden containing $\frac{2}{5}$ A. If the

children use equal portions, what part of an acre does each use?

6. 4 men own $\frac{5}{9}$ of an estate, equally. What part belongs to each man? If the estate is valued at \$36,000, how much money is represented by each part?

7. If $\frac{115}{8}$ A. is divided equally among 7 men, how many acres are there for each man?

8. Find the following quotients:

(1) $\frac{6}{17} \div 8 = ?$
(2) $\frac{7}{8} \div 35 = ?$
(3) $\frac{10}{21} \div 40 = ?$
(4) $\frac{12}{25} \div 48 = ?$
(5) $\frac{30}{17} \div 45 = ?$
(6) $\frac{4}{7} \div 26 = ?$
(7) $\frac{8}{5} \div 32 = ?$
(8) $\frac{9}{10} \div 9a = ?$
(9) $\frac{a}{x} \div 2a = ?$
(10) $\frac{b}{c} \div ba = ?$

§44. Dividing a Mixed Number by a Whole Number.

1. A rectangular ceiling 16 ft. wide contains $233\frac{1}{3}$ sq. ft. Find the length.

SOLUTION: $233\frac{1}{3}$ ft. $= \frac{700}{3}$ feet.

$$\frac{700}{3} \div 16 = \frac{\overset{175}{\cancel{700}}}{3} \times \frac{1}{\underset{4}{\cancel{16}}} = \frac{175}{12} = 14\frac{7}{12}.$$

$14\frac{7}{12}$ ft. $= 14'\ 7''$, the length of the ceiling.

2. Solve the following examples:

(1) $14\frac{3}{4}$ bu. $\div 8 = ?$
(2) $9\frac{7}{12}$ yd. $\div 6 = ?$
(3) $16\frac{2}{3}$ hr. $\div 12 = ?$
(4) $44\frac{9}{16}$ lb. $\div 9 = ?$
(5) $39\frac{5}{8}$ mi. $\div 7 = ?$
(6) $\$12\frac{5}{8} \div 5 = ?$
(7) $45\frac{9}{16} \div 5 = ?$
(8) $\frac{224}{5} \div 4 = ?$
(9) $14\frac{7}{21} \div 7 = ?$
(10) $66\frac{2}{3} \div 8 = ?$

NOTE: In examples like (7) and (9), in which the integral part of the dividend exactly contains the divisor, divide the whole number and the fraction separately, and add the quotients.

3. Find the lengths of the rectangular surfaces whose areas and widths are given here:

AREAS	WIDTHS	LENGTHS
(1) $212\frac{1}{2}$ sq. ft.	17 ft.	.....
(2) $250\frac{1}{4}$ "	10 "	.....
(3) $261\frac{1}{2}$ "	14 "	.....
(4) $275\frac{4}{5}$ "	25 "	.....
(5) $406\frac{1}{4}$ "	25 "	.....
(6) $164\frac{3}{4}$ "	13 "	.....

4. Five measures of last year's growth of twigs from the lower branches on the north side of an oak tree are: $5\frac{1}{2}''$, $6\frac{1}{4}''$, $4\frac{3}{4}''$, $5\frac{1}{2}''$ and $6\frac{1}{4}''$. What is the average growth?

5. From the lower branches on the east and west sides of the same tree the measures are: $6\frac{3}{4}''$, $4\frac{3}{4}''$, $6\frac{1}{4}''$, $6\frac{1}{4}''$, and $6\frac{1}{2}''$. What is the average growth of twigs on the east and west sides?

6. For twigs on the south side of the same tree the measures are: $6\frac{1}{4}''$, $6\frac{3}{4}''$, $6\frac{1}{4}''$, $8\frac{3}{4}''$ and $4\frac{1}{2}''$. What is the average growth of twigs on the south side of the tree?

7. From the middle branches of the oak tree 5 measures of branches from the north side are: $5\frac{3}{4}''$, $4\frac{3}{4}''$, $7\frac{1}{2}''$, $3\frac{3}{4}''$ and $5\frac{1}{8}''$; of branches from the east and west sides are: $4\frac{3}{4}''$, $7\frac{1}{2}''$, $5\frac{1}{2}''$, $7\frac{1}{4}''$ and $6\frac{3}{4}''$; of branches from the south side: $5\frac{1}{2}''$, $6\frac{1}{2}''$, $7\frac{3}{4}''$, $6\frac{5}{8}''$ and $7\frac{1}{2}''$. How much greater is the average growth of twigs on the south side than the average growth of those on the north side of the tree? Than twigs from the east and west sides?

8. Make problems based on these items:

(1) A ship sails $246\frac{3}{10}$ mi. in 36 hours.

(2) A man paid $\$195\frac{1}{4}$ for 72 days' board.

(3) A field contains $2172\frac{1}{6}$ sq. rd. One side is $40\frac{3}{4}$ rd. long.

(4) 15 lb. of fresh salmon cost $\$2.73\frac{3}{4}$.

§45. Dividing Any Number by a Fraction.

(*A*) *Dividend a whole number.*

1. How many $\frac{3}{4}$ lb. packages can be made from 3 lb. of tea?

SOLUTION: $1 \div \frac{3}{4} = \frac{4}{3}$

$3 \div \frac{3}{4} = 3 \times 1 \div \frac{3}{4} = \overset{1}{\cancel{3}} \times \frac{4}{\cancel{3}_1} = 4$. *Ans.* 4 packages.

2. A boy earns at the average rate of $\$\frac{4}{5}$ per day selling papers. How many days did it take him to earn \$12?

SOLUTION: $1 \div \frac{4}{5} = \frac{5}{4}$.

$12 \div \frac{4}{5} = 12 \times 1 \div \frac{4}{5} = \overset{3}{\cancel{12}} \times \frac{5}{\cancel{4}} = 15$ *Ans.* 15 days.

(*B*) *Dividend a fraction.*

3. Divide $\frac{4}{5}$ by $\frac{3}{7}$.

SOLUTION:

$$\frac{\frac{4}{5}}{\frac{3}{7}} = \frac{\frac{4\times7}{5\times7}}{\frac{3\times5}{7\times5}}; \text{ why?} \qquad \frac{\frac{4\times7}{5\times7}}{\frac{3\times5}{7\times5}} = \frac{28}{15}; \text{ why?}$$

(*C*) *Dividend and divisor both mixed numbers.*

4. I paid $\$49\frac{3}{5}$ for coal @ $\$7\frac{3}{4}$ per ton. How many tons did I buy?

SOLUTION:

$$49\tfrac{3}{5} = \frac{248}{5}; \quad 7\tfrac{3}{4} = \frac{31}{4}; \quad 49\tfrac{3}{5} \div 7\tfrac{3}{4} = \frac{\frac{248}{5}}{\frac{31}{4}}$$

$$\frac{\frac{248}{5}}{\frac{31}{4}} = \frac{\overset{8}{\cancel{248}} \times \cancel{5} \times 4}{\cancel{31} \times 5 \times \cancel{4}} = \frac{32}{5} = 6\frac{2}{5}$$

Ans. $6\frac{2}{5}$ T.

A general method of dividing *any number* by a fraction follows here.

5. How many times is each of the following fractions contained in 1?

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{7}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{15}$ $\frac{1}{25}$ $\frac{1}{50}$ $\frac{1}{100}$ $\frac{1}{a}$ $\frac{1}{x}$

6. How many times is each of these fractions contained in 1?

$\frac{2}{3}$ $\frac{3}{5}$ $\frac{5}{7}$ $\frac{3}{4}$ $\frac{5}{9}$ $\frac{9}{10}$ $\frac{7}{12}$ $\frac{13}{15}$ $\frac{21}{25}$ $\frac{33}{50}$ $\frac{37}{100}$ $\frac{b}{x}$

7. What number should stand in place of the letter x?

(1) $\frac{2}{3} \times \frac{3}{2} = x$ (4) $\frac{8}{7} \times \frac{7}{8} = x$ (7) $\frac{9}{13} \times \frac{13}{9} = x$

(2) $\frac{3}{4} \times \frac{4}{3} = x$ (5) $\frac{9}{10} \times \frac{10}{9} = x$ (8) $\frac{a}{b} \times \frac{b}{a} = x$

(3) $\frac{5}{6} \times \frac{6}{5} = x$ (6) $\frac{12}{17} \times \frac{17}{12} = x$ (9) $\frac{c}{d} \times \frac{d}{c} = x$

8. What number should stand in place of the letter y?

(1) $\frac{2}{3} \times y = 1$ (3) $\frac{7}{12} \times y = 1$ (5) $\frac{25}{12} \times y = 1$

(2) $\frac{3}{4} \times y = 1$ (4) $\frac{12}{25} \times y = 1$ (6) $\frac{a}{b} \times y = 1$

1 divided by $\frac{5}{6}$ equals $\frac{6}{5}$, and $1 \div \frac{6}{5} = \frac{5}{6}$; $1 \div 8 = \frac{1}{8}$, and $1 \div \frac{1}{8} = 8$.

Dividing 1 by any number, whole or fractional, is called inverting the number.

The reciprocal of any number is the number inverted.

9. To what number is the product of any number and its reciprocal equal?

10. Examine your answers to problem 7 and state how to find quickly the number of times any fraction is contained in 1.

11. If a fraction is contained in *one* $\frac{5}{3}$ times, how many times is it contained in 2? In 6? In 12? In $\frac{1}{2}$? In $\frac{3}{4}$? In $\frac{7}{8}$? In any number?

NOTE: 7 may be written $\frac{7}{1}$, 12 may be written $\frac{12}{1}$, and so on. Consequently 7 and $\frac{1}{7}$ are reciprocals, as are also 12 and $\frac{1}{12}$, 25 and $\frac{1}{25}$, and so on.

12. What is actually done by inverting the divisor in a problem in division of fractions? (See definition above.)

13. If, then, the reciprocal of a fraction is multiplied by 6, what operation is performed?

14. Make a rule for quickly dividing any whole number by a fraction?

The reciprocal of any fraction shows how many times the fraction is contained in 1.

Any number, mixed, integral, or fractional, may be divided by a fraction by multiplying the number by the reciprocal of the divisor.

15. Why is the divisor inverted? Why is the inverted divisor multiplied by the dividend?

§46. Problems.

1. $\frac{2}{9}$ yd. of silk costs $\$\frac{3}{4}$. Find the cost of 1 yard.

2. At $\$1\frac{1}{4}$ a yd. how much cloth can be bought for $\$\frac{3}{4}$?

3. At $\$6\frac{3}{10}$ per cwt. how much beef will $\$2\frac{1}{2}$ buy?

4. How many meals will $5\frac{1}{4}$ bu. of oats supply, if $\frac{3}{16}$ bu. are fed at a meal?

5. How many pieces of wire $\frac{3}{4}$ ft. long can be cut from a piece $49\frac{1}{2}$ ft. long?

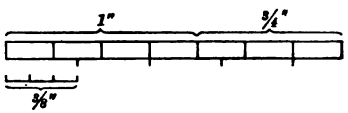


FIGURE 1

6. Measure $1\frac{3}{4}$ in. by $\frac{3}{8}$ in. (See Fig. 1.)

7. Divide $16\frac{1}{2}$ yd. by $2\frac{1}{2}$ yd.; by $1\frac{3}{4}$ yd.; by $4\frac{1}{2}$ yd.; by $1\frac{7}{8}$ yd.

8. Divide $\$6\frac{2}{5}$ by $\$\frac{3}{10}$; by $\$\frac{9}{20}$; by $\$\frac{2}{3}$; by $\$\frac{3}{4}$.

9. Find the number of strips of floor-covering running the long way of the room:

WIDTH OF ROOMS	FLOOR-COVERING
$22\frac{1}{2}$ ft.	$2\frac{1}{2}$ ft. wide.
$7\frac{2}{3}$ yd.	$\frac{5}{6}$ yd. "
$18\frac{3}{4}$ ft.	$2\frac{1}{2}$ ft. "
$8\frac{1}{4}$ yd.	$\frac{5}{6}$ yd. "

10. Make and solve problems based upon the following items:

(1) Portland cement $\$2\frac{3}{4}$ per bbl.; \$9 purchasing fund.

(2) Sidewalk having an area of $607\frac{3}{4}$ sq. ft.; flagstones $5\frac{1}{4}$ ft. square.

(3) $15\frac{3}{8}$ gal. maple syrup; $\frac{3}{4}$ gal. measures.

(4) $29\frac{5}{6}$ doz. oranges; $\frac{9}{12}$ doz. oranges.

(5) The ratio of two numbers is $\frac{3}{5}$; one of the numbers is $\frac{6}{7}$.

11. A man bought a house for \$1860. The house cost him $\frac{3}{5}$ as much as he paid for a mill. What was the cost of the mill?

12. My library contains 1075 volumes, which are $\frac{1}{9}$ as many as a friend's contains. How many volumes are in my friend's library?

13. Of a number of cars of grain inspected on a certain day 36 were rejected. This number was $\frac{1}{3}\frac{2}{5}$ of the entire number inspected. Find the number.

14. Let $\frac{a}{b}$ and $\frac{x}{y}$ denote two fractions, and state the following principle in symbols: The quotient of one fraction by another is equal to the product of the dividend by the inverted divisor.

§47. Complex Fractions.

Any problem in division of fractions may be written in fractional form, the dividend being written above and the divisor below the line.

EXAMPLES:

To divide $\frac{6}{7}$ by $\frac{5}{11}$ may be written $\dfrac{\frac{6}{7}}{\frac{5}{11}} = ?$

Or, to divide $18\frac{3}{8}$ by $6\frac{3}{5}$, may be written $\dfrac{18\frac{3}{8}}{6\frac{3}{5}} = ?$

Fractions that contain fractions in one or both terms, are called complex fractions.

The second principle of page 66, will enable us to solve all such problems. If either the divisor or the dividend, or both, are mixed numbers, they should first be reduced to improper fractions.

Following this principle:

$$(1)\quad \frac{\frac{6}{7}}{\frac{5}{11}} = \frac{6}{7} \times \frac{11}{5} = \frac{66}{35} = 1\frac{31}{35}$$

$$(2)\quad \frac{3\frac{2}{3}}{5\frac{3}{8}} = \frac{\frac{11}{3}}{\frac{43}{8}} = \frac{11}{3} \times \frac{8}{43} = \frac{88}{129}$$

The second step becomes unnecessary if we notice that the product of the outside terms (6 and 11), called *the extremes*, gives the numerator, and the product of the inside terms (7 and 5), called *the means*, gives the denominator of the result.

A complex fraction is simplified when its terms are freed of fractions, and the result is expressed in its simplest form.

Factors may often be cancelled from both numerator and denominator before multiplying.

Show that this is the same as dividing the product of *the extremes* by the product of *the means* in the given fractions.

Simplify these complex fractions, cancelling when possible:

1. $\frac{\frac{3}{4}}{\frac{1}{2}}$	**4.** $\frac{\frac{5}{16}}{\frac{4}{3}}$	**7.** $\frac{\frac{2}{7}}{3}$	**10.** $\frac{20\frac{2}{5}}{12\frac{3}{4}}$	**13.** $\frac{84\frac{2}{7}}{62\frac{4}{5}}$
2. $\frac{\frac{5}{6}}{23}$	**5.** $\frac{\frac{7}{12}}{\frac{3}{8}}$	**8.** $\frac{15\frac{2}{3}}{12\frac{1}{2}}$	**11.** $\frac{25\frac{2}{3}}{6\frac{3}{10}}$	**14.** $\frac{123\frac{5}{9}}{116\frac{2}{3}}$
3. $\frac{\frac{7}{9}}{\frac{4}{15}}$	**6.** $\frac{4}{6\frac{2}{3}}$	**9.** $\frac{1\frac{5}{9}}{\frac{3}{7}}$	**12.** $\frac{44\frac{4}{9}}{32\frac{2}{5}}$	**15.** $\frac{\frac{a}{b}}{\frac{c}{d}}$

§48. Joint Effects of Forces.

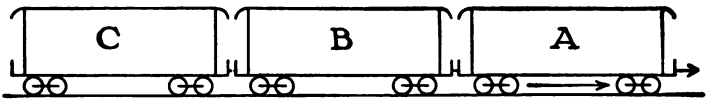


FIGURE 1

1. The rear car, *C*, pulls back on the middle car, *B*, with a force of 100 lb., and the front car, *A*, pulls it forward with a force of 250 lb. If car, *B*, were taken out of the train what single forward force would draw it the same as it is being drawn in the train?

2. A force of 23 lb. is pulling the car (Fig. 2) toward the right and another of 12 lb. is pulling it toward the left. If the car is free to move, in which direction will it move? What single force would move the car in the same manner as do both of these forces pulling at the same time?

FIGURE 2

3. What single force will move the car (Fig. 3) the same way as $8x$ lb. acting toward the right with $5x$ lb. acting toward the left?

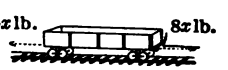


FIGURE 3

4. What single force will move the car (Fig. 4) in the same manner as $6m$ lb. pulling against $2m$ lb. will move it?

FIGURE 4

When a force of 18 lb. is supposed to be pulling a car toward the *right*, it will be written thus: R 18 lb. When the same force pulls toward the *left*, it will be written thus: L 18 lb. When two forces, R 16 lb. and L 12 lb., act at the same time, their joint effect is the same as the effect of the single force, R 4 lb.

5. Find the joint effect of R $28\frac{1}{2}$ lb. and L $21\frac{3}{4}$ lb.?

6. What would be the joint effect of each of these pairs of forces?

(1) R $48\frac{1}{4}$ lb. and L $12\frac{3}{8}$ lb.

(2) R $75\frac{3}{16}$ lb. and L $69\frac{5}{16}$ lb.

(3) R $181\frac{1}{8}$ lb. and L $129\frac{7}{10}$ lb.

A force of 25 T., pulling *forward,* may be written F 25 T., and a force of 18 T., pulling *backward,* may be written B 18 T. The joint effect of both would be written thus: F 7 T.

7. What is the joint effect of the three forces: F 17 T., F 12 T. and B 25 T.?

8. What is the joint effect of each of the following sets of forces?

(1) F $80\frac{1}{2}$ T., B $28\frac{5}{12}$ T., and B $96\frac{3}{8}$ T.

(2) F $49\frac{1}{2}$ T., B $84\frac{7}{8}$ T., and F $248\frac{7}{12}$ T.

(3) B $68\frac{9}{10}$ T., F $73\frac{2}{3}$ T., and B $12\frac{3}{8}$ T.

9. A force of 182 lb. pulling *upward* may be written U 182 lb. How would you write a force of 78 lb. pulling *downward?*

10. What is the joint effect of each of the following sets of forces?

(1) U $8\frac{2}{3}$ lb., D $16\frac{1}{2}$ lb., and U $3\frac{3}{8}$ lb.

(2) U $16\frac{1}{8}$ lb., U 28 lb., and D $29\frac{1}{2}$ lb.

(3) D 29 oz., D $32\frac{1}{2}$ oz., and D $16\frac{5}{8}$ oz.

(4) D $29\frac{3}{8}$ oz., U $65\frac{4}{5}$ oz., and D $42\frac{1}{2}$ oz.

11. If we call a force of 16 lb. pulling forward, or upward, or eastward, or northward, or to the right, a *positive* force, and write it + 16 lb., we should call the same force pulling backward, or downward, or westward, or southward, or to the left, respectively, a *negative* force. How should we write it?

12. Give the joint effect, with proper sign (+ or −) of each of these sets of forces:

(1) 2 forces, each + $8\frac{1}{4}$ lb., and one force, − $12\frac{7}{8}$ lb.

(2) 3 forces, each + $12\frac{1}{8}$ oz., and 4 forces, each − $8\frac{7}{8}$ oz.

(3) 5 forces, each − $16\frac{2}{3}$ T., and 12 forces, each + $9\frac{3}{4}$ T.

(4) 18 forces of − $16\frac{9}{16}$ lb. each, and 20 forces of + $25\frac{1}{5}$ lb. each.

§49. Exercises for Practice.

1. Find the following sums and differences as rapidly as you can work correctly:

(1) $\frac{7}{8} + \frac{6}{11} = ?$

(2) $\frac{12}{13} - \frac{7}{8} = ?$

(3) $\frac{11}{15} - \frac{5}{12} = ?$

(4) $\frac{16}{27} + \frac{15}{32} = ?$

(5) $\frac{25}{32} - \frac{16}{25} = ?$

(6) $8\frac{8}{9} + 4\frac{2}{27} = ?$

(7) $9\frac{4}{9} - 3\frac{14}{15} = ?$

(8) $16\frac{2}{9} - 10\frac{8}{15} = ?$

(9) $210\frac{5}{12} + 49\frac{9}{10} = ?$

(10) $37\frac{1}{2} - 18\frac{5}{21} = ?$

2. Find the products and quotients as rapidly as you can work correctly, using cancellation when it makes the work easier:

(1) $\frac{7}{8} \times \frac{16}{21} = ?$

(2) $\frac{6}{11} \times \frac{22}{27} = ?$

(3) $\frac{14}{17} \times \frac{17}{21} = ?$

(4) $\frac{16}{21} \div \frac{8}{21} = ?$

(5) $\frac{18}{23} \div \frac{9}{46} = ?$

(6) $\frac{12}{17} \div \frac{5}{18} = ?$

(7) $18\frac{9}{10} \div 16\frac{2}{3} = ?$

(8) $21\frac{3}{8} \div 14\frac{5}{16} = ?$

(9) $6 \div 15 = ?$

(10) $8\frac{1}{28} \div 12\frac{5}{7} = ?$

(11) $15\frac{6}{7} \div 3\frac{7}{8} = ?$

(12) $26\frac{11}{12} \div 80\frac{2}{3} = ?$

3. Find the simplest results for these exercises:

(1) $\frac{3}{4} \times \frac{2}{5} \times \frac{5}{14} = ?$

(2) $\frac{5}{8} \times \frac{9}{15} \times \frac{11}{12} = ?$

(3) $\frac{5}{12} + \frac{9}{10} + \frac{11}{15} = ?$

(4) $\frac{3}{14} \times \frac{7}{12} \times \frac{16}{21} = ?$

(5) $\frac{3}{14} \times \frac{7}{12} + \frac{16}{21} = ?$

(6) $12\frac{1}{2} \times 8\frac{1}{3} \div 6\frac{1}{4} = ?$

(7) $9\frac{3}{7} \times 4\frac{2}{5} \div 3\frac{7}{8} = ?$

(8) $16\frac{2}{3} + 70\frac{5}{6} - 87\frac{1}{2} = ?$

(9) $9\frac{3}{8} \div 6\frac{1}{4} \times 6\frac{2}{3} = ?$

(10) $16\frac{2}{3} \div 37\frac{1}{2} \times 6\frac{7}{8} = ?$

§50. Scale Drawings of Familiar Objects.

1. Notice the scale of the drawing of the playhouse (Fig. 1), and compute the lengths of the following dimensions:

(1) The width; (2) the height of the lower side; of the higher side; (3) the rise (difference of higher and lower sides); (4) the height of the door; the width; (5) the distance from the right side of the door to the right corner of the house.

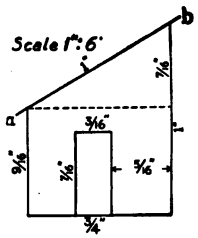


FIGURE 1

2. From the dimensions in Fig. 1, with ruler and triangles, make a drawing of the playhouse to a scale 4 times as large as that of the drawing (Fig. 1).

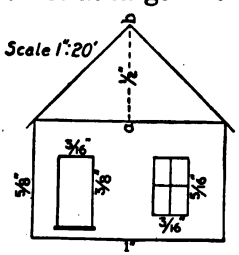


FIGURE 2

3. From the scale of the drawing (Fig. 2) find the following dimensions of the house:

(1) The width; (2) the height of the eaves; (3) the rise (*ab*); (4) the width and the height of the door; (5) the width and the height of the window.

4. Find the area of the end of the house, including the gable and excluding the areas of the door and the window.

5. Make an enlarged drawing of the house to a scale 8 times as large as the scale of the drawing (Fig. 2).

6. From the scale of Fig. 3, give the length and width of the door A; the length and the width of the panel. (Find the length by measurement).

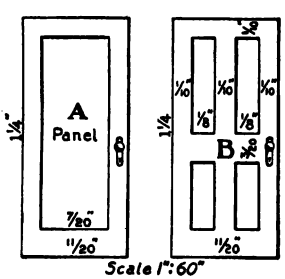


FIGURE 3

7. Similarly, give the length and the width of the door B, also the length and the width of the upper panels, and the widths of the strips enclosing the panels.

8. Make an enlarged drawing of each of the doors to a scale 5 times as large as the scale of the drawings (Fig. 3, p. 72).

9. How long are the cross-pieces of the kite represented by the drawing (Fig. 1)? How long is the binding-string of the kite? (Measure unmarked lines to the nearest $\frac{1}{16}''$.)

10. Make an enlarged drawing of the kite to a scale 8 times as large as that of Fig. 1.

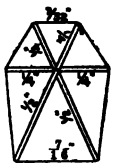


FIGURE 1

11. The cross-pieces divide the kite into 6 triangles. Call the altitude of the upper triangles in Fig. 1, $\frac{11}{50}''$, and that of the lower triangles, $\frac{11}{25}''$. How many square feet of paper are needed to cover the kite? (Allow 144 sq. in. for folding and pasting over the strings.)

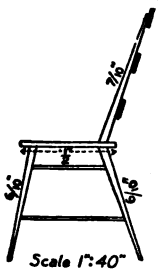


FIGURE 2

12. What is the length of each leg of the chair shown in the drawing (Fig. 2)? What is the length of the back? The depth of the seat to the extreme rear?

13. Make a drawing of the chair to a scale 5 times as large as that of Fig. 2.

14. Find from the drawing of the gate (Fig. 3) (1) the height of the high end of the gate; (2) of the lower end; (3) the length of the long brace; (4) the length of the gate; (5) the width of the strips.

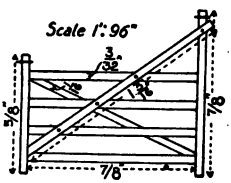


FIGURE 3

15. Make a drawing of the gate to a scale 8 times as large as that of Fig. 3, p. 73.

16. Fig. 1 is a scale drawing of the side and the end views of a large book. How long is the book? How wide? How thick?

17. Make an enlarged drawing of the book to a scale 4 times as large as that of Fig. 1.

18. From your own measurements make a drawing, to any convenient scale, of a thick object, as a block, a brick, a crayon-box, showing two views as in Fig. 1.

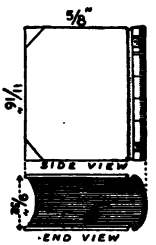


FIGURE 1

§51. Schoolhouse and Grounds.

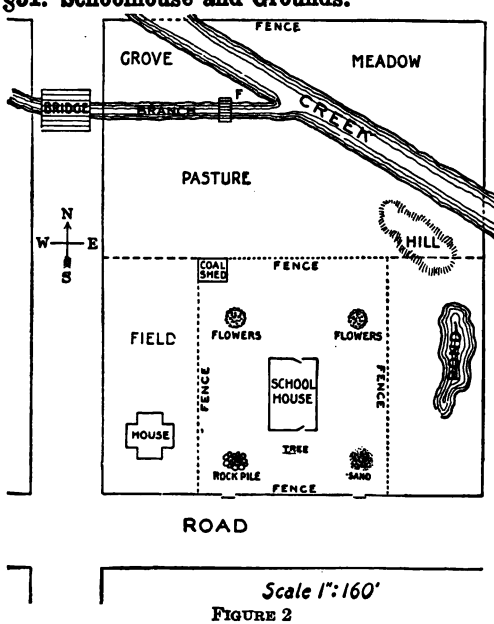


FIGURE 2

1. Using a foot rule, graduated to 16ths of an inch, and regarding the scale of the drawing (Fig. 2), find the width of the grounds; the length; the area in square rods. ($30\frac{1}{4}$ sq. yd. = 1 square rod.)

2. Find the length of the field; the width; the area in square rods.

3. Find the length and the width of the school yard; the area in square rods.

4. Find the length and the width of the schoolhouse; the area, in square yards, covered by it.

5. How far is the front door of the schoolhouse from the front fence? From the west front gate? From the sand pile? From the tree?

6. How far is it from the back door to the east flower bed? To the back fence? To the west fence? To the coal shed? To the northeast corner of the school yard? To the south end of the pond? To the hill? To the nearest point on the creek bank? To the foot bridge (F)?

7. How wide is the south road? The creek? The branch?

8. How many square rods in the south road in front of the grounds? In the crossing of the roads?

9. How many square rods in the meadow? In the grove? In the pasture?

10. How many square rods are covered by the creek and the branch together, within the fence lines?

11. How many rods of fence will be needed to enclose the grounds and to run along the lines indicated?

RATIO AND PROPORTION

§52. Ratio.

Expressions like 1 to 2, 3 to 1, 12 to 13, $\frac{1}{2}$ to $\frac{2}{3}$, a to x, are called ratios.

1. Other ways of writing the ratio 1 to 2 are 1:2, and $\frac{1}{2}$. Give the other ways of writing the ratios 3 to 1; 12 to 13; $\frac{1}{2}$ to $\frac{2}{3}$; a to x.

Read as ratios: 2:3; $\frac{4}{5}$; $\frac{5}{8}$; $\frac{6}{9}$; $\frac{10}{11}$; $\frac{a}{b}$.

The two numbers of a ratio are called terms of the ratio. The first term is the antecedent, and the second term is the consequent.

A ratio, like a fraction, is in its simplest form when the terms are the smallest possible whole numbers.

2. Express the following ratios in their simplest forms:

(1) 2:4 (3) 12:15 (5) $\frac{2}{3}:\frac{1}{3}$ (7) 120:12
(2) 8:12 (4) 75:90 (6) $\frac{7}{8}:\frac{4}{5}$ (8) $5a:25a$

The value of a ratio is the quotient of the antecedent divided by the consequent.

3. Compare the values of the following pairs of ratios:

(1) 6:3 and 4:2 (4) $\frac{3}{4}$ and $\frac{12}{16}$ (7) 7:14 and $\frac{14}{28}$
(2) 1:4 and 5:20 (5) $\frac{5}{7}$ and $\frac{25}{35}$ (8) 9:90 and $\frac{2}{20}$
(3) 2:9 and 8:36 (6) $\frac{2}{9}$ and $\frac{8}{36}$ (9) $\frac{3}{7}$ and $\frac{24}{50}$

§53. Proportion.

When two ratios are equal they make a proportion.

1. Which pairs of ratios in problem 3 above do not make a proportion?

The first, second, third, and fourth numbers of a proportion are called its first, second, third, and fourth terms. The first and fourth terms are the extremes and the second and third terms are the means of the proportion.

2. Compare the product of the means with the product of the extremes in all parts of problem 3 above that make proportions. What seems to be true of the products?

In a proportion, the product of the means equals the product of the extremes.

3. Tell which of the following are proportions, by comparing the products of the means and extremes:

(1) 2:3 =8:12 (5) $\frac{1}{2}$:8 =2:32 (9) $\frac{7}{8}=\frac{14}{16}$
(2) 4:16=1:5 (6) $\frac{1}{3}:\frac{1}{12}=\frac{1}{6}:\frac{1}{18}$ (10) $\frac{5}{7}=\frac{40}{56}$
(3) 7:21=9:27 (7) 3:9 $=\frac{1}{2}:\frac{3}{2}$ (11) $\frac{9}{10}=\frac{18}{22}$
(4) 1:40=2:81 (8) 2:$\frac{1}{2}$ =8:3 (12) $\frac{15}{25}=\frac{5}{1}$

4. The first expression of problem 3 is read "2 to 3 as (or equals) 8 to 12;" and the ninth is read "7 to 8 equals 14 to 16." Read the expressions of problem 3.

5. Find the value of a in the proportion $4:6=8:a$.

SOLUTION: As the product of the means equals the product of the extremes, $4\times a=6\times 8$, or $4a=48$; $a=\frac{1}{4}$ of $48=12$, Ans.

6. What does the letter stand for in each of these proportions?

(1) $\frac{3}{8}=\frac{12}{a}$ (2) $\frac{3}{7}=\frac{b}{35}$ (3) $\frac{8}{7}=\frac{x}{63}$ (4) $\frac{2}{11}=\frac{16}{m}$ (5) $\frac{3}{c}=\frac{21}{30}$

(6) $\frac{14}{a}=\frac{7}{12}$ (7) $\frac{a}{10}=\frac{16}{45}$ (8) $\frac{2a}{16}=\frac{9}{36}$ (9) $\frac{x}{18}=\frac{36}{324}$ (10) $\frac{3}{y}=\frac{1}{5}$

(11) $\frac{5}{z}=\frac{13}{65}$ (12) $\frac{11}{44}=\frac{d}{8}$ (13) $\frac{23}{161}=\frac{a}{14}$ (14) $\frac{32}{4}=\frac{1}{n}$ (15) $\frac{12}{3}=\frac{2}{m}$

§54. Problems.

Use pencil only when needed.

1. A boat runs 30 miles in 2 hours. How many miles will it run in 6 hours at the same rate? In 12 hours? In 20 hours?

Solutions for last part (1) As $2:20=30:x$, $\frac{2}{20}=\frac{30}{x}$; $2x=600$, or $x=300$. Ans. 300 mi.

Or (2) As 20 is 10 twos, the boat will run 10×30, or 300 mi. in 20 hours.

2. A steamboat runs 72 knots in 3 hours. How many knots will it run in 18 hours at the same rate? In 15 hours? In 30 hours? In $16\frac{1}{2}$ hours? In $1\frac{1}{2}$ hours?

3. 3 knots equal 3.45 miles. How many miles are there in 6 knots? In 24 knots? In 72 knots? In $\frac{1}{3}$ knot? In $33\frac{1}{3}$ knots? In 100 knots? In 1 knot?

4. If 10 acres of land yield 440 bushels of corn, at the same rate of yield how many acres would be needed to yield 220 bushels? 660 bushels? 1100 bushels? 4400 bushels?

5. A tree 65 feet high casts a shadow 60 feet long. How high is a tree which casts a shadow 30 feet long, at the same moment? 90 ft. long? 120 ft. long? 13 ft. long? 5 ft. long? 25 ft. long?

6. A clock ticks 7 times in 5 sec. How many times does it tick in 2 da. 6 hours?

7. If 60 A. cost \$3000, how many acres will cost \$2450?

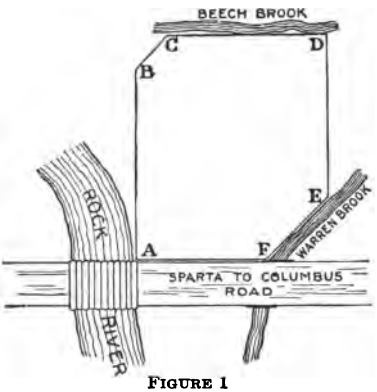


FIGURE 1

8. The line BC (Fig. 1) represents 20 rods. FE is twice as long, AF four times as long, CD and ED five times as long, and AB six times as long. Find the length of each side.

9. Find the value of x in the following:

(1) $BC : FE = AB : x$ (2) $BC : x = FE : CD$

(3) $AF : FE = x : BC$

10. When a foot-rule is held $2\frac{1}{2}$ ft. (arm's length) in front of the face as shown in Figure 2, 7 in. on the ruler seems just to cover the edge of a door 7 ft. high. If the ruler is held parallel to the edge of the door, how far is the eye from the door?

11. If the shadow cast by a 4-ft. stake is 5 ft. long, how high is a tree which casts a shadow 50 ft. long on the same day and hour?

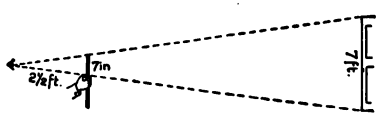


FIGURE 2

12. The shadow cast by a $3\frac{1}{2}$ ft. stake is $10\frac{1}{2}$ ft. long.

How high is a flagpole that casts a shadow 225 ft. long at the same time?

13. How high is a house that casts a shadow 120 ft. long at the moment when a stake 6 ft. high casts a shadow 20 ft. long?

14. A road runs 2 miles south, then 1 mile west, then $\frac{3}{4}$ mi. south, then $1\frac{1}{2}$ mi. southeast, and then $2\frac{1}{2}$ mi. south. In a scale drawing of the road the two-mile part is represented by a line 18 in. long. How long a line will represent each of the other parts of the road in the same drawing?

15. What is the scale of the drawing just mentioned?

16. What is the scale of a drawing in which a distance of 20 ft. is represented by a line $2\frac{2}{5}$ in. long? By a line $\frac{4}{5}$ in. long? By a line $2\frac{1}{2}$ ft. long?

17. The food parts of beef are water, fat, and protein. In 1 lb. of loin steak there are $8\frac{2}{5}$ oz. of water, $2\frac{4}{5}$ oz. fat, $2\frac{3}{5}$ oz. protein, and $2\frac{1}{5}$ oz. of waste material. How much of each of these parts are there in 3 lb. of loin steak? In $5\frac{1}{2}$ lb.? In a 12-pound loin steak?

18. In 4 lb. sirloin steak there are $34\frac{2}{5}$ oz. water, $10\frac{2}{5}$ oz. fat, $10\frac{2}{3}$ oz. protein, and $8\frac{8}{15}$ oz. waste matter. How much of each is there in an 8-lb. sirloin steak? In 24 lb. sirloin steak? In 30 pounds?

19. In a 10-lb. porterhouse steak there are $5\frac{1}{4}$ lb. water, $1\frac{3}{4}$ lb. fat, $1\frac{2}{3}$ lb. protein, and $1\frac{1}{3}$ lb. waste. How much of each is there in a 30-lb. porterhouse steak? In 15 lb.? In 18 lb.? In 48 pounds?

20. In 15 lb. of beef ribs there are $5\frac{1}{2}$ lb. water, $3\frac{1}{5}$ lb. fat, $2\frac{11}{20}$ lb. protein, $3\frac{3}{4}$ lb. waste. How much of each is there in 10 lb. beef ribs? In 25 lb.? In 16 lb.? In 75 pounds?

DECIMAL FRACTIONS

§55. To Reduce a Decimal to a Common Fraction.

1. How may $2\frac{1}{2}$ tenths be expressed as a decimal? $62\frac{1}{2}$ hundredths? $33\frac{1}{3}$ hundredths? 14 and $666\frac{2}{3}$ thousandths?

Pure decimals: .25; .038; .508; .0173.
Mixed decimals: $0.3\frac{1}{8}$; 4.675; $6.66\frac{2}{3}$; $4.444\frac{4}{9}$.

2. Read and give the meaning of these mixed decimals:
$.12\frac{1}{2}$ $3.33\frac{1}{3}$ $18.66\frac{2}{3}$ $2.1\frac{1}{2}$ $.0\frac{3}{4}$ $.04\frac{2}{9}$ $.00\frac{3}{5}$ $36.000\frac{5}{6}$

To reduce such an expression as $3.44\frac{5}{7}$ to a mixed number proceed thus:

$$3.44\tfrac{5}{7} = \frac{344\frac{5}{7}}{100} = \frac{\frac{2413}{7}}{100} = \frac{2413}{700} = 3\frac{313}{700}.$$

Or thus: $3.44\frac{5}{7} = 3\frac{44\frac{5}{7}}{100} = 3\frac{\frac{313}{7}}{100} = 3\frac{\frac{313}{7}\times 7}{100\times 7} = 3\frac{313}{700}.$

3. Reduce the decimals of problem 2 to mixed numbers, or to common fractions.

4. Make a rule for expressing any mixed decimal as a common fraction, or a mixed number.

Any decimal may be expressed as a common fraction in its lowest terms, or as a mixed number, by dropping the decimal point, writing the denominator, and reducing the resulting common fraction to its lowest terms.

5. Express as common fractions in their lowest terms:
.1 .24 1.75 $2.33\frac{1}{3}$ 3.175 1.875 5.218 $8.88\frac{8}{9}$

ADDITION AND SUBTRACTION

§56. Addition.

1. The following numbers denote the monthly rain or snowfall in a certain city for 1902: .66, 1.53, 4.16, 2.26, 5.08, 6.45, 4.25, 1.44, 4.83, 1.45, 2.03, 1.90.

What was the total rain or snowfall for the year?

2. Without rewriting the numbers, find the total yearly rain or snowfall for the years 1891–1901 inclusive, from these recorded data:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct	Nov.	Dec.	Total
1.99	1.95	2.13	2.14	2.09	2.42	2.47	4.52	.32	.36	3.83	1.32	.....
1.99	1.57	2.21	2.17	6.77	10.58	2.23	1.85	1.34	1.54	2.68	1.63	.....
2.08	2.44	1.69	4.16	1.93	3.59	3.08	.18	1.98	1.75	2.45	2.14	.....
1.55	2.13	2.66	2.65	3.35	1.96	.60	.60	8.28	.85	1.18	1.66	.....
2.15	1.60	1.32	.36	1.99	1.79	2.43	6.49	1.89	.51	5.60	6.76	.....
1.12	3.48	1.26	2.79	4.16	2.82	3.61	3.52	6.70	1.36	2.16	.16	.....
4.53	2.22	3.56	2.23	.84	3.60	1.47	1.70	.84	.18	3.06	1 62	.....
3.54	2.59	4.60	.76	2.23	5.30	1.94	3.03	3.16	3.26	2.25	1.11	.....
.58	1.60	2.11	.14	4.35	2.71	6.66	.91	2.39	2.09	1 14	6.81	.....
1.21	3.52	1.58	1.02	3.59	2.06	4.64	4.24	1.56	1.35	3.30	58	.....
1.15	2.05	3.38	.33	2.18	2.42	4.25	2.00	2.92	1.29	.85	1.70	.....

3. Foot and average the vertical columns and tell what the footings and averages mean.

4. Compare the total of the footings of problem 3, with the total of problem 2.

§57. Subtraction.

COMPOSITION OF NUTS (IN PERCENTAGES)

Nuts	Waste	Eatable Portion					Calories of heat per lb.
		Water	Protein	Fat	Carbo-hydrates	Ash	
Almonds	64.8	4.8	21.0	54.9	17.3	2.0	3030
Filberts	52.1	3.7	15.6	65.3	13.0	2.4	3432
Pecans	53.2	3.0	11.0	71.2	13.3	1.5	3633
Walnuts (English)	58.0	2.8	16.7	64.4	14.8	1.3	3305
Cocoanuts (shredded)	....	3.5	6.3	57.3	31.6	1.3	3125
Cocoanuts	48.8	14.1	5.7	50.6	27.9	1.7	2986
Roasted Peanuts	32.6	1.6	30.5	49.2	16.2	2.5	3177
Peanuts, raw	24.5	9.2	25.8	38.6	24 4	2.0	2560
Peanut Butter	....	2.0	29.3	46.6	17.1	5.0	2830
Pistachio	....	4.2	22.6	54.5	15.6	3.1	3010
Brazil nuts	49.6	5.3	17.0	66.8	7.0	3.9	3329
Hickory nuts	62.2	3.7	15.4	67.4	11.4	2.1	3495
Chestnuts, fresh	16.0	45.0	6.2	5.4	42.1	1.3	1125
Butternuts	86.4	4.5	27.9	61.2	3.4	3.0	3371

1. Find the total percentage of the eatable portion of

almonds; of filberts; of pecans; of walnuts; of cocoanuts (shredded).

2. Find the difference in percent between the waste and what is not waste of almonds; of filberts; of peanuts; of walnuts; of butternuts.

3. Find the difference in percents of fat and of water with the first 6 kinds of nuts. With the last 4 kinds.

4. Protein is muscle-food. Find the difference between the percentages of muscle-food in almonds and filberts; in almonds and walnuts; in peanut butter and raw peanuts; in roasted and raw peanuts.

5. The ash is bone-food. Find the differences in percent between the muscle-food and the bone-food of roasted peanuts; of hickory nuts; of pecans.

6. Carbohydrates and fats furnish heat and power. Find the difference between the combined percentages of fat and carbohydrate in pecans and almonds; cocoanuts, shredded and unshredded.

7. Make and solve other problems on the table.

MULTIPLICATION AND DIVISION

§58. Multiplication.

1. From the product $436 \times 328 = 143{,}008$, write the following products:

(1) $43.6 \times 328 = ?$ (3) $4.36 \times 3.28 = ?$ (5) $.436 \times 328 = ?$

(2) $4.36 \times 32.8 = ?$ (4) $436 \times 32.8 = ?$ (6) $.0436 \times .0328 = ?$

2. Make a rule for finding the number of decimal places in the product of decimal numbers.

With decimal numbers multiply as with whole numbers, and point off as many decimal places in the product as there are in both the multiplicand and the multiplier.

3. Find the products for the following:

(1) $426 \times 32.8 = ?$ (4) $654.1 \times 23.6 = ?$

(2) $42.6 \times .328 = ?$ (5) $6.541 \times 236 = ?$

(3) $4.26 \times .0328 = ?$ (6) $.6541 \times .236 = ?$

4. To draw a load in a road wagon at a slow walk over hard, level country roads a horizontal pull of .075 of the total weight of the wagon and load is required. What horizontal pulls will be required to draw the following:

	Weight of Wagon	Weight of Load	Total Weight	Horizontal Pull in Lb.
(1)	1068 lb.	2168 lb.	.....	.....
(2)	969.5 "	2408.3 "	.....	.....
(3)	1580.75 "	3675.6 "	.....	.....

5. Over fresh earth .125 of the total load as a horizontal pull will draw the load on a road wagon. Fill out the vacant columns, for these conditions:

	Weight of Wagon	Weight of Load	Total Weight	Horizontal Pull in Lb.
(1)	980 lb.	1260 lb.	.....	.....
(2)	940.8 "	768.78 "	.....	.....
(3)	1164 "	3890.85 "	.....	.....
(4)	1675 "	4060.75 "	.....	.....

§59. Division.

1. The 36 members of a society were assessed (made to pay) equally to meet a debt of \$1341 against the society. What was the amount assessed against each member?

Solution:

$$
\begin{array}{r}
37.25 \\
36\overline{)1341.00} \\
\underline{108} \\
261 \\
\underline{252} \\
9.0 \\
\underline{7.2} \\
1.80 \\
\underline{1.80}
\end{array}
$$

By writing zeros after the decimal point the quotient may be carried out in a decimal form.

What effect does writing zeros after the point have on a number?

Another way to point the quotient is: Imagine $36\overline{)1341}$ changed to $3.6\overline{)134.1}$. Then, $\frac{1}{3}$ of 134 is about 44. Point 3725 to get as nearly 44 as possible by pointing alone, thus: \$37.25.

The second way is to divide as with whole numbers, then imagine the point moved to the right (or left) over the same number of digits in *both* dividend and divisor until only *one* digit stands to the left of the point in the divisor. Then disregard the decimal parts and so point the quotient as to get about $\frac{1}{2}$, $\frac{1}{3}$, etc., to $\frac{1}{9}$ of the modified dividend.

2. A company divided 10138.8 acres of land into 426 equal parts. How many acres were in each part?

```
      23.8
426)10138.8
    852
   1618
   1278
    3408
    3408
```

Dividing as with whole numbers, the digits of the quotient are 238. Imagine the point moved in both 10138.8 and 426, to make 101.388 ÷ 4.26. Then $\frac{1}{4}$ of 101 is about 25. Point 238 to get as near 25 as possible. Thus: 23.8. Ans. 23.8 acres.

3. Find the following quotients as just explained:

(1) 2152.96 ÷ 23.2 = ? (3) 40195.6 ÷ 3170 = ?

(2) 4596.84 ÷ 226 = ? (4) 692.224 ÷ 41.6 = ?

4. The number of states and territories and the total areas in square miles of the land surface of the principal geographic divisions of continental United States are given in the following table. Find the average land surface of a state to the nearest .001,* for each division and for the whole United States.

Division	Number	Land Surface	Average
North Atlantic	9	168,665	
South Atlantic	9	282,535	
North Central	12	765,855	
South Central	8	620,290	
Western	11	1,187,535	
Continental United States			

**If the next decimal place after the last one required is less than 5, write the given quotient as the required decimal. If it is greater than 5, add 1 to the last figure of the decimal required. This rule applies to all cases of division of decimals.*

5. The 10 loads given in pounds in column 2 of the following table required the number of pounds of force given in the third column, to draw them on a common wagon over a good, level country road. For example, 1400 lb. required 98 lb. to pull it; 1616 lb. required 112 lb., and so on. Divide each load by the number of pounds of force needed to draw it, and, on a separate sheet, write the quotients to two decimal places in a column.

Experiment	Load	Pull	Ratio
1	1400	98	.....
2	1616	112	.....
3	1825	126	.....
4	2236	155	.....
5	2440	170	.....
6	2650	185	.....
7	2863	198	.....

§60. Ratio of Circumference of Circle to Diameter.

1. The distance around the rung of a chair was measured and found to be 3.625 in.; the diameter was found to be 1.153 in. Divide the distance around the rung by the diameter and find the quotient to the nearest .001.

The distance around a circle is the circumference of a circle.

2. The circumference of a circular rod, 1.875 in. in diameter, was measured and found to be 5.884 in. Find the ratio to the nearest .001, of the circumference to the diameter.

Object	Circumference	Diameter	Ratio
Ink bottle.....	5.5 in.	1.75 in.	
Tin box.....	6.154 "	1.953 "	
Globe of lamp.....	26.538 "	8.444 "	
Terrestrial globe ..	57.080 "	18.062 "	
Barrel-head.....	71.458 "	22.750 "	
Iron ring.....	18.125 "	5.675 "	
Average.....	.....	.....	

3. Measure the diameters and the circumferences of any circles in your schoolroom and find to the nearest .001, the ratio of their circumferences to their diameters. If no circles are at hand use the measures of above table.

4. Measure the circumferences and the diameters of the following objects and find the ratios to the nearest .001. If you are unable to make your own measures use those of the table:

OBJECT	CIRCUMFERENCE	DIAMETER	RATIO
Bicycle wheel.....	81.177 in.	26.025 in.	
Bicycle wheel.....	88.055 "	28.012 "	
Front carriage wheel.....	151.189 "	48.125 "	
Rear " "	176.333 "	56.125 "	
Locomotive driver.....	174.762 "	55.625 "	
Average.....	.....	.....	

5. It is proved in Geometry that the ratio of the circumference of any circle to its diameter is about $3\frac{1}{7}$, or more accurately 3.1416. If then, the diameter of a circle is known, how may the circumference be found without measurement?

6. Find the average of all the values of the ratios found in problems 1 to 4 and compare this last average with $3\frac{1}{7}$; with 3.1416. What is the difference in each case?

NOTE: For most practical purposes this ratio, denoted by the Greek letter π, and called *pi*, may be taken as $3\frac{1}{7}$. For greater accuracy use $\pi = 3.1416$.

§61. Specific Gravity.

The specific gravity of any solid or liquid substance is the ratio of its weight to the weight of an equal bulk of water. The weight of a cubic foot of water is 62.5 pounds.

1. The following table contains the weight in pounds of 1 cu. ft. of the substances mentioned. Find to the nearest .001 the specific gravities of these substances:

Metal	Weight of 1 Cu. Ft.	Specific Gravity	Wood	Weight of 1 Cu. Ft.	Specific Gravity	Liquid	Weight of 1 Cu. Ft.	Specific Gravity
Aluminum	166.5		Cork	15.00		Alcohol	50.0	
Zinc	436.5		Spruce	31.25		Turpentine	54.4	
Cast iron	450.0		Pine (yellow)	34.60		Petroleum	55.7	
Tin	458.3		Cedar	35.06		Olive oil	57.0	
Wr't iron	480.0		Pine (white)	28.00		Linseed oil	59.5	
Steel	490.0		Walnut	41.90		Sea water	64.1	
Brass	523.8		Maple	46.88		Milk	64.5	
Copper	552.0		Ash	52.80		Acetic acid	66.5	
Silver	655.1		Beech	53.25		Muriatic acid	75.0	
Lead	709.4		Oak	65.00		Nitric acid	95.0	
Gold	1200.9		Ebony	76.00		Sulphuric acid	115.1	
Platinum	1347.0		Lignum vitæ	83.30		Mercury	850.0	

2. The following familiar substances may be compared with water as to weight. Find their specific gravities to the nearest .01:

Substance	Weight, in Pounds of 1 Cubic Foot	Specific Gravity
Glass (average)	175.8	
Chalk	174.5	
Marble	169.2	
Granite	166.4	
Stone (common)	158.2	
Salt "	133.4	
Soil "	124.5	
Clay	121.8	
Brick	118.3	
Sand	113.9	

§62. Original Problems.

Make and solve problems based on the following facts:

1. A man walks about 3.5 mi. per hour. It is 85 mi. from Chicago to Milwaukee.

2. A horse trots about 7.5 mi. per hour.

3. A horse runs about 18 mi. per hour for short distances.

4. A steamboat runs 18 mi. per hour.

5. A slow river flows 3 mi. per hour.

6. A rapid river flows 7 mi. per hour.

7. A crow flies 25 mi. per hour; a falcon, 75 mi.; a wild duck, 90 mi.; a sparrow, 92 mi.; a hawk, 150 mi., and a carrier pigeon, 80 mi. per hour.

8. Sound travels through air 1134 ft. per second; through water, 5000 ft. per second; and through iron or steel, 17,000 ft. per second.

9. A rifle ball travels 1460 ft. per second at starting; and a 20-lb. cannon ball, 16,000 ft. per second at starting.

10. Light travels 186,650 mi. per second; electricity, 288,000 mi. per second. The sun is about 93,000,000 mi. from the earth; the moon, about 240,000 miles.

11. One horse-power raises 33,000 lb. through a height of 1 ft. in 1 minute.

12. The equatorial diameter of the earth is 7925.6 miles, the polar diameter, 7899.1 miles.

§63. Area of a Circle.

1. Draw a circle and diameter AB (see Fig. 1).

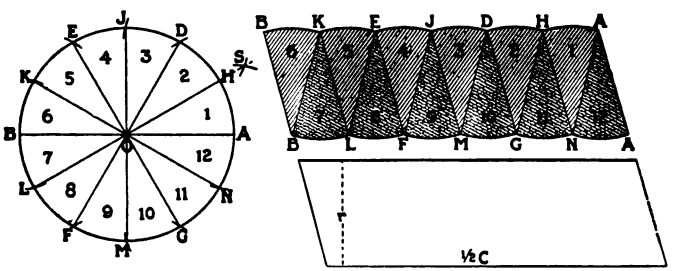


FIGURE 1

With an opening of the compasses equal to the radius of the circle and with A as center draw short arcs at D and G, also, with same radius and with B as center, draw short arcs at E and F. Draw radii OD, OE, OF, and OG.

With compasses opened wider than the distance, DH, and with A as a center, draw one of the arcs at S. With the same opening, draw the other arc with A as a center.

Connect SO. SO divides the angle, AOD, and the arc, AD, into two equal parts.

With an opening of the compasses equal to the distance between D and H and with center D draw an arc at J; with center B draw arcs at K and L; also, with center G draw arcs at M and N. Draw radii OJ, OK, OL, OM, and ON.

Cut the circle into the twelve parts (sectors) thus formed and place these sectors as shown in the second part of the figure. How long is the base AB of the *approximate* parallelogram thus formed? How wide is the approximate parallelogram?

2. If each of the 12 sectors were split into halves and the 24 sectors fitted together how long and how wide would the new *approximate* parallelogram be? What would be its area?

3. Having the circumference and the radius of a circle how can you find the area of the circle?

4. Having the radius of any circle how can you find the area of the circle (See pp. 85 and 88)?

Let r be the length of the radius, c the circumference, and A, the area of a circle.

Work problems 5 to 7 in two ways: (1) using both r and c, and (2) using r and π $(= 3\frac{1}{7})$ and the equation, $A = \pi \times r^2$.

5. Find the areas, A, of these circles:

(1) $r = 12.5'$, $c = 78.57'$ (3) $r = 20''$, $c = 125.714''$

(2) $r = 6.25'$, $c = 39.29'$ (4) $r = 60''$, $c = 377.143''$

6. The diameter of a drum-head is $2.5'$ and the circumference is $7.857'$. How many square inches of skin are in the two heads?

7. The diameter of a circular window is $18.5''$ and its circumference is $58.143''$. What is its area?

NOTE: In the following problems π is taken as 3.1416. Results should be correct in the nearest .0001.

8. The wind is blowing squarely against a circular signboard, whose diameter is 32.75 ft., with a pressure of 25.5 lb. to the square foot. Find the total wind pressure against the board.

9. The circumference of a cylindrical chair rung is 5.5″; find the diameter and area of the right section.

NOTE: A right section is the section that would be made by sawing the rung square across.

10. Steam passes from the boiler of an engine through the passage, P, into the cylinder, and pushes against the circular piston, C, with a pressure of 65 lb. to the square inch. If the diameter of the circular head, C, is 12.5″, what is the total pressure against the left side of the piston?

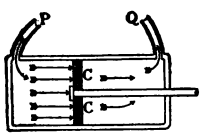


FIGURE 1

11. Answer similar questions for pistons having diameters with steam pressures per square inch as indicated:

	DIAMETER OF PISTON	PRESSURE PER SQUARE INCH		DIAMETER OF PISTON	PRESSURE FOR SQUARE INCH
(1)	10.85″	86	(4)	16.45″	175
(2)	22.35″	120	(5)	20.75″	125
(3)	14.85″	180	(6)	15.85″	160

12. Find the area, A, of a circle whose radius is r ft. long and whose circumference is c ft. long.

13. Find the circumference, c, of a circle whose radius is r rods long. Find the area.

14. A cow is tied to the corner of a corn crib, 18′ × 18′, with a rope 18′ long. Over how many square feet can she graze?

§64. Exercises for Practice.

1. Add the following:

(1)	(2)	(3)	(4)	(5)	(6)
37.8	248.001	.96	7.68	862.01	328.
8.65	16.25	3.004	86.043	2.8106	.032
12.08	9.018	10.	190.608	843.7	7.001½
.75	24.	7.	9.	1200.	29.080¾
16.	3.	.206	.9	.16	109.
14.01	.03	200.	8.3	9.	9000.
.6	.4	860.03	60.5	.0068	.08
10.	17.003	.2	3.7	.0309	.0007

2. Subtract the following:

(1)	(2)	(3)	(4)	(5)	(6)
84.96	98.01	100.01	106.038	67.083	800.00
6.38	1.97	87.08	69.879	29.187	98.68

(7)	(8)	(9)	(10)	(11)	(12)
810.02	3.786	26.	3.	68.	1.0101
86.983	.989	.748	2.983	9.8789	.9098

3. Find the following products:

(1) 89.3×42.1
(2) 68.01×9.82
(3) 86.3×8.761
(4) 100.001×86.8
(5) 76.9×.93
(6) 7.69×.093
(7) 8.08×.008
(8) 16.8×.072
(9) 9.001×1.801
(10) 1.68×7.2
(11) 168×.072
(12) 3.1416×25.18
(13) 31.416×2.518
(14) 314.16×251.8
(15) 785.4×2.781
(16) .7854×.2781
(17) 3.1416×16.84
(18) 314.16×.1684

4. Find the following quotients:

(1) 120.716÷26.82
(2) 123.1498÷26.82
(3) 276.496÷3.142
(4) 268.928÷8.4
(5) 109.624÷3.86
(6) 548.12÷14.2
(7) 1774.08÷.384
(8) 8.8704÷192
(9) 443.52÷.96
(10) 14.784÷.032
(11) .3696÷.0008
(12) 9.24÷.00004

5. Find the following quotients to 3 decimal places:

(1) 8÷.3
(2) 631÷.072
(3) 9÷.007
(4) 11÷.0009
(5) 12÷36
(6) 1÷3
(7) 2÷3
(8) 4÷9
(9) 44÷.09
(10) 13÷3
(11) 7.7÷9
(12) 123÷9.99
(13) 4÷.011
(14) 2.5÷.033
(15) 260÷3.3
(16) 2.6÷33
(17) 11.5÷3.33
(18) 10.4÷.909
(19) 1.74÷9.09
(20) .77÷1.21
(21) 1.111÷.099
(22) 60.6÷1.818
(23) .625÷3.36
(24) .1÷.09

COMPOUND DENOMINATE NUMBERS

§65. Definitions.

A denominate number is a number whose unit is concrete; as, 13 mi., 8 hr., 40 A., $125. etc.

A concrete unit is a unit having a specific name; as 1 yd., 1 lb., $1, 1 hat, 1 horse, etc.

A compound denominate number is a number expressed in two or more units of the same kind; as, 11 hr., 25 min. 15 sec.; 12 gal. 3 qt. 1 pt. 3 gills.

TABLES OF MEASURES

NOTE: Read carefully the tables that follow and fix them in mind by solving the problems beginning at §74, p. 99.

§66. Measures of Value.

The *standards of value* of the United States and of some of the European countries are here given with both their rough and their accurate equivalents in U. S. money.

COUNTRY	STANDARD	SYMBOL	ROUGH EQUIVALENT	ACCURATE EQUIVALENT
United States	Dollar	$	$1.	$1.
Great Britain	Pound (Sterling)	£	$5.00	$4.8665
Germany	Mark (Reichsmark)	M.	$ .25	$ .2385
France	Franc	F.	$ .20	$ .193
Russia	Ruble	R.	$ .75	$ .772
Austria-Hungary	Crown, or Filler	C. or F.	$ .20	$ .203
Italy	Lira	L.	$ .20	$ .193

TABLE OF U. S. MONEY

10	mills (m.)	= 1 cent (¢)
10	cents	= 1 dime (d.)
10	dimes	= 1 dollar
10	dollars	= 1 eagle
5	dollars	= ½ eagle
2½	dollars	= ¼ eagle
20	dollars	= 1 double eagle

The coins of the United States are bronze, nickel, silver, and gold.

TABLE OF ENGLISH MONEY

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
21 shillings	= 1 guinea

TABLE OF MONETARY UNITS OF OTHER NATIONS

Germany,	1 mark (M.)	= 100 pfennige (pf.)
France,	1 franc (fr.)	= 100 centimes (c.)
Russia,	1 ruble (r.)	= 100 copecks (c.)
Austria-Hungary,	1 crown, or filler	= 100 heller.
Italy,	1 lira	= 100 centessimi.

§67. Measures of Weight.

Three systems of weight units are used in the United States, viz.: troy weight, avoirdupois weight, and apothecaries' weight.

Troy weight is used in weighing gold, silver, and jewels. Avoirdupois weight is used for weighing all ordinary articles, and apothecaries' weight is used by druggists in mixing medicines.

The standard of weight in the U. S. is the *troy pound.* The grain is the same in all three systems; the pound is the same in troy and in apothecaries' weight; but different in avoirdupois.

TABLE OF TROY WEIGHT		TABLE OF EQUIVALENTS	
24 grains (gr.)	= 1 pennyweight (dwt.)	5760 gr.	= 1 lb.
20 pennyweights	= 1 ounce (oz.)	240 dwt.	
12 ounces	= 1 pound (lb.)	12 oz.	

TABLE OF AVOIRDUPOIS WEIGHT

7000 grains (gr.)	= 1 pound (lb.)
16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton (L. T.)

TABLE OF APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (lb.)

§68. Measures of Length, or Distance (Linear Measure).

The standard unit for measures of length, or distance, is the *yard.*

TABLE OF COMMON LINEAR MEASURE		TABLE OF EQUIVALENTS	
12 inches (in.)	= 1 foot (ft.)	63360 in.	= 1 mi.
3 feet	= 1 yard (yd.)	5280 ft.	
5½ yards, or 16½ ft.	= 1 rod (rd.)	1760 yd.	
320 rods	= 1 statute mile (mi.)	320 rd.	

TABLE OF SURVEYORS' LINEAR MEASURE

7.92 inches = 1 link (li.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)
1.15+common, or statute, miles = 1 geographical, or nautical, mile.

§69. Measures of Surface.

The unit upon which surface measure is based is the *square yard,* which is a square each of whose sides equals 1 yard.

TABLE OF COMMON SURFACE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)

TABLE OF SURVEYORS' SURFACE MEASURE

625 square links (sq. li.) = 1 square rod (sq. rd.)
16 square rods = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres (a section) = 1 square mile (sq. mi.)
36 square miles = 1 township (Tp.)

TABLE OF EQUIVALENTS

3686400 sq. rd.
230400 sq. ch.
36 sq. mi. } = 1 Tp.

§70. Measures of Volume.

TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

TABLE OF EQUIVALENTS

46656 cu. in. } = 1 cu. yd.
27 cu. ft. }

A *perch* of stone is a square-cornered mass, $1' \times 1\frac{1}{2}' \times 16\frac{1}{2}'$, or $24\frac{3}{4}$ cubic feet.

Fire wood is measured by the *cord*. A cord of wood is a straight pile, $4' \times 4' \times 8'$, or 128 cubic feet. A *cord foot* is a straight pile of wood, $4' \times 4' \times 1'$. How many cubic feet are there in a cord foot?

§71. Measures of Capacity.

Liquid measure is used in measuring liquids, the capacity of cisterns, tanks, etc.

TABLE OF LIQUID MEASURE		TABLE OF EQUIVALENTS	
4 gills (gi.)	= 1 pint (pt.)	32 gi. }	
2 pints	= 1 quart (qt.)	8 pt. }	= 1 gal.
4 quarts	= 1 gallon (gal.)	4 qt. }	

NOTE: A liquid gallon contains 231 cubic inches.

Dry measure is used in measuring grain, fruit, vegetables, etc. The standard of dry measure is the *Winchester bushel*, which is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep, containing 2150.42 cubic inches.

TABLE OF DRY MEASURE		TABLE OF EQUIVALENTS	
2 pints (pt.)	= 1 quart (qt.)	64 pt. }	
8 quarts	= 1 peck (pk.)	32 qt. }	= 1 bu.
4 pecks	= 1 bushel (bu.)	4 pk. }	

NOTE: A dry gallon = 4 qt. or $\frac{1}{2}$ pk., or 268.8 cubic inches.

§72. Measures of Time.

The standard unit for measuring time is the *mean solar day*. The mean solar day is the average time interval from the instant when the sun crosses the meridian of a place (noon) to the next instant of crossing the meridian (the next noon).

TABLE OF TIME MEASURE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days	= 1 calendar month (mo.)
12 calendar months	= 1 calendar year (yr.)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

NOTE: One mean solar year = 365 da. 5 hr. 48 min. 46 sec. = 365¼ da. (nearly).

TABLE OF EQUIVALENTS

31536000 sec.
525600 min.
8760 hr.
365 da.
52 wk. 1 da.
12 mo.
} = 1 common yr.

According to the *Julian calendar*, adopted by Julius Cæsar (whence the name), every year whose date number is divisible by 4 (as 1896, 1904) must contain 366 da. and all other years contain 365 days. Years containing 366 da. are called *leap years;* those containing 365 da. are called *common years.*

According to the *Gregorian calendar*, which is now used by nearly all civilized nations, every year whose date number is divisible by 4 is a leap year, *unless the date number ends in two zeros* (as 1600, 1900), *in which case the date number must be divisible by* 400 *to be a leap year.*

The extra day of the leap year is added to February, giving this month 29 da. in leap years.

§73. Measurement by Counting.

TABLE FOR COUNTING

12 things	= 1 dozen (doz.)
20 things	= 1 score
12 dozen	= 1 gross
12 gross	= 1 great gross

TABLE FOR MEASURING PAPER

24 sheets	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

The operations to be performed in compound denominate numbers are the following:

(1) To change from higher to lower units, or denominations;

(2) To change from lower to higher units, or denominations;

(3) To add compound denominate numbers;

(4) To subtract such numbers;

(5) To multiply such numbers;

(6) To divide such numbers.

These processes will now be illustrated in order.

1. Express 16 bu. 3 pk. 3 qt. 1 pt. as pints.

CONVENIENT FORM

16 bu. 3 pk. 3 qt. 1 pt.
4 = No. pk. in 1 bu.

64 pk. = 16 bu.
3 pk.

67 pk. = 16 bu. 3 pk.
8 = No. qt. in 1 pk.

536 qt. = 67 pk.
3 qt.

539 qt. = 67 pk. 3 qt.
2 = No. pt. in 1 qt.

1078 pt. = 539 qt.
1 pt.

Ans. 1079 pt. = 16 bu. 3 pk. 3 qt. 1 pt.

2. Express (1) 1079 pt. in higher units; (2) 2 pk. 3 qt. 1 pt. in bushels.

CONVENIENT FORM

(1) 2) 1079 pt.
8) 539 qt. + 1 pt. remaining
4) 67 pk. + 3 qt. remaining
16 bu. + 3 pk. remaining

Ans. 16 bu. 3 pk. 3 qt. 1 pt.

(2)

3 qt. 1 pt. = 3.5 qt. = $\frac{3.5}{8}$ pk. = .4375 pk.

2 pk. 3 qt. 1 pt. = 2.4375 pk. = $\frac{2.4375}{4}$ bu. = .609375 bu. *Ans.*

3. The three sides of a triangular grass plot were: 8 yd. 2 ft. 10 in.; 12 yd. 1 ft. 9 in., and 9 yd. 2 ft. 7 in. How far is it around the plot?

CONVENIENT FORM

8 yd.	2 ft.	10 in.
12	1	9
9	2	7
Ans. 31 yd.	1 ft.	2 in.

EXPLANATION: First, adding the inches, we obtain 26 in. = 2 ft. 2 in. Write the 2 in. in "inches" column, and add the 2 ft. to the numbers in the "feet" column, giving 7 ft. = 2 yd. 1 ft. Write 1 ft. in "feet" column and add 2 yd. to the numbers in "yards" column, giving 31 yd.

4. From a vessel containing 25 gal. 2 qt. 1 pt. 2 gi. of oil, 8 gal. 3 qt. 1 pt. 3 gi. were drawn out. How much oil remained in the vessel?

CONVENIENT FORM

25 gal.	2 qt.	1 pt.	2 gi.
8 gal.	3 qt.	1 pt.	3 gi.
Ans. 16 gal.	2 qt.	1 pt.	3 gi.

EXPLANATION: 3 gi. can not be taken from 2 gi. But the 1 pt. of the minuend equals 4 gi. and this added to 2 gi. gives 6 gi. 6 gi. − 3 gi. = 3 gi. 1 pt. from 0 pt. can not be taken. But 1 qt. is taken from 2 qt., and changed to pints, giving 2 pt. 2 pt. − 1 pt. = 1 pt. 1 gal. = 4 qt. 4 qt. + 1 qt. = 5 qt. 5 qt. − 3 qt. = 2 qt. Finally, 24 gal. − 8 gal. = 16 gal. Prove the work by adding the remainder to the subtrahend and comparing the result with the minuend.

5. A man built an average of 45 ft. 8 in. of fence a day for 16 days. How much fence did he build in 16 days?

CONVENIENT FORM

45 ft. 8 in.

16

10 ft. 8 in. = 8 in. × 16 = 128 in. = 10 ft. 8 in.

720 = 45 ft. × 16 = 720 ft.

Ans. 730 ft. 8 in.

6. An iron rod, 4 yd. 2 ft. 8 in. long, was cut into 5 equal pieces. How long was each piece?

CONVENIENT FORM

5)4 yd. 2 ft. 8 in.

Ans. 2 ft. $11\frac{1}{5}$ in.

EXPLANATION: 4 yd. = 12 ft. 12 ft. + 2 ft. = 14 ft. 14 ft. ÷ 5 = 2, with a remainder of 4 ft. 4 ft. = 48 in. 48 in. + 8 in. = 56 in.

§74. Exercises on Denominate Number Tables.

STANDARDS OF VALUE

1. How many cents are there in 23 dimes? In \$23.00? In 23 eagles?

2. How many dollars are there in 248 cents? In 248 dimes? In 248 eagles?

3. How many half-eagles are there in \$245.00? In 240 dimes?

4. About what is the value of \$50.00 in pounds sterling In marks? In francs? In rubles? In crowns? In lira?

5. What are the more accurate equivalents of \$100.00 in these same units?

6. I owe a debt of 32 M. 50 pf. (pfennig = $\frac{1}{100}$ of a mark). How much U. S. money will it take to pay this debt if I have to pay 24.5 cents for a mark?

7. If I have to pay \$4.94 for a pound sterling, how much U. S. money will it take to pay a debt in London of £65, 8s.?

WEIGHT

8. How many grains (troy) are there in

(1) 12 dwt.? (3) $8\frac{1}{2}$ oz.? (5) 9 oz. 12 dwt.?

(2) 3 oz.? (4) 8 oz. 10 dwt.? (6) 4 lb. 8 oz.?

9. A carat (of weight) usually means 32 troy grains. How many carats are there in a diamond weighing 13 dwt. 8 gr.?

10. How many carats weight are there in a diamond weighing 2 troy ounces?

11. The avoirdupois table is used for weighing iron, coal, and all coarser (less costly) articles. How many grains heavier is a pound of coal than a pound of silver? How many troy ounces?

12. Express 8 lb. (av.) as ounces. Express 8 lb. (troy) as ounces.

13. How many ounces are there in 24 lb. of iron? In 48 lb. 6 oz. of iron?

14. How many ounces are there in 3 cwt.? In 1 cwt. 40 lb.? In 5 cwt. 8 oz.?

15. Express as avoirdupois ounces the following avoirdupois weights:

(1) 3 lb. 8 oz. (3) 86 lb. 15 oz. (5) 8 cwt. 13 lb.
(2) 18 lb. 12 oz. (4) 1 cwt. 8 lb. (6) $9\frac{1}{2}$ cwt.

16. Express as tons the following:

(1) 40 cwt. (3) 16000 lb. (5) 12500 lb.
(2) 30 cwt. (4) 9000 lb. (6) 13500 lb.

17. Express the following as long tons:

(1) 6720 lb. (3) 112 cwt. (5) 56 tons
(2) 19040 lb. (4) 952 cwt. (6) 39 cwt. 20 lb.

18. How many pounds are there in $\frac{1}{4}$ of a ton of coal? In $\frac{1}{2}$ of a long ton of pig iron?

LINEAR MEASURE

19. How many inches in a rod? A chain?

20. How many feet are there in 1 chain?

21. Express 3 rd. $2\frac{1}{3}$ yd. in inches. Express 2 rd. 1 yd. 2 ft. in inches.

22. Express 6408 inches in rods and yards.

23. A toy railroad track is 10 yd. long. How many feet of toy rails are needed for it?

24. A 12-rod fence is made of 10-inch boards set vertically with no cracks between them. How many boards are needed?

25. The four sides of a garden are 10 rods, 1020 ft., 162½ ft., and 1019 ft. 4 in. How many 10-inch boards will be needed to enclose it with a fence like that described in problem 24?

26. The tire of a bicycle is 88 inches long. How many times will the wheel turn in going 40 rods?

27. The tire of a wagon wheel is 15 ft. 9 in. long. How many turns will the wheel make in going 5 miles?

SURFACE MEASURE

28. Express as square inches:

(1) 3 sq. ft.
(2) 3 sq. ft. 8 sq. in.
(3) 1 sq. yd.
(4) 2 sq. yd. 3 sq. ft.
(5) 6 sq. yd. 48 sq. in.
(6) 3½ sq. yd.
(7) 45¼ sq. ft.
(8) 1 sq. rd.

29. Express:

(1) 121 sq. yd. as sq. rd.
(2) 640 sq. rd. as A.
(3) 40 A. as sq. rd.
(4) 80 sq. rd. as A.
(5) 1 sq. rd. as sq. ft.
(6) 1 A. as sq. yd.

30. Express:

(1) 10 sq. ch. as A.
(2) 20 A. as sq. ch.
(3) 1 sq. mi. as sq. ch.
(4) 1 Tp. as A.
(5) 1 A. as sq. rd.
(6) A. section as sq. rd.

31. How wide must a rectangle 80 rd. long be to contain 1 A.? 5 A.? 20 A.? 80 A.?

32. How wide must a rectangle 16 rd. long be to contain 1 A.? How many square rods more than 1 A. are there in a 14-rd. square?

33. How many square yards more than 1 A. are there in a 70-yd. square?

34. How many square yards less than 1 A. are there in a 69-yd. square?

35. How many square feet are there in 1 A.?

36. How many square yards more than 1 A. are there in a 213-ft. square?

CAPACITY

37. Express:

(1) 2 qt. 1 pt. as pt.
(2) 1 qt. ½ pt. as gi.
(3) 3 qt. 1 pt. 2 gi. as gi.
(4) 2 gal. as pt.
(5) 12 gal. as qt.
(6) 11 gal. 3 qt. as pt.

AVOIRDUPOIS POUNDS IN A BUSHEL PRESCRIBED BY VARIOUS STATES

COMMODITIES	*California*	*Connecticut*	*Illinois*	*Indiana*	*Iowa*	*Kansas*	*Kentucky*	*Louisiana*	*Maine*	*Massachusetts*	*Michigan*	*Minnesota*	*Missouri*	*New Hampshire*	*New Jersey*	*New York*	*Ohio*	*Pennsylvania*	*Tennessee*	*Texas*	*Vermont*	*Virginia*	*Washington*	*Wisconsin*
Barley	50	48	48	48	48	48	47	48	48	48	48	48	48	..	48	48	48	47	48	48	48	48	48	48
Beans	..	60	60	60	60	60	60	60	62	..	60	..	60	60	60	62	60	..	60	60	60	60	60	60
Blue Grass Seed	..	..	14	14	14	14	14	14	..	..	14	..	14	..	..	..	..	..	14	..	..	14	..	..
Buckwheat	40	48	52	50	52	50	56	..	48	48	48	50	52	..	50	48	50	48	50	42	48	52	42	48
Castor Beans	..	..	46	46	46	46	..	..	..	..	..	..	46	..	..	..	..	..	..	..	..	..	..	..
Clover Seed	..	..	60	60	60	60	60	60	..	..	60	60	60	..	64	60	60	62	..	60	60	60	60	60
Coal (Anthracite)	..	..	80	..	80	80	76	..	..	..	..	..	..	..	..	..	80	..	..	80	..	80	..	..
Corn on the Cob	..	..	70	68	70	70	70	..	..	..	70	..	..	..	..	..	70	..	70	70	..	70	..	70
Corn, Shelled	52	56	56	56	56	56	56	56	56	56	56	56	56	56	56	58	56	56	56	56	56	56	56	56
Cornmeal	..	50	48	50	..	50	50	..	50	50	50	..	..	50	..	..	..	..	50	..	..	50	..	..
Dried Apples	..	..	24	25	24	24	24	..	..	..	22	28	24	..	25	..	24	..	24	28	..	28	28	25
Dried Peaches	..	..	33	33	33	33	39	..	..	..	28	28	33	..	33	..	33	..	..	28	..	32	28	28
Flax Seed	..	..	56	..	56	56	56	..	..	..	56	..	56	..	55	55	56	..	56	56	..	56	..	56
Hemp Seed	..	..	44	44	44	44	44	..	..	..	..	..	44	..	..	..	..	..	..	44	..	..	..	..
Millet	..	..	50	..	48	50	50	..	..	..	..	..	..	..	..	..	50	..	..	50	..	50	..	55
Oats	32	32	32	32	32	32	32	32	32	32	32	32	32	32	30	32	33	30	32	32	32	32	32	32
Onions	..	50	57	48	57	57	57	..	52	52	54	..	57	..	57	..	55	..	56	57	52	57	50	57
Peas	..	60	..	..	..	..	60	60	60	..	60	..	..	60	60	60	60	..	60	..	60	60	60	..
Potatoes	..	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Rye	54	56	56	56	56	56	56	56	50	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
Sweet Potatoes	..	...	50	..	46	50	55	..	..	54	56	..	..	..	54	..	50	..	50	55	..	56	..	..
Timothy Seed	..	..	45	45	45	45	45	45	..	..	45	..	45	..	..	44	45	..	45	45	45	45	40	45
Turnips	..	50	55	..	..	55	60	..	60	..	58	..	..	..	..	..	..	..	..	55	60	55	50	42
Wheat	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	30	60	60	60	60

38. Express:

(1) 24 pt. as qt.
(2) 48 gi. as qt.
(3) 76 pt. as gal.
(4) 64 gi. as gal.
(5) 17 pt. as qt. and pt.
(6) 84 pt. as gal. and qt.

39. There are 231 cu. in. in a liquid gallon. How many cubic inches make a quart? A pint?

40. There are 537.6 cu. in. in a peck. How many more cubic inches are there in a dry than in a liquid quart?

41. Express:

(1) 3 bu. as pk.
(2) $2\frac{1}{4}$ bu. as pk.
(3) $2\frac{1}{2}$ bu. as pk.
(4) 4 bu. as dry gallons.
(5) 18 dry gallons as bu.
(6) 20 pk. as pt.

42. What is the difference between the weight of a quart of barley in California and in Illinois?

43. How many bushels are there in a long ton (2240 lb.) of anthracite in Illinois? In a short ton?

44. My coal bin is $7' \times 12' \times 12'$. How many bushels of anthracite will it hold? How many tons? (Count $1\frac{1}{4}$ cu. ft. per bushel.)

45. One cu. ft. is about what part of a Winchester bushel?

46. Express 3.5 bu. as pints; 160 pt. as bushels.

47. Find the error in the adage "A pint's a pound" if there are $7\frac{1}{2}$ gal. in a cubic foot of water weighing 1000 ounces.

TIME

48. Express:

(1) 1 hr. as sec.
(2) 1 da. as sec.
(3) 1 wk. as sec.
(4) 30 sec. as hr.
(5) 7200 sec. as hr.
(6) 604800 sec. as da.
(7) 1209600 sec. as wk.
(8) 1000000 sec. as wk., da. and hr.
(9) 1 wk. as minutes.

49. How many weeks in a common year? July 4 fell on Tuesday in 1905. On what day of the week did July 4, 1906, fall? Why?

50. How many months old are you this month?

51. How many days old are you today? How many hours in this many days?

52. A clerk earned $15 a week for 28 calendar months. How much did he earn in all?

53. A clerk saved $6 a week for 7 yr. How much did he save in all?

54. Illinois was admitted as a state in December, 1818. How many years and months old is the state of Illinois this month?

55. On the Saturdays of April, 1905, the sun rose and set at Chicago at the times here given:

Day	Date	Sun Rose	Sun Set
Saturday	April 1	5:46 A.M.	6:23 P.M
Saturday	April 8	5:34 A.M.	6:30 P.M.
Saturday	April 15	5:24 A.M.	6:37 P.M.
Saturday	April 22	5:14 A.M.	6:43 P.M.
Saturday	April 29	5:05 A.M.	6:50 P.M.

Give the length in hours and minutes of each of these 5 days "from sun to sun," and also give in minutes the change per week of the days' length during April, 1905.

56. It is noon at 12:00. How long in hours and minutes was the forenoon of April 1, 1905? The afternoon? How many minutes longer was the afternoon than the forenoon?

57. Answer similar questions for Apr. 8; 15; 22; 29.

58. From the numbers of a common almanac compare the days' length of April with the same dates for October.

Date	Morning Twilight		Evening Twilight	
1905	Begins	Ends	Begins	Ends
Jan. 1st	5:46 A.M.	7:25 A.M.	4:42 P.M.	6:21 P.M.
April 1st	4:10 A.M.	5:45 A.M.	6:24 P.M.	7:58 P.M.
July 1st	2:27 A.M.	4:32 A.M.	7:35 P.M.	9:40 P.M.
Oct. 1st	4:26 A.M.	5:56 A.M.	5:44 P.M.	7:14 P.M.

59. From the table find how long morning twilight lasted on Jan. 1, 1905; on Apr. 1; July 1; Oct. 1.

60. Answer similar questions for evening twilight.

COUNTING THINGS

61. A stationer pays 25¢ per dozen for lead pencils and sells them at 5¢ apiece. What is his profit on a gross of pencils?

62. How many are "three score and ten"? How many years are "a score of centuries"? How many months?

63. Lincoln's Gettysburg address was spoken in November, 1863. The address begins "Four score and seven years ago," etc. To what date did Lincoln refer?

64. What is the value of 1 bale of paper at 12¢ per quire?

65. What is the value of 3 great gross of steel pens at the rate of 2 pens for 5¢?

66. What is the value of 10 great gross boxes matches at the rate of 2 boxes for 5¢?

§75. General Exercises.

1. Reduce 9 great gross to units.

2. How many great gross are there in 79,636 units?

3. A dealer bought 3600 lead pencils at $2.00 per gross. He sold them at 25 cents per dozen. What was his profit?

4. Bought 2 gross foot-rules at $.12 a doz.; 7 gross Spencerian pens No. 1 at $.07 a doz.; 8 gross Eagle pencils No. 3 at $.30 a dozen. Find the amount of my bill.

5. A stationer bought 3 gross boxes of paper, each box containing 6 reams. How many sheets did he buy?

6. A stationer sold 3 quires, 20 sheets of paper to one man; 18 quires to another; 4 reams, 15 quires to another. How many sheets did he sell in all?

7. There are 2 reams, 9 quires of paper in one package; 3 times as much in a second package, and 7 times as much in a third package as in the second package. How much paper is there in the second package? In the third?

8. In £28 there are how many shillings? How many pence? How many farthings?

9. In 18s. there are how many pence? How many farthings?

10. In £28, 18s. 9¾d. there are how many farthings?

11. In £342, 15s. 6d. there are how many pence?

12. How many pounds, shillings, and pence are there in 28,643 pence?

13. How many feet are there in 78 yd.? How many inches?

14. How many inches are there in 78 yd. 2 ft. 6 inches?

15. How many yards, feet, and inches are there in 2838 inches?

16. How many ounces are there in 12 cwt.? In 10 pounds?

17. How many ounces are there in 12 cwt. 10 lb. 11 ounces?

18. How many cwt. lb. and oz. in 19,360 ounces?

There are two classes of problems to be solved in reducing denominate numbers to their equivalents in different units.

In one class, numbers expressed in larger units are to be expressed in smaller units. This is called *reduction descending*, or reduction from *higher to lower denominations*.

In the other class, numbers expressed in smaller units are to be expressed in larger units. This is *reduction ascending*, or reduction from *lower to higher denominations*.

Problems 15 and 18 are examples of the second class, and problems 13, 16, and 17 are examples of the first class.

19. Reduce 160 yd. 1 ft. 9 in. to inches.

20. Reduce 5781 in. to yards, feet, and inches.

21. Reduce 8 bu. 2 pk. 5 qt. to quarts.

22. Reduce 27 T. 18 cwt. 15 lb. to pounds.

23. Reduce 23 cu. yd. 14 cu. ft. to cubic feet.

24. Change £395, 15s. sterling to dollars and cents.

United States gold and silver coins are .9 pure gold or silver and .1 copper.

25. What weight of pure silver is there in a silver dollar weighing 412½ grains?

26. The U. S. standard 5-dollar gold piece weighs 129 gr. What is the number of grains of pure gold in the standard five-dollar gold piece?

27. The 5-cent piece weighs 73.16 gr. Of the weight of the coin, .75 is copper and .25 is nickel. What is the weight of the copper in the 5-cent piece? Of the nickel?

28. The eagle weighs 258 gr. What is the weight of pure gold in the eagle?

29. How many standard gold dollars can be coined from 1 oz. of pure gold?

30. Express the following ratios:

1 franc : $.25	$.25 : 1 franc
1 mark : $.25	$.25 : 1 mark.

31. If .52 oz. of gold is worth 43s. 4d., how many ounces can be bought for £35, 18s.?

32. A boy laid by a certain sum of money each week. At the end of 1 yr. 3 mo. 2 wk. he had saved $88.50. How much did he save each week? (52 wk. = 1 yr.)

33. A man changed $350, half into English and half into German money. How much English money did he have? How much German money?

34. How many feet are 4 ch. 30 links?

35. What is the ratio of 1760 yd. to 1 mi. 32 rods?

36. The mast of a ship was 78 ft. 4 in. high. During a storm .3 of it was broken off. How high was the remaining piece?

37. A four-sided field had sides of the following lengths: 63 ch. 2 rd.; 49 ch. 14 li.; 53 ch. 1 rd. 16 li. and 38 ch. 24 li. How far is it around the field?

38. A man walked $\frac{1}{5}$ of the length of a breakwater, which was 1 mi. 243 rd. 5 yd. long. How far did he walk?

39. A knot, or geographic mile, equals 6080.27 ft. What is the speed in common or statute miles per hour of a vessel that runs 21 knots per hour?

40. A wheel, 12 ft. in circumference, makes how many revolutions in $1\frac{1}{2}$ miles?

41. A telegraph wire is 14 mi. 140 rd. long, and is supported by 386 poles, which are placed at equal distances apart, a pole being at each end of the wire. How many feet apart are the poles?

42. What is the cost of 18 A. 120 sq. rd. of land at $52 per acre?

43. A man owned 14 A. of land, and sold 1428 sq. rd. How many acres did he have left?

44. If 1000 shingles are needed to cover 100 sq. ft. of roof, how many shingles are required to cover a roof 40 ft. long and 25 ft. wide, at the same rate?

45. A lawn tennis court 78 ft. long and 36 ft. wide is to be surrounded by a strip of sod 15 ft. wide at each end and 8 ft. wide at each side. What will the sodding cost at $.35 per square yard?

46. Find the cost of a half section of land at $45 per acre?

47. How many square rods are there in a rectangular field 24 ch. 45 li. long by 16 ch. 34 li. wide?

48. How many cubic inches are there in a tank containing 120 gal.? How many cubic feet?

49. How many cubic feet are there in a block of stone 5 ft. $\times$ 4 ft. $\times$ 6 inches?

50. How much must I pay for a board 16 ft. long and 8 in. wide, the board being 1 in. thick and lumber costing $35 per thousand? (A board foot means 144 sq. in. of surface, not over 1 in. thick.)

51. I bought 3 boards 12′ long, 6″ wide; 8 boards 16′ long, and 9″ wide, and 2 boards 10′ long, 12″ wide. All were 2″ thick. What did the whole cost at $30 per thousand?

52. What is the value of a straight pile of wood 16 ft. long, 8 ft. wide, and 6 ft. high, at $7.50 per cord?

53. What is the weight of a pile of oak boards 14 ft. long, 8 ft. 4 in. wide, and 5 ft. high, at an average weight of 54 lb. per cubic foot?

54. At $.75 a load of 1 cu. yd. what will be the cost of removing a pile of earth 60 ft. long, 24 li. wide. and 1 yd. high?

55. In one year the quarries of Minnesota yielded 4,000,000 cu. ft. of sandstone. What was this worth, at \$.76 a perch?

56. Find the cost of building a stone wall 150′ × 10′ × 2½′, at \$3.50 a perch.

57. A bin 12′ × 8′6″ × 5′ is filled with wheat. What is the weight of the wheat at 60 lb. per bushel?

58. What is the value of a straight pile of pine slabs 32′ × 7′ × 4′, at \$3.25 per cord?

59. A grocer paid \$4.50 for a barrel of vinegar (31½ gal.), and sold it at 5¢ per quart. What was his profit?

60. A jug contained 214 cu. in. of molasses. How much did it lack of containing 1½ gallons?

61. What will 1 mi. of right of way for a railroad cost at \$60 per acre, the width of the right of way being 100 feet?

62. What will be the cost per mile for railroad ties at \$.45 apiece, the ties being laid one every 2 feet?

63. Find the cost of the rails for 1 mi. (single track) of the road, the rails weighing 77 lb. per linear yard and costing \$35 per long ton.

64. Find the cost of fencing 1 mi. on both sides of the track, placing posts costing \$.25 apiece 16 ft. from center to center, and using wire weighing 1080 lb. per mile at 4¢, and 30 lb. of staples @ 4¢. The fence is to be 4 wires high, and labor costs 21¢ per rod of fence. (Count the odd post.)

65. Reduce .865 gal. to smaller units.

SOLUTION: .865 × 4 qt. = 3.46 qt.; .46 × 2 pt. = .92 pt.; .92 × 4 gi. = 3.68 gills. *Ans.* 3 qt. 3.68 gills.

66. Reduce .168 gal. to smaller units.

67. If oil is \$.11 per gallon, what will be the cost of three 42-gallon barrels of kerosene?

68. If a certain spring regularly yields 25 gal. daily, how many barrels of the capacity of $31\frac{1}{2}$ gal. would it fill in 20 days?

69. A merchant bought 16 gal. 3 qt. of syrup at 38¢ a qt., and sold 27 qt. for \$12; 18 qt. for \$9, and the remainder at 35¢ per quart. Did he gain or lose, and how much?

70. Reduce 2 pk. 7 qt. 1 pt. to the decimal of a bushel.

SOLUTION: 1 pt. = .5 qt. 7.5 qt. = $\frac{7.5}{8}$ pk. = .9375 pk. 2.9375 pk. = $\frac{2.9375}{4}$ bu. = .734375 bushels.

71. Reduce 3 pk. 5 qt. 1 pt. to bushels.

72. How many bushels will a bin hold that is 14 ft. long, 7 ft. 6 in. wide, and 5 ft. 8 in. high?

73. A farmer sold 6 loads of corn, each load averaging 36 bu. 2 pk. at 35 cents per bushel. What did he receive for the whole?

74. A boy had a bushel of hickory nuts, and sold 3 pk. 7 qt. 1 pt. What fraction of a bushel had he left?

75. If 9 bu. of potatoes cost \$4.80, what is the average cost per peck?

76. A bin contained 5376.25 cu. in. of rye. How much did it lack of containing $3\frac{1}{2}$ bushels?

77. How many oz. of quinine will be required to prepare 12 gross of 3-grain capsules?

78. A coal dealer buys two car-loads of coal each weighing 67,200 lb., at \$4.50 a long ton, and sells it at \$5.75 a short ton. What is his gain?

79. How many acres of land are there in a road 4 rods wide extending around a tract of land 80 chains square?

80. How many bushels do 2240 lb. of buckwheat make in Kentucky? In California? (See table p. 102.)

THE METRIC SYSTEM

§76. Historical.

The metric system is a decimal system of weights and measures adopted by the French Government soon after the French Revolution of 1789. The aim of the system is to base all measures upon an invariable standard, and to secure the simplest possible relations between the different units of the system.

The units of length, which is fundamental to the whole system, is called the *meter*. It was attempted to make the meter 1 ten-millionth of the length of the part of a meridian of the earth, which reaches from the equator to the pole, called a *quadrant* of the earth's meridian. The meridian of the earth was measured, and $\frac{1}{10000000}$ of the quadrant was obtained. A platinum bar equal to this length was very accurately cut and stored in the Government archives as the official standard of reference.

Later measurements of the earth's meridian showed the former length of the meridian to be incorrect. The length of the meter as obtained from the erroneous measures was, however, retained, and the length of this bar is the standard meter. From it all the other units of the system are derived.

The metric system is used for all purposes in France, and for most scientific purposes in Germany, England, and the United States. It is legalized in nearly all civilized countries.

The length of the meter is 39.37 in., or about 1.1 yards.

§77. Tables of Metric Measures.

Fig. 1 shows a scale graduated along the upper edge to 16ths of an inch, and along the lower edge to 1000ths of a meter, called *millimeters*.

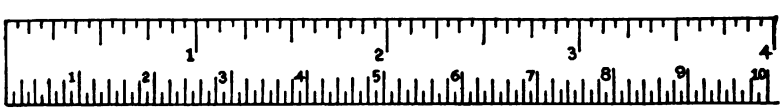


FIGURE 1

Decimal parts of the standard units are denoted by the Latin prefixes, abbreviated in each case to a single, small letter:

milli, meaning 1000th written m.
centi, meaning 100th written c.
deci, meaning 10th written d.

Multiples of the standard units are denoted by the Greek prefixes, abbreviated in each case to a single, capital letter:

deka, meaning 10 times written D.
hekto, meaning 100 times written H.
kilo, meaning 1000 times written K.
myria, meaning 10000 times written M.

Table of Measures of Length

10 *millimeters* (mm.)	= 1 *centimeter* (c.m.)	= about .4 in.
10 centimeters	= 1 decimeter (dm.)	= about 4.0 in.
10 decimeters	= 1 METER (m.)	= about 1.1 yd.
10 meters	= 1 dekameter (Dm.)	= about 32.8 ft.
10 dekameters	= 1 hektometer (Hm.)	= about 328 ft.
10 hektometers	= 1 *kilometer* (Km.)	= about .62 mi.
10 kilometers	= 1 myriameter (Mm.)	= about 6.21 mi.

Table of Surface Measure

100 *sq. millimeters* (mm².)	= 1 *sq. centimeter* (cm².)
100 sq. centimeters	= 1 sq. decimeter (dm².)
100 sq. decimeters	= 1 SQ. METER (m².)
100 sq. meters	= 1 sq. dekameter (Dm².)
100 sq. dekameters	= 1 sq. hektometer (Hm².)
100 sq. hektometers	= 1 *sq. kilometer* (Km².)

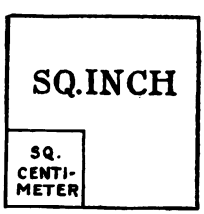


FIGURE 1

The cm². = .155 sq. in.
The m². = 10.764 sq. ft.
The Km². = 247.107 A.
1 m² = 1 centare (ca.)

Table of Land Measure

100 centares = 1 âre (pronounced âir) (a.)
100 ares = 1 *hektare* (Ha.)

Table of Measures of Volume

The standard unit of volume is the cubic meter = 35.314 cu. ft. = about 1.3 cubic yards.

1000 cubic millimeters (mm^3.) = 1 cu. centimeter (cm^3.)
1000 cubic centimeters = 1 cu. decimeter (dm^3.)
1000 cubic decimeters = 1 cu. meter (m^3.)
etc., etc.

Table of Measures of Capacity

The standard unit of capacity is the *liter* (leeter). It is equal to 1 cu. decimeter, and equivalent to .908 dry quarts.

10 milliliters (ml.) = 1 *centiliter* (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 *liter* (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 *hektoliter* (Hl.)
10 hektoliters = 1 kiloliter (Kl.)

Table of Measures of Weight

The standard unit of weight is the gram, which is the weight of 1 cu. centimeter of distilled water at its temperature of greatest density (39.1° F.).

10 milligrams (mg.) = 1 *centigram* (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 *gram* (g.)
10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 *kilogram* (Kg.)
10 kilograms = 1 myriagram (Mg.)
10 myriagrams = 1 quintal (Q.)
10 quintals = 1 *metric ton* (T.)

1 centigram = .15432 grain
1 gram = 15.432 grains
1 kilogram = 2.20462 lb.
1 metric ton = 2204.621 lb.

§78. Metric and U. S. Equivalents.

The equivalents will be of assistance in changing the metric to the common system of measures.

1 m. = 39.37 in.	1 mile = 1.6093 Km.
1 Km. = .6214 mi.	1 yard = .9144 m.
1 m^2. = 1.196 sq. yd.	1 square yard = .8361 m^2.
1 Km^2. = .3861 sq. mi.	1 square mile = 2.59 Km^2.
1 are = .0247 A.	1 acre = 40.47 a.

1 m³.	= 1.308 cu. yd.	1 cubic yard	= .7645 m³.
1 stere	= .2759 cord	1 cord	= 3.624 st.
1 liter	= 1.0567 liquid qt. / .9081 dry qt.	61.022 cu. in.	= 1 l.
		1 liquid qt.	= .9436 l.
1 Hl.	= 2.8376 bu. / 26.417 liq. gal.	1 dry qt.	= 1.101 l.
		1 bushel	= .3524 Hl.
1 g.	= 15.432 grains	1 grain	= .0648 g.
1 Kg.	= 2.2046 lb. avoir.	1 lb. avoir	= .4536 Kg.
1 metric ton	= 1.1023 ton	1 ton	= .9072 ton.

PROBLEMS WITH THE METRIC UNITS

1. Take a smooth lath, or plane one, lay a straight strip of paper beside the lower edge of the metric rule, Fig. 1, p. 112, and with a sharp pencil transfer the graduations from the rule to the strip of paper, and then from the strip of paper to the lath. Continue the *centimeter* graduation marks along your lath until you get a meter stick (= 100 centimeters). How does this meter stick compare in length with a yard stick?

2. Explain which digit in 6.8752 m. stands for m.; which for dm.; which for cm., and which for millimeters.

3. Find by measurement the length and the width of your schoolroom in meters, and write the result of your measurements in meters and decimals of a meter.

4. How many centimeters long, wide, and thick is your arithmetic?

5. Express the length of your pencil in centimeters.

6. A sidewalk is 2112 m. long; how many kilometers long is it?

7. What is the cost of 158 cm. of ribbon at $.45 a meter?

8. How many steel rods, each 16 cm. long, can be cut from a rod 7.68 m. long?

9. From a piece of wire, 98.52 m. long, a piece .047 Km. was cut. How long was the remainder?

10. A bicycle track is .807 Km. long. How many meters would one go in riding around the track 5 times?

11. Express 1 Km^2. in square dekameters; in square meters.

12. Find the number of square meters in the surface of your schoolroom floor. Express the same in square dekameters.

13. Express the area of your desk in square decimeters; in square inches. How do the two compare?

14. Using the decimeter as a measure, find the number of square centimeters in the surface of your geography.

15. Express the same in square decimeters; in square inches.

16. Find the area of the door in square meters. What part of an *are* is this area?

17. A certain plot of ground contains 20 m^2. How many square feet in the plot?

18. Lay off a square meter upon the floor. Within this area, lay off a square yard. Note results.

19. With the aid of your decimeter measure, draw a square decimeter. In the corner, draw a square centimeter. How many square centimeters could be drawn within the square decimeter?

20. What is the area of a floor 8.4 m. by 5 meters?

21. A lot is 7.64 m. by .033 Km. Find the area?

22. What is the value of a piece of land 8 Dm. long and 6.5 Dm. wide, at $32 an *are*?

23. How many *ares* in a street 1.5 Dm. wide and 2.48 Km. long?

24. How many square meters in a floor 700 cm. long and 500 cm. wide?

25. A man had 3 pieces of land as follows: 8.4 Ha.; 3846 m^2., and 2.5 Km^2. How many hektares of land are there in all?

26. 20 liters equal how many centiliters? How many Dl.? What part of a Kl.? Find the difference between 12 l. and 12 dry quarts.

27. Using your decimeter measure as a basis, model a cubic decimeter or liter.

28. Using your square centimeter as a basis, model a cubic centimeter. What is the relation of a cubic centimeter to a gram? How many cubic centimeters in a liter?

29. How many cubic meters are there in the volume of your schoolroom?

30. What is the difference between an ordinary ton and a metric ton (tonneau)?

31. A gram is the weight of 1 cm^3. of distilled water. What is the weight in grams of 1 l. of distilled water?

32. A liter of water weighs nearly $2\frac{1}{5}$ lb. What is the weight of 6 l. of water in pounds? In grams?

33. A package of silver weighs 2.58 grams. What is its weight in grains?

34. How many pounds in 1 Q.? In 5 quintals?

35. How many 2 gr. capsules will 5 g. of quinine fill?

36. Find the cost of the quinine at 10¢ a doz. capsules?

37. What is the cost of 5500 Kg. of coal at $6.50 per ton?

38. What is the cost of 2 Q. of sugar at $.08\frac{1}{2}$ a kilogram?

39. If a book weighs 3.2 Dg., how many such books will weigh 1.792 kilograms?

40. A box contains 2500 packages of quinine, each package weighing 1.25 g. How many hektograms does the contents of the box weigh?

41. A vessel, 18 cm. long, 14 cm. wide, and 22 cm. high, is filled with lead, which weighs 11.35 times as much as water. What is the weight of the lead? (1 cu. ft. of water weighs 62.5 lb.)

PERCENTAGE AND INTEREST

§79. Percentage.

Solve all you can orally.

PERCENT AS HOW MANY IN 100

In a city block there are 100 buildings. 50 of them are stores; 40 are dwelling houses; 5 are warehouses; 3 are hotels, and 2 are banks. 10 of them are built of brick; 30 are stone, and 60 are frame houses; 35 of them are 3 stories high; 25 are two stories, and the rest are one story high.

1. What part of the buildings are stores? How many hundredths of the buildings are stores?

NOTE: "How many percent?" "What percent?" means "How many in a hundred?" or "How many hundredths?"

2. How many percent of the buildings in the block are stores?

3. $\frac{1}{2}$ equals how many percent?

4. What part of the buildings are 2 stories high? What percent of them are 2 stories high?

5. What percent are warehouses? Hotels? Banks? Dwellings?

6. What percent are brick? Stone? Frame?

7. What percent are 3-story buildings? 1-story buildings?

8. What part of the buildings are warehouses? $\frac{1}{20}$ equals how many percent?

9. What part of the buildings are dwellings? $\frac{2}{5}$ equals how many percent?

10. What part of the buildings are frame houses? $\frac{3}{5}$ equals how many percent?

A sidewalk 200 rd. long is made up of 10 rd. of stone walk, 25 rd. of cement walk, 40 rd. gravel walk, 50 rd. board walk, and 75 rd. cinder walk.

11. What percent of the entire walk is stone? What percent cement? Gravel? Board? Cinder?

12. What percent of the whole was the stone and the cement? The cement and the cinder? The gravel and the board? The stone and the board?

In a certain nursery there are 100 oak, 75 ash, 50 maple, 30 elm, 25 hickory, 15 poplar, and 5 walnut trees.

13. What percent of the trees are oak? What part are oak?

14. What percent of the trees are ash? What part are ash?

15. What percent of the trees are maple? Elm? Hickory? Poplar? Walnut?

16. Of 100 kinds of animals of a neighborhood, 27 die, 18 migrate and 10 hibernate as winter comes on. What percent die? Migrate? Hibernate?

17. 10 kinds store food, 20 remain and feed abroad and 10 kinds appear only in winter. What percent store food? Feed abroad? Appear only in winter?

PERCENTS AS FRACTIONAL PARTS AND AS HUNDREDTHS

18. \$2 equals what part of \$8? 9 bu. equals what part of 12 bu.? 8 hr. equals what part of 24 hr.? \$6 of \$9? 18 of 27? 27 of 36?

19. To how many hundredths of a number are the following fractional parts of that number equal?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{1}{20}$ $\frac{3}{20}$ $\frac{4}{25}$

20. Express the following as hundredths:

$\frac{3}{50}$ $\frac{9}{25}$ $\frac{7}{20}$ $\frac{9}{10}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{3}{16}$

The word *percent* means *hundredth*, or *hundredths*.

The sign, "%," is a short way of writing *percent*, or *hundredths*: thus, 2%, 8%, 33⅓%, mean $\frac{2}{100}$, $\frac{8}{100}$, $\frac{33\frac{1}{3}}{100}$, and are read, "2 percent," "8 percent," "33⅓ percent."

The number (as 2, 8, 33⅓), written before the sign, "%," is called the *rate percent*.

It is well to recall that we have the following ways of writing such numbers as 6 percent: (1) 6 percent; (2) 6%; (3) 6 hundredths; (4) $\frac{6}{100}$; and (5) .06. The sign, "%," is merely an abbreviation for $\frac{1}{100}$, or .01.

21. In a schoolroom containing 40 pupils, $\frac{3}{5}$ of the pupils were girls. How many girls were there?

22. In the last problem, what percent of the pupils were girls?

23. In the month of September, 12 days were cloudy. What percent of the days were cloudy?

24. Express the equivalents of the following fractions as percents, or hundredths:

$\frac{5}{6}$ $\frac{5}{8}$ $\frac{11}{12}$ $\frac{1}{7}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{11}$ $\frac{6}{13}$ $\frac{9}{19}$ $\frac{18}{37}$

25. In a sample of soil weighing 37 oz., 18 oz. were sand. What percent of the soil was sand?

26. A sample stick of timber, weighing 19 lb., contained 2 lb. of water. What percent of the weight of the timber was due to the water it contained?

SOLUTION: (1) Annex zeros after the numerator and divide thus:

$$19\overline{)2.00} \qquad .10\tfrac{10}{19}.$$

But $.10\frac{10}{19} = 10\frac{10}{19}\%$. Why?

27. What percent of a yard is 1 ft.? 9 in.? 15 in.? 28 in.? 35 in.? 1 inch?

28. The first number of these pairs is what percent of the second?

(1) 8 of 150	(4) $4\frac{8}{8}$ of 60	(7) 32.91 of 263.28
(2) 13 of 700	(5) $27\frac{3}{5}$ of 600	(8) $\frac{1}{4}$ of $\frac{1}{2}$
(3) $7\frac{1}{2}$ of 12	(6) 38.45 of 769	(9) $\frac{3}{5}$ of $12\frac{1}{2}$

29. Change the following percents to their fractional equivalents (fractions in their lowest terms):

5% 8% $8\frac{1}{3}\%$ $12\frac{1}{2}\%$ 12% 16% $16\frac{2}{3}\%$ $22\frac{1}{2}\%$ $67\frac{1}{2}\%$ $87\frac{1}{2}\%$

30. Change to their decimal equivalents the following:

7% $6\frac{1}{4}\%$ $83\frac{1}{3}\%$ $87\frac{1}{2}\%$ $1\frac{1}{2}\%$ $2\frac{3}{4}\%$ $\frac{7}{8}\%$ $\frac{1}{10}$ of 1% $\frac{1}{100}$ of 1%

To find the percentage.

31. At the beginning of a school year, the lung capacity of a boy was 161 cu. in. By the middle of the year it was 7% larger. How many cubic inches had his lung capacity increased?

32. The lung capacity of a boy at the beginning of the year was 160 cu. in., and at the middle of the year it was 166.4 cu. in. What was the percent of increase?

To find the rate percent.

33. What percent of the farm, Fig. 1, p. 30, is the cornfield? the wheatfield? the meadow? the south oatfield? the pasture? the lot occupied by the house and grounds?

34. 60% of the value of a mill is \$5400; what is the value of the mill?

35. The standing of the several clubs in the National League during one season was determined from this table:

Club	Games Won	Games Lost	Percent Won
Pittsburg	90	49	64.7%
Philadelphia	83	57	
Brooklyn	79	57	
St. Louis	76	64	
Boston	69	69	
Chicago	53	86	
New York	52	85	
Cincinnati	52	87	

Find for each club what percent of the total number of games played were won. Carry results to the first decimal place, as shown for Pittsburg.

To find the base.

36. I pay 12% of the value of the property I occupy as rent every year. My rent is $240 a year; what is the value of the house?

37. A house was damaged by fire, and an insurance company paid the owner $840 damages, which was 40% of the original cost of the house; what was the original cost?

38. The season's record of games lost for a certain association of 7 baseball clubs was as follows:

Club	Games Lost	Percent Lost
First	14	20
Second	17	25
Third	18	30
Fourth	27	$33\frac{1}{3}$
Fifth	28	35
Sixth	28	40
Seventh	32	$45\frac{5}{7}$

The last column of the table contains the percent of all the games played that were lost. Find the number of games played by each club.

§80. General Number.

Answer all you can orally.

1. 50 equals what percent of 100? Of 200? Of 150? Of 500?

2. 2 equals what percent of 4? Of 8? Of 6? Of 20?

3. Any number equals what percent of a number twice as large? 4 times as large? 3 times as large? 10 times as large?

4. x equals what percent of $2x$? Of $4x$? Of $3x$? Of $10x$?

5. 13 equals what percent of 17? Of 28? Of 45? Of 55?

SOLUTION: $\frac{13}{17}=\frac{13\times100}{17\times100}=\frac{13\frac{9}{17}}{100}=\frac{76\frac{9}{17}}{100}=76\frac{9}{17}\%$.

Use pencil only when needed.

To find the percentage, p.

6. What is 2% of \$200? Of \$375? How is 2% of any number of dollars found?

7. How is 2% of \$ a found? Of \$ b? Of \$ x?

8. How is 5% of a number found? What is 5% of n?

9. How is $12\frac{1}{2}\%$ of a number found? Find $12\frac{1}{2}\%$ of a.

10. How is *any* percent of a number found?

11. How can a be expressed as hundredths? x? $9x$?

12. Express as hundredths: 16; 29; a; m; $16x$.

13. What is $r\%$ of 160? Of 350? Of a? Of R?

The result of finding a given percent of any amount or number, is called the percentage. The amount, or number, on which the percentage is computed is often called the base.

14. Calling the percentage, p, the rate percent, r, and the base, b, show by an equation how to find p, from b and r.

15. Replacing the symbols in your equation by the words for which they stand, translate the equation into the common language of percentage. This translation, properly made, is the fundamental principle of percentage.

The percentage equals the product of the base and rate percent divided by 100, or more briefly,

$$(I)\quad p=\frac{b\times r}{100}$$

ALL OF PERCENTAGE IS IN THIS EQUATION, CALLED A FORMULA, BECAUSE IT FORMULATES (EXPRESSES IN BRIEF FORM) A LAW.

To find the rate percent, r.

Multiplying both sides of the formula by 100, we have

$$(A)\quad 100\ p=b\times r$$

Now divide both sides of this equation by b, and write the second number first, and obtain (note that $\frac{br}{b}=r$),

$$(II)\quad r=\frac{100\times p}{b}$$

16. Translate (II) into words. How may the rate percent (r) be found, the *percentage* (p) and *base* (b) being given?

To find the base, b.

17. Divide both sides of (A) by r, write the second number first, and make a rule for finding the base, b, when p and r are given.

18. Show the rule to be the same as

$$(III)\quad b=\frac{100\times p}{r}$$

§81. Gain and Loss.

1. A merchant paid \$16 for a suit of clothes. At what price must he sell the suit to gain 25 percent?

2. A dealer pays \$1.80 a dozen for blank books, and retails them at a profit of $66\frac{2}{3}\%$. Find selling price.

3. A grocer buys eggs at wholesale at 10¢ a doz., and retails them at a profit of 30%. Supposing there is no loss due to breakage or spoiling, at what price per dozen does he sell them?

4. Allen paid \$1 for a sled, and sold it at a loss of 20%. How much did he receive for the sled?

5. From a box containing 200 oranges, 80% were sold in one day. How many oranges were sold?

6. What is the percentage on \$640 at 20%? At 28%? At 35%? At $6\frac{1}{4}$%? At $12\frac{1}{2}$%? At $87\frac{1}{2}$%?

7. A farm, costing \$4400, was sold at a gain of 18%; for how much did the farm sell?

8. A farm sold for \$5192, which was 18% more than was paid for it. How much was paid for it?

SOLUTION: The 18% here means 18% of what was paid for the farm (the cost price). \$5192 is then equal to what percent of the cost? The statement may be written thus:

1.18 times the cost price = \$5192,

or, more briefly, thus: $1.18x = \$5192$. Hence, $x = \frac{\$5192}{1.18}$. Divide and find the value of x.

9. A center fielder threw a ball 90 ft., which was 12% farther than the third baseman threw it. How far did the third baseman throw it?

SUGGESTION: First answer the question, 12% of what?

10. A boy bought oranges at the rate of 5 for 3¢, and sold them at the rate of 3 for 5¢. What was his rate percent of gain or loss?

11. A merchant in shipping 150 crates of eggs lost 25 crates by freezing. What percent did he lose?

12. Hats, costing \$2.75, sold for \$3.50. What was the percent of profit?

13. An automobile, costing \$750, sold for \$1250. What was the percent of profit?

14. In a certain battle, 32,000 men were engaged, and 35% of all engaged lost their lives. How many lost their lives?

15. A horse, costing \$88, sold at a loss of 40%. For how much did the horse sell? (40% of what?)

16. I paid \$35 for a bicycle, and sold it the next season for 60% less than I paid for it. What was the selling price?

17. A cow gave 13 lb. of milk Nov. 1, and 15.6 lb. on Nov. 15. What was the rate % of gain in 14 days, in her daily yield of milk?

18. The increase in population in one year in a certain town was 2000, which was $6\frac{1}{4}\%$ of the population at the end of the year? What was this population?

19. The death rate the same year was 1.8% of this population. How many deaths occurred?

20. A man lost 27% of the books of his library by fire. He lost 540 books. How many books did his library contain before the fire?

21. Anthracite coal cost \$8.50 a ton in Nov., and rose 15% in 1 mo. What was the price of anthracite coal after the rise in price?

22. I sold a lot for \$600, at a loss of 20%. What did the lot cost?

23. Find the cost of a piano which sold for \$187.50 at a loss of 25%.

24. I bought a tennis outfit for \$35, and sold it at a loss of 15%. Find the selling price.

25. Carpet, marked at \$1.25, sold at a reduction (discount) of $6\frac{2}{3}\%$. Find the selling price.

26. A man lost 20% of his money and after losing 10% of the remainder had \$3,600 left. How much had he at first?

27. A dealer sells a cask of 30 gal. of oil, agreeing to deliver it and receives \$30. In delivering it 4.5 gal. were spilled. What percent was spilled, and how much should the dealer pay back?

28. Goods damaged by fire were sold at a loss of 40%. The amount received was \$555. What was the cost?

29. A suit of clothes was marked at \$45, which was 50% more than the cost. It sold at a reduction of 20% from the marked price. What was the percent of profit on the first cost of the suit?

§82. Meteorology.

To find the rate, r; given b and p. Use $r = \frac{100 \times p}{b}$.

The *base* is the number the percentage is reckoned on.

The number of clear, cloudy, partly cloudy, and rainy or snowy days for the first 4 mo. of 1903, at Chicago, are here tabulated:

1903	Clear	Cloudy	Partly Cloudy	Rain or Snow
Jan.	9	15	7	9
Feb.	10	11	7	9
Mar.	12	14	5	11
Apr.	10	13	7	13

1. What percent of the number of days of Jan. were clear? Cloudy? Partly cloudy? Rainy or snowy?

2. Answer similar questions for Feb.; for March; for April.

3. What percent of the total number of clear days of 1903 to May 1st fell in Jan.? In Feb.? In March? In April?

4. Answer similar questions for cloudy, partly cloudy and rainy, or snowy days.

5. The highest velocities (speeds) in miles per hour of the wind in Chicago for each of the 12 mo. (beginning with Jan.) of a certain year were: 50; 48; 57; 70; 54; 45; 56; 47; 52; 58; 52; 58. What percent of the highest velocity for April was the highest velocity for Jan.? For Feb.? For each of the remaining months?

6. The total movement, in miles, of the wind in Chicago for each of the 12 mo. of a certain year was: in Jan. 12,736; 10,279; 13,999; 12,820; 12,356; 10,900; 11,231; 9839; 11,834; 13,148; 13,583; 15,476. What percent of the total wind movement in Dec. was the total movement for each of the other months?

7. 100 green oak leaves weighed 100 grams. When thoroughly dried they weighed 39.5 grams. The dried leaves were then burned, and the ash weighed 2.4 grams. What percent of the leaves was dry solid? Mineral matter (ash)?

8. 100 green elm leaves weighed 60 grams. $38\frac{1}{3}\%$, by weight, was dry solid, and $2\frac{2}{3}\%$ was mineral matter. What was the weight of the dry solid? Of the mineral matter?

9. A school garden, $15' \times 40'$, was seeded as follows: 25%, potatoes; $16\frac{2}{3}\%$ of the remainder, peas; 20% of the remainder, beans; 25% of the remainder, lettuce; $33\frac{1}{3}\%$ of the remainder, radishes; 20% of the remainder, parsley, and the rest in beets. How many square feet were seeded to beets?

10. The precipitation (rainfall) at Chicago, in inches, for Jan., Feb., Mar., and Apr. for 1903 was 1.09, 3.03,

1.67 and 3.77, and the averages for these same months for 33 yr. are 2.05, 2.11, 2.52, and 2.73. Find the rate percent of excess, or deficiency, for each of the four months of 1903, on the 33-yr. average.

11. Make and solve problems on the following table of data relating to Chicago weather:

1902	Temperature		Precipitation		Weather		
	Monthly Mean	Mean for 31 Years	In Inches	Average for 32 Yr.	Clear Days	Fair Days	Cloudy Days
Jan.....	25.2	23.8	.66	2.08	10	14	7
Feb.....	20.8	25.9	1.53	2.30	12	9	7
Mar.....	38.6	34.3	4.16	2.56	6	13	12
Apr.....	46.4	46.4	2.26	2.70	10	12	8
May.....	59.0	56.5	5.08	3.59	10	19	2
June.....	64.2	66.6	6.45	3.79	7	16	7
July.....	72.4	72.3	5.78	3.61	12	13	6
Aug.....	68.4	71.1	1.44	2.83	15	10	6
Sept ...	60.8	64.4	4.83	2.91	9	8	13
Oct.....	55.2	53.1	1.45	2.63	14	10	7
Nov.....	47.0	38.5	2.03	2.66	8	13	9
Dec.....	26.5	29.0	1.90	2.71	7	8	16

§83. The Almanac.

Note: In this section "length of the day" means the length of daylight, or the time interval from sunrise to sunset.

1. On Jan. 1, at Chicago, the sun rose at 7 hr. 29 min., and set at 4 hr. 38 min., and on July 1, it rose at 4 hr. 28 min., and set at 7 hr. 39 min. The length of the day on Jan. 1 equals what percent of the length on July 1?

2. On Jan. 1, the length of the day in St. Louis was 103.8%, and in St. Paul 96.4%, of the length of the same day in Chicago. Find the day's length in each of these cities.

3. On July 1, in the same places, the day's lengths were, respectively, 97.9% and 102.2% of its length in Chicago. Find the day's length in each of these cities on this date.

4. Make and solve similar problems from the calendar pages of a common almanac.

5. On the first of Feb., of March, of April, of May, of June, of July, of Aug., of Sept., of Oct., of Nov., and of Dec., the day's lengths in Chicago were 108%, 122.3%, 137.8%, 152.6%, 163.2%, 163.5%, 157.9%, 144.6%, 129.2%, 132.5%, and 102.6% of the day's length on Jan. 1. Find the day's length on the dates named.

6. On vertical lines, one for each month, plot these rates to scale and draw through the points a smooth free-hand curve.

7. From the almanac find what percent the period from "moon rises" to "moon sets" on the first of some month is of the same period on the 15th or 20th.

8. Find what percent the time from full moon to new moon is of the time from new moon to first quarter; from new moon to second quarter, or full moon.

§84. Geography.

The following table shows the growth in territory of the United States and the dates of the additions.

Acquisition	Date	Area in Sq.Mi.
Original territory	1783	827,844
Louisiana purchase	1803	1,182,752
Florida	1819	59,268
Texas	1845	371,063
Mexican purchase	1848	522,568
Texas purchase	1850	96,707
Gadsden purchase	1853	45,535
Alaska	1867	590,884
Hawaii	1898	6449
Porto Rico	1899	3600
Philippine Islands		114,000
Guam		200
Isle of Pines	1899	882

1. By what percent was the territory of the United States increased by the purchase of Louisiana? By the purchase of Texas?

2. State and solve other similar problems on the table.

3. The area in millions of square miles of the surface of the earth is 148.18; of this surface 108.77 is water, and 39.41 land. Find what percent of the surface of the earth is water? What percent is land? The land surface equals what percent of the water surface?

To find the base.

4. If the coast line of Asia were smooth the length would be 13,860 miles, which is 45% of the actual length. Find the actual length.

5. If the coast line of Australia were smooth the length would be 5852 miles, which is 77% of the actual length. Find the actual length.

6. If the following continents had smooth coast lines the lengths, and the percents that these lengths are of the actual lengths, are here given:

CONTINENT	COAST-LINE IF SMOOTH	% EQUIVALENT OF ACTUAL LENGTH
North America	10387 mi.	43.2%
South America	9034.4	66.4
Africa	11771	83.6
Europe	6622	38.5

Find the actual lengths of coast of these continents.

7. The Amazon River is 3612 miles long, which is 86% of the length of the Mississippi River. How long is the Mississippi?

8. The Ohio River is 3000 mi. long, which is 77% of the length of the Nile. How long is the Nile?

9. The Congo is 3000 mi. long, which is $90\frac{10}{11}\%$ of the length of the Yangtsekiang. How long is the Yangtsekiang?

10. The area of Lake Michigan is 22,450 sq. mi. which is 71.9% of the area of Lake Superior. Find the area of Lake Superior.

11. The area of Victoria Lake is 22,167 sq. mi., which is 93.14% of the area of Lake Huron. Find the area of Lake Huron.

12. The area of Lake Ontario is 7244 sq. mi., which is 72.8% of the area of Lake Erie. Find the area of Lake Erie.

13. The greatest depth of Lake Superior is 1008 ft., which is 115.2% of the greatest depth of Lake Michigan. Find the greatest depth of Lake Michigan.

14. The Mississippi River drains 1,250,000 sq. mi., which is 7.7% of the area of North America. Find the area of North America.

15. Other problems may be made and solved from data furnished by a Geography.

§85. Commission.

Commission is a sum of money paid by a person or firm called the principal, to an agent for doing business for the principal.

Commission is usually reckoned as some percent of the amount of money received or expended for the principal.

1. An agent sells 250 bu. @ 80¢ on a 2% commission. How much money was due the agent?

2. An agent bought for his firm 1000 bbl. of apples @ $1.25, and received a commission of 5%. How much money did he receive?

3. What is the commission on 100 baskets of peaches @ 50¢, the rate of commission being 3%?

4. What is the commission on the following sales?

(1) 80 A. land @ $75, rate of commission being 5%.

(2) City property sold at $8500, rate of commission $4\frac{1}{2}\%$.

(3) 40 bu. tomatoes @ 75¢, rate of commission 10%.

(4) 1000 bu. wheat @ 95¢, rate of commission $1\frac{1}{2}\%$.

(5) 100 doz. eggs @ 25¢, rate of commission 12%.

(6) 1000 bu. green vegetables @ 90¢, rate of commission $12\frac{1}{2}\%$.

(7) $500 collected debts, rate of commission 15%.

(8) $1200 bad debts collected, rate of commission 20%.

5. Find the commission on $1850 at $2\frac{1}{2}\%$; at $3\frac{1}{2}\%$.

6. A clothing house sent its agent $1224, which included a sum of money to be invested and 2% commission on this sum for the agent. What sum was to be invested?

7. 5% commission on a certain amount of money was $684.20. What was the amount? (Statement: $.05x = \$684.20$. Find x.)

A shipment of goods sent to an agent to be sold is called a consignment.

8. A consignment of 4560 bu. of wheat was sold by an agent at $78\frac{3}{8}$¢ per bushel. What was the agent's commission at $1\frac{1}{2}$ percent?

9. An agent sold the following consignment of goods:

20 doz. eggs at $14\frac{3}{4}$¢	16 boxes oranges at $2.25
40 lb. creamery butter at 21¢	4 boxes oranges at $2.50
36 lb. cheese at $13\frac{1}{4}$¢	12 boxes grape fruit at $2.75
80 lb. chicken at $12\frac{1}{2}$¢	25 bunches bananas at $1.25
8 doz. live chickens at $3.75	12 boxes lemons at $2.76
4 bbl. apples at $3.25	16 bbl. potatoes at $4.25

Find the agent's commission at $6\frac{1}{4}$ percent.

§86. Trade Discount.

A discount is a certain rate percent of reduction (sum thrown off) from the prices at which articles are listed or quoted.

The discount is usually allowed for cash payments or for payment, within a specified time.

1. A retail merchant buys silk at $1.20 a yard. He is allowed a discount of 10% for cash. He pays cash. How much does the silk cost him?

SOLUTION: $1.20 − 10% of $1.20 = $1.08.

2. A publisher sells 50 books at $1.50 and allows 20% discount. How much does he receive for the books?

Merchants and manufacturers often publish expensive catalogues containing their price lists of articles, whose prices change rapidly. When prices fall, instead of publishing new lists, they mark off an additional discount. For example, the price of a certain article may be catalogued thus: $60, discount, 20%, 10%, 5%. This means a 20% reduction, then a 10% reduction on the reduced price, and then a 5% reduction on the second reduced price. It would be computed thus:

$60	$48	$43.20
.20	.10	.05
$12.00	$4.80	$2.1600

$60 − $12 = $48. $48 − $4.80 = $43.20. $43.20 − $2.16 = $41.04.

NOTE: Notice this is not the same as a discount of 20% + 10% + 5% (= 35%). Wherein is it different?

3. A New York merchant sells to a customer goods marked thus: Price, $3500; disc't, 10% 60 da. and 5% for cash. What must the customer pay to settle the bill by cash payment?

NOTE: The customer gets the benefit of both discounts.

What would the customer have paid if he had been given a single discount of 15 percent?

4. A bill of plumber's supplies was marked thus:

Price: \$7.50, disc't. 20% and 7% and 5%.

How much did the supplies really cost?

5. Compute the amount of money needed to settle the following bills if paid in cash or within the shortest time mentioned:

(1) \$45; discount 25% 60 da. and 10% 5 days.

(2) \$180; discount $16\frac{2}{3}\%$ 30 da., and 10% 10 da., and 5% cash.

(3) \$1800; discount 30% and 10% and 7% cash.

(4) \$54; discount $12\frac{1}{2}\%$, and 8%, and 5% 30 days.

(5) 46 T. coal at \$5.75; discount 10% and 2% cash.

(6) 50 men's suits at \$18; discount 20% and 5% 10 days.

(7) 4 gross tablets at 50¢ per doz.; discount 20% and 7% cash.

6. To what single rate of discount is a discount of 20% and 5% equivalent?

SUGGESTION: Take a base of \$100.

§87. Marking Goods.

In marking his goods a merchant uses the *key* word "harmonizes." He writes the *selling price* above a horizontal line, and the *cost price* below it, using the letters *h-a-r-m-o-n-i-z-e-s* in order for the digits 1-2-3-4-5-6-7-8-9-0. For example a book sells at \$2.50 and costs \$1.75. The mark would be $\frac{a\,o\,s}{h\,i\,o}$.

1. Using the key "harmonizes," interpret the following cost marks and find the percent of profit for each cost mark:

(1) $\frac{n\,s}{m\,a}$ (2) $\frac{i\,o}{o\,r}$ (3) $\frac{a\,a\,o}{h\,i\,o}$ (4) $\frac{m\,r\,z}{r\,s\,s}$ (5) $\frac{m\,a\,o}{r\,r\,n}$

2. Complete these marks so that the selling price may be 40% greater than the cost price:

(1) $\frac{\cdot\,\cdot\,\cdot}{a\,o}$ (2) $\frac{\cdot\,\cdot}{m\,s}$ (3) $\frac{\cdot\,\cdot\,\cdot}{i\,o}$ (4) $\frac{\cdot\,\cdot\,\cdot}{z\,s}$

(5) $\frac{\cdot\,\cdot\,\cdot}{h\,a\,o}$ (6) $\frac{\cdot\,\cdot\,\cdot}{r\,a\,o}$ (7) $\frac{\cdot\,\cdot\,\cdot}{m\,o\,s}$

3. Articles bearing the following selling marks are marked to sell at a profit of $33\frac{1}{3}\%$; fill in the cost mark, using the same key as above:

(1) $\frac{m\,s}{\cdot\,\cdot\,\cdot}$ (2) $\frac{z\,s}{\cdot\,\cdot\,\cdot}$ (3) $\frac{m\,a\,s}{\cdot\,\cdot\,\cdot}$ (4) $\frac{h\,a\,s}{\cdot\,\cdot\,\cdot}$ (5) $\frac{r\,n\,s}{\cdot\,\cdot\,\cdot}$ (6) $\frac{z\,n\,s}{\cdot\,\cdot\,\cdot}$

4. Using the same key, mark articles costing the following prices to sell at a profit of $37\frac{1}{2}\%$:

(1) 40¢ (2) 72¢ (3) \$1.68 (4) \$2.88 (5) \$6.40 (6) \$8.24

5. Supply complete cost marks for articles sold at 50% profit the selling prices of which are as follows:

(1) 90¢ (2) \$1.50 (3) \$2.50 (4) \$3.66 (5) \$7.50 (6) \$8.10

6. Solve problems 4 and 5, using the key "black horse."

7. Choose a key word and solve problems 4 and 5.

8. A merchant wishes to mark his goods so that he may drop 10% below the marked price and still make 20% of the cost price. Using the key "harmonizes," how must he mark articles costing the following prices:

(1) 50¢? (2) 60¢? (3) \$1.20? (4) \$2.50? (5) \$6.40?

§88. Interest.

Answer all you can orally.

In reckoning interest the year is commonly regarded as containing 12 mo. of 30 da. each.

1. A man is charged \$6 for the use of \$100 for 1 yr. What percent of the sum borrowed (\$100) equals the sum (\$6) he is charged for its use?

2. A man is charged \$21 for the use of \$350 for 1 yr. The sum charged equals what percent of the sum borrowed?

Interest is money charged for the use of money.

Interest is reckoned at a certain rate percent of the sum borrowed for each year it is borrowed.

When money earns 3, 6, 7, or 10 cents on the dollar *annually* (each year) the *rate* is said to be 3%, 6% 7%, or 10% *per annum* (by the year) and the *rate percent* is said to be 3, 6, 7, or 10.

The only difference between calculating percentage and calculating simple interest is that after the percentage is calculated it must be multiplied by the *time expressed in years.*

To find the simple interest on \$480 at 5% for $2\frac{1}{4}$ years—

To find percentage	To find simple interest
\$480 *base*	\$480 *principal*
.05 *rate*	.05 *rate*
\$24.00 *percentage*	\$24.00 *interest for* 1 *yr.*
	$2\frac{1}{4}$
	\$48.00
	\$ 6.00
	\$54.00 *interest for* $2\frac{1}{4}$ *yr.*

The numbers used in percentage are named differently in simple interest, though they are used the same way.

It may help to notice that—

in percentage		in simple interest
base	corresponds to	*principal*
rate	corresponds to	*rate*
percentage	corresponds to	*interest for* 1 *yr.*

3. At 6% per annum, how much interest does $360 earn in 1 yr.? In 3 yr.? In $4\frac{1}{2}$ yr.? In $2\frac{2}{3}$ yr.? In t years?

4. Make a rule for computing the interest on any sum of money at 6% when the time is in years.

SIX PER CENT METHOD

5. At 6% per annum, how much interest does $1 earn in 1 yr.? In 2 mo.? In 1 mo.? In 6 da.? In 12 da.? In 18 days?

6. When the time is in months, how may the interest on any sum of money at 6% per annum be computed?

7. How may the interest at 6% per annum on any sum of money be computed when the time is given in months and days? In years, months, and days?

The sum of money on which the interest is computed is called the principal. The principal plus the interest is called the amount.

Since the borrower must not only pay the interest on the borrowed principal, but also return the principal, the debt he must discharge (pay) is the *amount.*

8. How long will it require any principal ($1) to amount to twice its value ($2), or to double itself at 6%?

9. Give the reasons for these statements:

Any principal, at 6%, (1) doubles itself in 200 months; (2) earns $\frac{1}{10}$ of itself in 20 months; (3) earns $\frac{1}{100}$ of itself in 2 mo. or 60 days.

10. Let I stand for the interest on $\$p$ at $r\%$ for t yr. and let i denote the interest on $\$p$ at 6% for t yr. Explain the meaning of the equations:

$$i=\frac{6\times p\times t}{100}; \qquad I=\left(\frac{r}{6}\right)\times i$$

To find the interest, I.

GENERAL METHOD

11. What is .07 of $450? What is the interest on $450

at 7% for 1 yr.? For 2 yr.? For $3\frac{1}{2}$ yr.? For t years?

12. What is .08 of \$1250? What is the interest at 8% on \$1250 for 1 yr.? For 3 yr.? For $5\frac{10}{12}$ yr.? For x years?

13. Find the interest at 5% on \$640, for 4 years.

14. \$1800 was on interest at 7% for 3 yr. 7 mo. 21 da. Find the interest.

CONVENIENT FORMS

General Method

I. Interest computed first at the given rate.

	\$1800	, principal
	.07	rate
	\$126.00	int. for 1 yr.
	3	whole years
	\$378.00	int. for 3 yr.
$\frac{1}{2}$ of \$126.00 =	63.00	int. for 6 mo.
$\frac{1}{6}$ of 63.00 =	10.50	int. for 1 mo.
$\frac{1}{2}$ of 10.50 =	5.25	int. for 15 da.
$\frac{1}{5}$ of 10.50 =	2.10	int. for 6 da.
	\$458.85	int. for 3 yr. 7 mo. 21 da.

Six Percent Method

II. Interest computed first at 6%.

\$.18	, int. on \$1 for 3 yr. at 6%
.035	int. on \$1 for 7 mo. at 6%
.0035	int. on \$1 for 21 da. at 6%
.2185	int. on \$1 for 3 yr. 7 mo. 21 da. at 6%
1800	
1748000	
2185	
393.3000	, int. on \$1800 for given time at 6%
65.55	int. on \$1800 for given time at 1%
\$458.85	int. on \$1800 for given time at 7%

SUGGESTIONS FOR II.—

(1) If the interest on \$1 for 1 yr. is 6¢, what is the interest for 3 years?

(2) If the interest on \$1 for 2 mo. is 1¢, what is the interest for 7 mo.?

(3) If the interest on \$1 for 6 da. is 1 mill, what is the interest for 21 da.?

15. How much interest must be paid for the use of—

(1) \$600 for 1 yr. 5 mo. 24 da. at 7 percent?

(2) \$350 for 2 yr. 4 mo. 21 da. at 8 percent?

(3) \$575 for 2 yr. 8 mo. 18 da. at $6\frac{1}{2}$ percent?

(4) \$860 for 3 yr. 9 mo. 15 da. at $5\frac{1}{2}$ percent?

16. Find the amount of—

(1) \$300 for 2 yr. 4 mo. 25 da. at $4\frac{1}{2}$ percent?

(2) \$160 for 3 yr. 7 mo. 20 da. at $6\frac{1}{2}$ percent?

(3) \$850 for 3 yr. 2 mo. 14 da. at $5\frac{1}{2}$ percent?

17. Find the interest and the amount for the following:

(1) \$700 for 1 yr. 7 mo. 15 da. at 3 percent.

(2) \$400 for 2 yr. 9 mo. 27 da. at 3 percent.

(3) \$210 for 2 yr. 5 mo. 28 da. at 6 percent.

(4) \$150 for 1 yr. 11 mo. 13 da. at 7 percent.

(5) \$280 for 1 yr. 6 mo. 19 da. at $4\frac{1}{2}$ percent.

(6) \$360 for 1 yr. 4 mo. 5 da. at 4 percent.

(7) \$260 for 2 yr. 3 mo. 11 da. at $3\frac{1}{2}$ percent.

(8) \$300 for 2 yr. 7 mo. 12 da. at 5 percent.

To find the principal, P.

18. What principal at 8% will furnish \$16 interest in 2 years?

SUGGESTION: What interest will \$1 produce at 8% in 2 yr.? How many dollars will yield \$16 at 8% in 2 years?

19. What principal at 8% will produce \$30 interest in $2\frac{1}{2}$ years?

20. What principal at 6% will amount to \$112 in 1 yr. 6 months?

SUGGESTION: What is the amount of \$1 at 6% interest for 1 yr. 6 mo.?

21. Find the principal which will yield \$61.25 interest in 3 yr. 6 mo. at 7 percent.

22. Find the principal which will amount to \$972.40 in 3 yr. 2 mo. 12 da. at $4\frac{1}{2}$ percent.

23. Make a rule for finding the principal when the rate, time, and interest are given.

24. Make a rule for finding the principal when the rate, time, and amount are given.

25. Supply the value for the letter in the following:

	Rate	Time	Interest	Principal	Amount
(1)	6 %	1 yr. 7 mo. 15 da.	\$19.50	*P*	*A*
(2)	8 %	3 yr. 3 mo. 18 da.	\$169.02	*P*	*A*
(3)	$8\frac{1}{3}$%	4 yr. 7 mo. 21 da.	*I*	*P*	\$998.50
(4)	7 %	6 yr. 8 mo. 16 da.	*I*	*P*	\$198.42
(5)	4 %	2 yr. 6 mo. 18 da.	\$122.50	*P*	*A*
(6)	5 %	1 yr. 10 da.	\$176.00	*P*	*A*
(7)	$4\frac{1}{2}$%	90 da.	\$32.50	*P*	*A*

To find the rate, r.

26. At what rate percent will \$320 yield \$39.60 interest in 2 yr. 9 months?

Suggestion: How much interest will \$320 yield in 2 yr. 9 mo. at 1%? At what rate percent then will the same sum yield \$39.60 interest in 2 yr. 9 months?

27. At what rate will \$780 yield \$512.72 in 5 yr. 8 mo.?

28. A man invested \$2000 for 2 yr. 7 mo. 27 da. and received \$2638 at the end of this time. What rate percent of interest did his investment earn for him?

29. A man bought 120 A. of land at \$85 and sold it 2 yr. 8 mo. later for \$100 per A., after having received \$900 in rents and having twice paid taxes at 75 cents per acre. What was his annual rate per cent of profit?

Suggestion: Add the difference between the rent and the taxes to the selling price to find the net receipts.

30. Make a rule for finding the rate when the principal, time, and interest are known.

31. Find the correct value to the nearest .1 for the letter in each of the following problems:

	Principal	Time	Interest	Rate
(1)	$580.00	1 yr. 5 mo.	$46.50	$r\%$
(2)	$798.45	2 yr. 8 mo. 15 da.	$258.65	$r\%$
(3)	$3698.50	1 yr. 5 mo. 19 da.	$568.75	$r\%$

To find the time, t.

32. How long will it take $80 to earn $14 interest at 4% annually?

Suggestion: How much interest will $80 earn in one year at 4%? In how many years then will $80 earn $14 at the same rate?

33. At 8% how long will $125 take to earn $57.50?

34. At 7%, how long will it take $648 to yield $69.84?

Note: When the time results in decimals of a year the decimal may be reduced to months and days by the method of problem 65, p. 110.

35. At 8% how long will $750 take to yield $750.

36. How long will it take $10 to double itself at 6%?

37. How long will $975 take to amount to $1225 at 5%?

Suggestion: What is the total interest? What is the interest on $975 at 5% for 1 year?

38. Make a rule for finding the time, t, required for a given principal, P, to amount to a given sum, A, at a given rate percent, r.

39. Make a rule for finding how long it will take a given principal to earn a given interest at a given rate percent.

40. Find the time, t, in years, months and days under the conditions stated in each of the following problems:

	Principal	Rate	Interest	Time
(1)	$66.00	7 %	$28.60	t
(2)	$460.00	6 %	$31.05	t
(3)	$750.00	$5\frac{1}{2}\%$	$147.47	t

41. Supply the value for which each letter stands in the problems of the following table:

	Principal	Rate	Time	Interest	Amount
(1)	\$60.00	6%	3 yr. 3 mo. 8 da.	*I*	*A*
(2)	\$175.00	5%	4 yr. 9 mo. 15 da.	*I*	*A*
(3)	\$800.00	7%	*t*	*I*	\$926.00
(4)	\$475.00	8%	*t*	\$142.50	*A*
(5)	\$1266.00	4%	*t*	*I*	\$1349.85
(6)	*P*	6%	5 yr. 7 mo. 27 da.	\$509.25	*A*
(7)	\$1575.30	*r*%	1 yr. 4 mo. 18 da.	*I*	\$1662.46
(8)	\$728.25	8%	*t*	\$209.736	*A*

42. Complete, to mills, the following interest table:

Interest Table: Principal \$100.

Time	Rate				
	3%	4%	5%	6%	7%
1 da.	\$.008	\$.011	\$.014	\$.017	\$.019
2 da.					
3 da.					
4 da.					
5 da.					
6 da.					
1 mo.					
2 mo.					
3 mo.					
6 mo.					
1 yr.					

43. Compute by the table the interest on \$758 at 7% for 4 mo. 12 da.; for 7 mo. 8 da.; for 1 yr. 3 mo. 10 days.

44. Calculate by the table the interest on \$865 at 4% for 3 yr. 10 mo. 18 da.; for 8 yr. 11 mo. 24 days.

45. Calculate by the table the amount on \$1280 at 6% for 4 yr. 4 mo. 17 da.; for 9 yr. 8 mo. 16 days.

Given amount, principal, and rate; to find time.

46. The amount is \$180.80, the principal is \$160, and the rate is 4%. Find the time.

$I = \$180.80 - \$160 = \$20.60$; $.04 \times \$160 = \$6.40$ interest for **1 year.**

$$\text{time} = \frac{\text{total interest}}{\text{interest for 1 yr.}} = \frac{\$20.80}{\$\ 6.40} = 3.25 \text{ yr.} = 3 \text{ yr. } 3 \text{ mo.}$$

47. A principal of \$80 loaned at 6% amounted to \$102.40. How long was it loaned?

48. How long will it take \$100 to double itself at 5% simple interest?

Given amount, principal, and time; to find rate.

49. At what rate will \$50 amount to \$60 in 4 years?

$I = \$60 - \$50 = \$10$. Interest on \$50 at 1% for 4 yr. = \$2.00.

$\frac{\$10}{\$2} = 5$. Answer, 5%.

50. At what rate will \$600 amount to \$700 in $4\frac{1}{6}$ years?

51. At what rate will \$100 double itself in 5 years?

Given amount, rate, and time; to find principal.

52. The amount is \$15.60, the rate 6%, and the time 5 years. Find the principal, p.

The amount of \$1 at 6% for 5 yr. is \$1.30.

$$\frac{\text{amount on } p \text{ dollars}}{\text{amount on 1 dollar}} = \frac{\$15.60}{\$1.30} = 12; \text{ principal, } \$12.$$

53. What principal at 7% will amount to \$595 in 10 years?

54. What principal at 3% will amount to \$625 in 8 yr. 4 mo.?

55. What principal at 2.5% will amount to \$362.50 in 18 years?

§89. General Number.

1. Compute the interest on \$750 at 5% for each of the following times: (1) 1 yr. (2) 2 yr. (3) $6\frac{1}{2}$ yr. (4) $3\frac{2}{3}$ yr. (5) $25\frac{5}{6}$ yr. (6) x yr. (7) t years.

2. Find the interest and the amount on \$1250 for each of the following:

(1) 5%, 1 yr. (4) 6%, $3\frac{1}{3}$ yr. (7) r%, t yr.
(2) 6%, $2\frac{3}{4}$ yr. (5) 4%, $7\frac{3}{8}$ yr. (8) r%, x yr.
(3) 7%, 6 yr. (6) 3%, t yr. (9) r%, n yr.

3. Denote the interest on a certain principal, P, by I, the rate by r, and the time (in years) by t. Write an equation showing how to find I from P, r, and t, and translate into words the meaning of the equation.

SOLUTION:]

$$\text{(I)} \quad \begin{cases} \text{(1) } I = P \times \frac{r}{100} \times t, \text{ or} \\ \text{(2) } I = \frac{P \times r \times t}{100}, \text{ or (3) } I = \frac{Prt}{100}. \end{cases}$$

Translated into words: (1) means, "Interest equals the product of the principal, the rate divided by 100, and the time (in years)."

(2) and (3) mean, "Interest equals the product of principal, rate, and time, divided by 100."

4. Calling A the amount, I the interest, and P the principal, write an equation showing how to find A from I and P.

5. Write an equation to show how to find P from I, r, and t, and state in words the meaning of the equation.

If we multiply both sides of the equation, $I = \frac{Prt}{100}$, by 100 we have the equation, $100\,I = Prt$. To show how to find P from I, r, and t, divide both sides by rt, and write the second member on the left. We then have:

$$\text{(II)} \quad P = \frac{100\,I}{rt}.$$

6. State in words the meaning of formula (II).

7. Show, by proper multiplications and divisions of equation (I) (3), that the rate r may be found from P, I, and t, by the equation,

$$\text{(III)} \quad r = \frac{100\,I}{Pt}.$$

8. State in words the meaning of formula (III).

9. Show from (I) (3) that the time t may be found from I, P, and r by the formula,

$$\text{(IV)} \quad t = \frac{100\,I}{Pr}.$$

10. State in words the meaning of formula (IV).

11. Solve by formulas I-IV the following problems:

(1) $P = \$64$, $r = 8$, and $t = 2\frac{1}{4}$; find I.

(2) $I = \$24$, $r = 6$, and $t = 1\frac{1}{2}$; find P.

(3) $I = \$75$, $P = \$850$, and $t = 2$; find r.

(4) $I = \$120$, $P = \$600$, and $r = 10$; find t.

(5) $I = \$230.13\frac{3}{4}$, $P = \$722$, and $t = 3\frac{3}{4}$; find r.

§90. Promissory Notes.

A *promissory note* is a written promise, made by one person or party, called the *maker*, to pay another person or party, called the *payee*, a specified sum of money at a stated time.

The sum of money for which the note is drawn is called *the face value*, or the *face*, of the note.

The date on which the note falls due is called *the date of maturity*, and *the time to run* is the time yet to elapse before the note falls due.

$250.00 *Kankakee, Ill.*, June 1, 1903.

Ninety days *after date I promise to pay to the order of* A. M. Baker, Two Hundred Fifty and $\frac{00}{100}$ *Dollars, for value received, with interest at* six *percent per annum from date.*

Due Aug. 30, 1903. H. R. Newman.

1. Who is the maker of the above note? The payee? What is the face of the note? The date? The rate of interest? The date of maturity? The time to run?

2. A promissory note, unless otherwise specified in the note, draws interest on its face value at the rate mentioned in the note from the date of the note until it is paid. Compute the interest and the amount on the foregoing note if it was paid Sept. 1, 1903.

3. Find the interest on the following note, paid Aug. 30, 1903:

\$875. URBANA, ILL., May 18, 1897.

One year after date I promise to pay to James Black, or order, Eight Hundred Seventy-Five and $\frac{0\,0}{1\,0\,0}$ Dollars, at Busey's Bank, for value received, with interest at the rate of seven percent per annum from date.

Due May 18, 1898. HENRY OSBORN.

NOTE: To find the time for which interest is to be computed, proceed thus:

CONVENIENT FORM

	yr.	mo.	da.
Date of payment,	1903	8	30
Date of note,	1897	5	18
Time,	6	3	12

4. Find the amount of each of the following notes:

	FACE	RATE	DATE OF NOTE	DATE OF PAYMENT	INTEREST	AMOUNT
(1)	\$250	6 %	Mar. 12, 1899	Jan. 1, 1902	.....	.....
(2)	\$635	7 %	Nov. 20, 1896	July 15, 1900	.....	.....
(3)	\$2400	5½%	Dec. 12, 1899	Jan. 8, 1903	.....	.....
(4)	\$3865	4½%	Oct. 13, 1901	Sept. 7, 1903	.....	.....
(5)	\$3640	8⅓%	Aug. 17, 1900	Mar. 3, 1903	.....	.....

§91. Discounting Notes.

Discount is a deduction from the amount due on a note at the date of maturity.

In some cases promissory notes do not draw interest. The following is an example:

John C. Cannon purchased a self-binding harvester from A. R. Crow for $120 and gave him the following note in payment:

> $120. Peoria, Ill., June 20, 1906.
> Eighteen months after date, for value received, I promise to pay to A. R. Crow, or order, One Hundred Twenty and $\frac{0}{100}$ Dollars, without interest until due.
> Due Dec. 20, 1907. JOHN C. CANNON.

1. On September 20, 1906, Crow sold the note to Adams at such a price that Adams received his purchase money and 8% interest on it until the date of maturity (Dec. 20, 1907). How much did Adams pay for the note?

SUGGESTIONS: Any principal at 8% will amount to 110% of itself in 1 yr. 3 mo. Why? Hence, $120 = 110% of what number? Or, better, $1.10x = $120. Find the value of x.

The sum of money which, at the specified rate and in the time the note is to run before falling due, will amount to the value of the note, when due, is called the present worth of the note.

The difference between the value of the note, when due, and the present worth is called the true discount.

The bank discount of a note is the interest upon the value of the note when due, from the date of discount until the date of maturity.

2. If Crow had sold the above note to a banker at a discount of 8%, the banker would have computed the interest at 8% on $120 from the date of sale (Sept. 20, 1906) until the date of maturity. How much would he have received for the note? Find the difference between the true and the bank discount of the note.

3. C. A. Thomas bought a road wagon of J. K. Duncan, giving the following note in payment:

> $65. Pekin, Ill., Sept. 10, 1900.
> Two years after date I promise to pay J. K. Duncan, or order, Sixty-five Dollars, value received, with interest at 7% per annum.
> Due Sept. 10, 1902. C. A. THOMAS.

On March 10, 1901, Duncan sold this note to a bank at 7½% discount. How much did he receive for it?

NOTE: Remember, bank discount is computed on the *amount* of the note *when due*, for the time to run from date of sale.

4. Counting money worth 7%, how much did Duncan receive for the wagon?

SUGGESTIONS: First calculate the amount due at maturity on Sept. 10, 1902. Next, calculate the discount at 7½% for 1½ years on this amount, and deduct it from the amount.

Then find what sum of money at 7% will amount to this difference in ½ year. $1.035x = \$65.76$, find x.

5. A man bought a horse, giving in payment his note for 1 yr. for $85, dated Feb. 26, 1903, and drawing interest at 7% from date. Two months later the holder of the note discounted it at a bank at 6%. What was the discount? What did the bank pay for the note?

6. Find the bank discount and proceeds on the following notes:

	FACE	RATE	DATE OF NOTE	DATE OF MATURITY	DATE OF SALE	RATE OF DISC.
(1)	$60	6 %	Apr. 6, 1897	Oct. 6, 1898	June 15, 1897	7 %
(2)	$275	6½%	Aug. 7, 1899	Nov. 7, 1901	Mar. 13, 1900	6 %
(3)	$350	7½%	Sep. 10, 1900	Dec. 10, 1902	Jan. 15, 1901	6 %
(4)	$700	8 %	Feb. 15, 1901	Nov. 15, 1903	June 20, 1901	6 %
(5)	$858	6 %	Jan. 1, 1900	July 15, 1903	May 19, 1900	7 %
(6)	$1260	5 %	Feb. 28, 1896	Aug. 31, 1900	Aug. 8, 1896	6 %
(7)	$1800	5½%	June 19, 1897	Aug. 19, 1901	Dec. 29, 1898	6 %
(8)	$2450	7 %	Nov. 18, 1899	Feb. 28, 1902	Dec. 1, 1900	5½%
(9)	$3865	6 %	Dec. 20, 1901	Nov. 20, 1902	Feb. 12, 1902	5 %
(10)	$8600	4½%	Oct. 18, 1900	Jan. 18, 1904	Apr. 2, 1901	4 %

PART II

PRACTICAL APPLICATIONS OF NUMBER

§92. Pressure of Air.

Answer all you can orally.

I. When a glass is filled level with water, covered with a piece of writing paper, and carefully inverted, why does not the water fall out when the glass is held mouth downward?

II. When a soft leather sucker is moistened and spread out on a smooth, flat surface, why does it cling to the surface even when the sucker is raised by being lifted at its middle?

III. When one end of a glass tube is placed in water and you draw with the mouth at the other, why does the water rise in the tube?

IV. Fill a bottle with water, leave the cork out, and invert the bottle in a vessel of water. Why does the water not run out?

V. When a glass tube with one end closed is partly filled with mercury and the open end dipped under the surface of mercury in a cup, why does not all the mercury run out of the tube?

NOTE: In connection with V, examine a mercury barometer.

VI. When a sheet of thin rubber, or paper, is held over the mouth of a funnel, and the air is sucked out of the funnel through its neck, why does the rubber curve inward? Will it do this in all positions of the funnel? Why?

These experiments show that the air presses downward, upward, and in all directions upon surfaces. Careful measurements have shown that the pressure of the air on every square inch of surface is about 15 pounds.

NOTE: This pressure is equal on all sides of surfaces, upper sides and lower sides, outside and inside, toward the right and toward the left.

Use pencil only when necessary.

1. Measure the length and the width of the cover of your book and find the downward pressure in pounds on the upper surface of your book when it lies flat upon the desk. Why is the book so easily moved about under this pressure?

2. Measure and compute the downward pressure of the air on the top of your desk. Can you lift this number of pounds? Why can you lift the desk, if it is not fastened?

3. A room is $10' \times 20' \times 25'$. Find the pressure of the air, in pounds, on the floor, on the ceiling, and on each of the four walls of the room. Why does not this pressure tear the walls apart?

4. The average surface of the human body equals 20.6 sq. ft. What is the total pressure of the air on the outside surface? Why does not this pressure crush the body?

5. Measure the length and the width of the door of your room, and find the air pressure on one side of the door. Why can you open and close the door so easily?

6. Measure and find the air pressure in pounds on the outside surface of a pane of window glass. Why does not this pressure break the glass?

7. Calculate the air pressure on some regular surface in your room.

§93. Passenger and Freight Trains.

1. A fast train runs from Chicago to a station 356.4 mi. distant in exactly 9 hr. What is the average rate (miles per hour) of the train?

2. The train left Chicago at 6:10 A.M. At what time did it arrive at the station?

3. Another train runs $10\frac{1}{2}$ hr. at an average rate of 36 mi. per hour, including stops. How far does it run?

4. If the train (problem 3) started at 2:30 A.M., at what time did it reach the end of the run?

5. A traveler went by rail from Chicago to Los Angeles, California, in 4 da. 19 hr., at the average rate of 37.4 mi. an hour, including stops. How far is it from Chicago to Los Angeles by this route?

6. A train ran 361 mi. in $9\frac{1}{2}$ hr. What was its average rate?

7. A freight train is running 21 mi. an hour while a brakeman on top of the cars walks toward the engine at the rate of $2\frac{1}{2}$ mi. an hour. How fast does the brakeman actually move forward? How fast, if he walks from front to rear at the rate of $2\frac{1}{2}$ mi. an hour?

8. A conductor walks from the front to the rear of a train at the rate of $3\frac{1}{2}$ mi. an hour while the train is running 38 mi. an hour. At what rate per hour does the conductor actually move forward? At what rate does he move in the direction of the running train if he walks from the rear to the front at the rate of $3\frac{1}{2}$ mi. an hour?

9. How may the conductor (problem 8) suddenly change his rate of motion from $34\frac{1}{2}$ mi. to $41\frac{1}{2}$ mi. an hour? To how many miles an hour does this change of rate amount?

10. A passenger train running 32 mi. an hour meets a freight train running in the opposite direction on a parallel track $18\frac{1}{2}$ mi. an hour. At what rate do the trains approach each other?

11. At what rate does the passenger train (problem 10) pass the freight if both are running in the same direction on parallel tracks?

12. A boy tries to overtake a street car by running up behind it. If the boy runs 10 ft. a second and the car runs 6 ft. a second, how soon will the boy overtake the car if he is now 20 ft. behind it?

13. Two friends, coming from opposite directions, have arranged to meet at a certain railway station. Their trains are running at 35 mi. and 25 mi. an hour. Not allowing for time lost in stops, how soon will their trains be together at the station, if they are now 30 mi. apart and both trains reach the station at the same time?

14. A passenger train is made up of a postal and baggage car, an express car, 3 common coaches, 2 chair cars, a dining car, and 2 sleepers. The average length of a car is $61\frac{2}{5}'$, and the length of the engine and tender together is $65'$. How long is the train?

15. The weight of the engine (problem 14) is 142,780 lb.; of the tender, 43,200 lb., and the average weight of a car is 83,480 lb. What is the total weight of the train, in tons?

16. To draw a train on straight, level track at a speed of 40 mi. an hour, requires a horizontal pull of $\frac{1}{200}$ of the weight of the train. What force in tons is needed to draw the train of problem 14 at a speed of 40 mi. an hour?

The weight of a train includes the weight of the engine and tender.

17. An engine, 62′ long and weighing 240,000 lb., draws a tender, 22′ long, weighing 64,500 lb., and a train of 72 empty freight cars, averaging $36\frac{1}{2}$′ in length and 32,700 lb. in weight. How long is the train? How heavy is the train including engine and tender?

18. If it requires a force of $\frac{3}{1250}$ of the weight of a train to draw it on straight, level track at a speed of 20 mi. an hour, how many pounds of force must the engine of problem 17 exert to draw the train, on such track, 20 mi. an hour? How many tons of force?

19. How much force would be needed to draw 38 cars, each weighing $16\frac{1}{2}$ T., and each loaded with $18\frac{1}{2}$ T. of coal, on straight, level track at a speed of 20 mi. an hour? (See problem 18.)

20. When a train is moving 5 mi. an hour it takes a horizontal force of about $\frac{1}{700}$ of the weight of the train to draw it along straight, level track. On such track what force must an engine exert to draw a train of 68 empty freight cars, each weighing 34,300 lb., at a speed of 5 mi. an hour?

21. Under the same conditions as in problem 20, what force would be needed to draw a train of 38 cars, each weighing 34,000 lb. and carrying a load of 58,600 lb. of coal in addition to its own weight?

22. A railroad company purchased 6 locomotive engines, weighing, in pounds: 142,780; 142,630; 158,670; 139,790; 146,890; and 138,960. The tenders weighed, in pounds: 43,750; 44,200; 45,280; 42,920; 43,650, and 44,280. The cost per pound of the combined weight was $13\frac{4}{5}$¢. What was the total cost?

23. The greatest horizontal pulling force that an engine

can exert on dry, unsanded steel track in starting a train is about $\frac{2}{9}$ of the weight of the part of the engine that is carried by the driving wheels. What is the greatest horizontal pull the engine of problem 17 can exert in starting a train on such track, supposing the entire weight of the engine is carried by the driving wheels?

24. Answer a similar question for the engine of problem 15, supposing 105,750 lb. of the weight of the engine is carried by the driving wheels.

25. An engine pulls with a force of $\frac{5}{3\,6}$ of the weight that is borne by its driving-wheels; $\frac{5}{7}$ of its weight of $94\frac{1}{2}$ tons is on these wheels. Find the pulling force in pounds of this engine.

26. If a pull of 8 lb. per ton of weight draws a train 30 miles an hour, how many pounds of force are needed to draw at this speed 40 loaded cars, weighing 60 tons each?

§94. Train Despatcher's Report.

A copy from a train despatcher's sheet, for a test run, showing the distance between stations, the schedule time and the exact running time of the train, the number of the cars in the train, the number of passengers carried each trip for 7 week days is given on the next page.

NOTE.—The problems are based on the numbers of the table.

1. By "running time" is meant the difference between the time of leaving* station 1 and the time of arriving at station 15. On a separate sheet write the "running time" for each day.

2. Find the difference between "schedule time" and the time the train actually reached station 6 on the 2d, 4th, 6th, 7th, 8th.

*The train not stopping at stations, leaving and arriving times are the same.

PERFORMANCE OF TRAIN NO. 25, FROM JULY 1 TO 8, 1898.

STATION	DISTANCE MI.	SCHEDULE TIME	1ST	2D	4TH	5TH	6TH	7TH	8TH
1...	0	3:50	3:50	3:49¾	3:50¾	3:50	3:50½	3:50½	3:51¼
2...	3.1	3:55	3:55	3:55½	3:56	3:55	3:55	3:55¼	3:56
3...	5.5	3:57	3:57	3:58	3:58½	3:57¼	3:58¾	3:57¾	3:58¼
4...	7.9	3:59	3:59	4:00	4:00½	3:59¼	4:00¼	4:00	4:00
5...	12.0	4:02	4:02	4:03¾	4:03¾	4:03¼	4:04	4:03	4:03
6...	17.0	4:06	4:06	4:07¼	4:07½	4:06¼	4:08½	4:07½	4:07½
7...	19.9	4:09	4:08	4:10	4:09½	4:08½	4:11	4:10	4:10
8...	24.5	4:12	4:11½	4:14	4:13	4:12	4:14½	4:14	4:13¾
9...	27.6	4:15	4:13½	4:16½	4:15	4:14	4:17¾	4:16	4:15
10...	33.8	4:20	4:18	4:21¼	4:20	4:18½	4:22	4:20¾	4:20¾
11...	38.7	4:24	4:22	4:25	4:24	4:22	4:26½	4:24¼	4:23
12...	43.5	4:28	4:25½	4:28½	4:27½	4:26	4:30	4:28	4:27
13...	50.5	4:33	4:30½	4:34½	4:32½	4:31	4:35½	4:33	4:32
14...	53.8	4:38	4:32½	4:37½	4:35	4:34	4:38	4:36	4:35½
15...	55.5	4:40	4:35¼	4:40	4:37¾	4:36	4:40¼	4:37¾	4:37
Number of cars ...			5	7	6	5	5	5	5
Passengers carried..			201	441	11	79	107	113	118
Running time.....									
Mi. per hour (average)									

3. The train is "on time" if it reaches a station just at "schedule time." Find how much the train was ahead of or behind time in reaching station 12 on each of the 7 da. Mark "ahead of time" results with an "A," thus: A. $2\frac{1}{2}$ min., and "behind time" results with a "B," thus: B. $1\frac{1}{4}$ minutes.

4. Make and answer questions for other stations.

5. How far is it from station 1 to station 15? From 2 to 9? From 4 to 11? From 3 to 15? From 7 to 15?

6. What is the difference in schedule times between stations 1 and 15? 2 and 9? 4 and 11? 3 and 15? Any other two stations?

7. From your answers to problems 5 and 6 find the average* rate of running (in miles per minute) between stations 1 and 15, by a train running exactly on schedule time; between stations 2 and 9; 4 and 11; 3 and 15.

8. Find the average rates of running between the same stations (as in problem 7) on the 6th.

9. Make and solve other problems on the table.

§95. Areas of Common Forms.

1. If one side of an inch square represents 80 rd., how many square rods will the inch square represent? How many acres?

2. In a drawing of a rectangular farm to a scale 1 in. = 80 rd., how could you find the number of acres in the farm?

3. If a square field containing 40 A. is cut across diagonally by a railroad, how many acres does each of the triangular parts contain, supposing the railroad itself covers 3 acres?

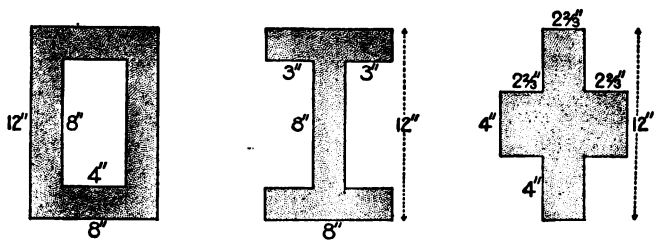


FIGURE 1

4. Give the areas of the shaded portions of the 3 forms of Fig. 1.

* "Average rate" here means *distance run divided by time of running.*

§96. Cost of Living.

Use pencil and paper only when needed.

1. If the living expenses of a family are \$70 per mo., what is the expense per year?

2. If house rent is \$25 per mo. and other expenses are \$50, what part of the total expense is house rent?

3. How many work days are there in a month? How many weeks in a year?

4. Find out at home or from your neighbor how much hard coal is required to supply some furnace or stove 1 mo. Supposing hard coal costs \$7.50 per T., how much will the coal cost for the 6 winter months from November to May?

5. If soft coal costs \$3.75 a T., and $1\frac{3}{4}$ T. goes about as far as 1 T. of hard coal, about how much would it cost to supply this furnace with soft coal for the 6 winter months?

The prices given below are fair averages for a large city. They would be less in smaller places, and prices current in your community are to be preferred to these.

GROCER'S PRICES

Apples	pk. \$.35	Olives	pt. \$.30
Apricots	can .25	Onions	pk. .20
Baking powder	½ lb. can .20	Oranges	doz. .30
Bread	loaf .05	Peaches	can .25
Butter	lb. .28	Pears	can .25
Cabbage	lb. 2¢., head .10	Peas	can .12½
Cauliflower	head .15	Pickles	doz. .10
Celery	bunch .10	Potatoes	pk. .25
Cheese	lb. .15	Prunes	lb. .10
Coffee	lb. .30	Rice	lb. .08⅓
Crackers	lb. .10	Rolls	doz. .10
Eggs	doz. .25	Salt	lb. .05
Flour	25 lb. sack .50	Soap	bar .05
Lemons	doz. .25	Starch	lb. .10
Lettuce	bunch .05	Sugar	lb. .05
Lima beans	can .10	Tea	lb. .60
Milk	qt. .06¼	Tomatoes	can .15
Oatmeal	2 lb. pkg. .10		

BUTCHER'S PRICES

Bacon	lb.	$.20	Pork tenderloin	lb.	.20
Chicken	lb.	.14	Porterhouse steak	lb.	.18
Fish, fresh	lb.	.12½	Round steak	lb.	.10
Fish, salt	lb.	.10	Salmon	lb.	.20
Ham	lb.	.18	Sausage meat	lb.	.10
Lard	lb.	.13	Sirloin steak	lb.	.16
Lobsters, shrimps, etc.	lb.	.25	Spare ribs	lb.	.08⅓
Mutton	lb.	.14	Turkey, duck, etc	lb.	.12½
Oysters	qt.	.30	Veal	lb.	.14
Pork	lb.	.10			

NOTE: It will be assumed that rates are the same for large as for small quantities.

Make and solve such problems as the following:

6. If you purchase of a grocer 2 loaves of bread, 1 lb. of butter, 1 lb. of coffee, and 1 pk. of potatoes, and give him $1, how much change should you receive, if prices are as quoted (named) in the table above?

7. If you buy a 2-lb. sirloin steak and give the butcher 50¢, what change should you receive, prices being as in the table?

8. Pupils may make out lists in the form of a statement and find the change due, if the account is paid with $2, $5, or $10, thus:

BOUGHT			PAID	
2 doz. rolls	@ 10¢.	$0.20	Cash	$5.00
1 doz. eggs	25¢.	.25		
1 doz. oranges	30¢.	.30		
2 bunches celery	10¢.	.20		
1 lb. tea	60¢.	.60		
2 cans peas	12½¢.	.25		
1 lb. starch	10¢.	.10		
5 bars soap	5¢.	.25		
1 pkg. oatmeal	10¢.	.10		
Total cost				
Balance due in change				

Foot and balance the statement above.

9. Make out a bill of supplies such as a hotel keeper might purchase at the grocer's, or the butcher's.

10. Pupils may make and solve problems similar to the above, using prices obtained from local dealers, or newspapers.

Jan., 1902	
Meat	$18.50
Groceries	27.75
Milk	1.88
Butter	2.60
Soft coal	9.65
Clothing	15.00

11. The actual living expense account of a family for January is as shown in the table. If this is a fair average monthly expense account for the family, what will be its living expense for one year?

12. The family pays $3000 for a home and is thereby saved $25 per mo. in rent. The yearly saving in rent amounts to what part of the cost of the home?

13. If the home is in the suburbs of a city and the man must pay 10¢ morning and evening 6 da. per wk. for car fare, how much does this add to his yearly expense account?

The table given here shows the average yearly cost to one person for food, clothing, and other supplies from 1897 to 1901 inclusive:

For Year Ending	Bread-stuffs	Meat	Dairy & Garden	Other Food	Cloth-ing	Metals	Miscel-laneous	Totals
Dec. 31, 1897	$13.51	$7.34	$12.37	$8.31	$14.65	$11.57	$12.11	
Dec. 31, 1898	13.82	7.52	11.46	9.07	14.15	11.84	12.54	
Dec. 31, 1899	13.25	7.25	13.70	9.20	17.48	18.09	16.31	
Dec. 31, 1900	14.49	8.41	15.56	9.50	16.02	15.81	15.88	
Dec. 31, 1901	20.00	9.67	15.25	8.95	15.55	15.38	16.79	
Sums								
Averages								

Note.—In the above table breadstuffs includes wheat, corn, oats, rye, barley, beans, and peas; meat includes lard and tallow; dairy

and garden products include vegetables, milk, butter, eggs, and fruit; the miscellaneous articles include a variety of things which make a part of the cost of living for the average family.

14. Fill out a column like the totals-column, thus finding the total cost of living for 1 person for each year.

15. Find the increase in cost of living from 1897 to 1901.

16. Has any item decreased in cost during this period? What has been the change in cost for each year of breadstuffs? Of meat? Of dairy and garden?

17. What fractional part of the whole cost of living for 1897 is the cost of breadstuffs? For 1899? For 1901?

18. What fractional part of the whole cost of living for 1897 is the cost of clothing? For 1899? For 1901?

19. Find the average yearly cost of living for the 5 years.

20. Find the average cost of each item.

21. Give the increase, or decrease, from year to year, in the total cost of foods and clothing for one person.

§97. Miscellaneous Problems for Review.

The table below gives the numbers of men that were employed from month to month during 1900, in factories making pins, needles, hooks and eyes.

MONTH	Penn.	N. Y.	N. H.	Conn.	All other States	Total The U. S.
Jan.....	44	75	195	1180	339	1833
Feb.....	41	75	201	1223	327	
Mar.....	41	76	199	1237	331	
Apr.....	38	76	200	1213	321	
May.....	38	76	195	1209	314	
June.....	37	76	187	1207	314	
July.....	37	74	189	1196	314	
Aug	40	74	169	1259	328	
Sept.....	40	75	178	1279	331	
Oct.....	40	75	181	1271	328	
Nov.....	42	74	185	1265	326	
Dec.....	42	74	189	1257	327	

1. Add horizontally, check, and tell what the totals mean. Time yourself.

2. Find the average number of men employed per month in the several states and in "All other states."

3. What percent to the nearest 1%, of the total number of men employed in the United States were employed in Connecticut in January? In February? In March? In September?

The corresponding table for women employed in these factories is here given:

Month	Penn.	N. Y.	N. H.	Conn.	All other States	Total The U. S.
Jan.....	188	185	166	940	358	1837
Feb.....	188	185	169	967	363	
Mar.....	188	185	175	973	366	
Apr.....	188	185	163	970	370	
May.....	188	185	161	964	362	
June.....	188	185	155	951	376	
July.....	188	185	148	963	368	
Aug	188	185	139	954	357	
Sept.....	188	185	156	970	372	
Oct.....	188	185	159	978	360	
Nov.....	188	185	159	984	352	
Dec.....	188	185	158	966	352	

4. Add horizontally, check the work, and tell what the totals mean. Time yourself.

5. Give a quick way of finding the sums for Pennsylvania, and for New York. Check the work in this quick way for these two columns, and by casting out the nines for the rest of the columns.

6. Find the average number of women employed per month in the several states and in "All other states."

7. Compare the monthly average number of men with the monthly average number for women in the whole United States.

§98. Exercises for Practice and Review.

1. Solve the following problems:

(1) $\frac{1}{2}+\frac{2}{3}=?$ (6) $\frac{2}{3}-\frac{1}{2}=?$ (11) $\frac{5}{6}-\frac{1}{2}+\frac{1}{8}=?$

(2) $\frac{1}{4}+\frac{3}{5}=?$ (7) $\frac{4}{5}-\frac{2}{3}=?$ (12) $\frac{7}{8}-\frac{3}{4}+\frac{5}{12}=?$

(3) $\frac{2}{3}+\frac{3}{4}=?$ (8) $\frac{4}{7}-\frac{2}{9}=?$ (13) $\frac{11}{12}-\frac{7}{8}+\frac{4}{11}=?$

(4) $\frac{4}{5}+\frac{6}{7}=?$ (9) $\frac{12}{13}-\frac{3}{7}=?$ (14) $8\frac{2}{3}+6\frac{1}{2}-10\frac{9}{11}=?$

(5) $\frac{4}{7}+\frac{3}{14}=?$ (10) $\frac{7}{8}-\frac{3}{5}=?$ (15) $16\frac{3}{4}-4\frac{1}{6}+7\frac{1}{9}=?$

2. Solve the following mentally:

(1) $9\times\frac{5}{36}=?$ (6) $16\times\frac{7}{8}=?$ (11) $2\frac{1}{5}\times 5=?$

(2) $21\times\frac{9}{14}=?$ (7) $24\times\frac{11}{12}=?$ (12) $6\frac{1}{4}\times 4=?$

(3) $10\times\frac{11}{15}=?$ (8) $\frac{7}{9}\times 18=?$ (13) $5\times 1\frac{2}{5}=?$

(4) $\frac{5}{9}\times 9=?$ (9) $\frac{5}{18}\times 36=?$ (14) $8\times 1\frac{7}{8}=?$

(5) $\frac{25}{64}\times 8=?$ (10) $75\times\frac{2}{25}=?$ (15) $6\frac{2}{3}\times 3=?$

3. Solve the following, using pencil only when necessary:

(1) $3\frac{1}{3}\times 4=?$ (6) $5\times\frac{3}{4}=?$ (11) $25\times 8\frac{3}{5}=?$

(2) $\frac{2}{3}\times\frac{3}{4}=?$ (7) $8\times\frac{3}{5}=?$ (12) $\frac{7}{8}\times\frac{3}{10}\times\frac{8}{9}=?$

(3) $\frac{7}{8}\times\frac{8}{14}=?$ (8) $\frac{4}{5}\div\frac{2}{5}=?$ (13) $2\frac{1}{2}\times 6\frac{2}{5}=?$

(4) $10\frac{5}{6}\times 6=?$ (9) $\frac{9}{10}\div\frac{8}{11}=?$ (14) $3\frac{1}{4}\times\frac{7}{13}\times\frac{6}{7}=?$

(5) $50\times 5\frac{9}{10}=?$ (10) $\frac{7}{4}\div\frac{14}{30}=?$ (15) $\frac{3}{4}\div 5=?$

4. Solve the following:

(1) $4\frac{4}{5}\div 8=?$ (6) $21\div\frac{7}{11}=?$ (11) $3\frac{1}{8}\div 4\frac{5}{7}=?$

(2) $12\frac{4}{7}\div 11=?$ (7) $50\div 4\frac{3}{7}=?$ (12) $\frac{9}{49}\div\frac{2}{11}=?$

(3) $3\frac{2}{3}\div 7=?$ (8) $56\div 5\frac{4}{9}=?$ (13) $27\frac{1}{2}\div 2\frac{3}{4}=?$

(4) $6\frac{1}{3}\div 8=?$ (9) $3\frac{3}{7}\div\frac{9}{14}=?$ (14) $12\frac{2}{5}\div 4\frac{1}{6}=?$

(5 $9\frac{1}{9}\div 6=?$ (10) $3\frac{3}{4}\div 5\frac{5}{8}=?$ (15) $19\frac{7}{8}\div 3\frac{2}{3}=?$

5. In the following exercises first divide as with whole numbers. Then move the decimal points in dividend and divisor until but *one digit stands on the left of the point in the divisor.* Neglect the fractional parts and locate the decimal point by estimating about what the quotient must be:

(1) $211.248 \div 32.6 = ?$
(2) $2112.48 \div 64.8 = ?$
(3) $21.1248 \div 32.4 = ?$
(4) $21124.8 \div 6.48 = ?$
(5) $1768.96044 \div 24.831 = ?$
(6) $176.896044 \div 24.831 = ?$
(7) $17689.6044 \div 248.31 = ?$
(8) $1768.96044 \div 71.24 = ?$

6. Divide and point the quotients as explained in problem 5, and check the work by multiplication, or by casting out the nines:

(1) $989.440 \div 25.6 = ?$
(2) $182.428 \div .472 = ?$
(3) $3398.340 \div 83.6 = ?$
(4) $13690.355 \div 416.5 = ?$
(5) $1551.0138 \div 27.51 = ?$
(6) $338.51584 \div .6032 = ?$

Problem 7 shows another way to point the quotient.

7. $933.0188 \div 36.82 = ?$

```
      25.34
36.82)933.01'88
      7364
      19661
      18410
       12518
       11046
        14728
        14728
```

Mark off as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. The quotient is a whole number until this mark is reached, and the rest is a decimal.

Check: $36.82 \times 25.34 = 933.0188$, or, casting the nines from dividend, divisor, and quotient, the product (1×5) of the excesses of divisor and quotient, equals the excess (5) of the dividend.

8. Divide the following, pointing the quotient as in 7, and check the work, by casting out the nines, or by multiplication:

(1) $98.944 \div 38.65 = ?$
(2) $1824.28 \div 386.5 = ?$
(3) $339.834 \div 4.065 = ?$
(4) $1369.0355 \div 4.165 = ?$
(5) $155.10138 \div 56.38 = ?$
(6) $3385.1584 \div 5612 = ?$

9. Divide the following, pointing as you prefer, and check the work:

(1) $4062.824 \div 105.2 = ?$
(2) $792.4824 \div 38.62 = ?$
(3) $8348.970 \div 116.2 = ?$
(4) $13876.778 \div 328.6 = ?$
(5) $138978.93 \div 42.23 = ?$
(6) $2234.3893 \div .5291 = ?$

§99. To Model a 3-Inch Cube.

The cube is much used by architects in buildings and parts of buildings. See the lower part of the tower of Old Independence Hall, Fig. 1. Some crystals found in nature also have the cubical form. Can you suggest any uses or occurrences of the cube?

FIGURE 1
Old Independence Hall, Philadelphia

To draw the development, or pattern of the outside of a 3″ cube, Fig. 2, draw a line as *ac* nine inches long, and mark it off into 3-inch spaces as at *p* and *b*. At *p* draw a perpendicular line to *ac*, prolong it.

EXERCISE: *To draw a perpendicular to a line as ac, at a point, as p, on the line:*

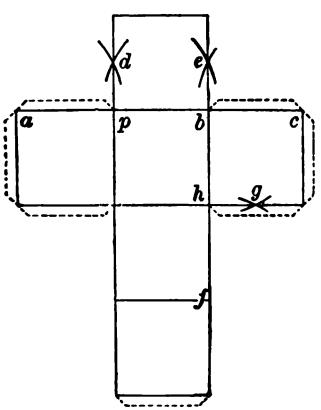


FIGURE 2
Development of 3″ cube

Measure off equal distances, as *pa* and *pb*, on opposite sides of *p*.

Put the pin-foot of the compasses, at *a*, and open the points farther than the distance *ap* and draw one of the arcs at *d*.

Using the same distance of compass points, put the pin-foot on *b* and draw the other arc at *d*.

With a ruler draw a line through *p* and the crossing point of the arcs at *d*.

The line *dp* is the perpendicular.

Another way is to place the square corner of a box, or book, at *p* and with one edge of the box or book, along *ap*, draw a line *dp*, along the other edge.

Through *b* draw another perpendicular, as *eb*, and divide it into 3-inch spaces. Mark the division points.

Through the point *h* draw a perpendicular, as *hg* to the line *ef*.

Set off 3-inch spaces on these lines and complete the development as shown in the figure. Leave flaps as indicated.

Crease the paper over the edge of a ruler along the lines and fold and paste a 3-inch cube. Leave the top unpasted.

1. What is the area of each surface? All surfaces?

2. How many edges has the cube? How many faces meet in each edge?

The surfaces of the cube meet each other in the edges forming lines. Call the corners vertices (ver'-ti-ses). A single corner is a vertex.

3. How many vertices has the cube? How many edges meet at each vertex?

The edges meet each other in the corners of the cube forming points.

4. In how many faces does each vertex lie?

5. Do cubes have length? Breadth? Thickness? Do surfaces? Do lines? Do points?

6. What are the limits of cubes? Surfaces? Lines?

Patterns of the surfaces of figures like those of Fig. 2, p. 165, will hereafter be called *developments*.

7. Place an inch-cube in the corner of the 3-inch cube. How many inch-cubes will fill the row along one edge of the larger cube?

8. How many such rows form a layer on the bottom?

9. How many such layers will fill the cube?

10. How many inch cubes will fill a 3-inch cube?

11. How many cubic inches are there in a 4-inch cube? In a 10-inch cube?

Pupils should make their own models, different pupils using different lengths of lines and even devising different forms of pattern. For example, let one pupil use a 3-inch fundamental distance instead of an inch, another a 4-inch distance, and so on, each comparing his completed model with other models.

§100. To Model a Square Prism.

Draw a straight line *ad* (Fig. 1) 12 inches long, and make *ab* and *dc* each 2 inches long. Draw *en* and *fo* perpendicular to *ad* at *b* and *c*. Lay off distances *eb*, *bg*, *gl*, *ln*, *fc*, *ch*, *hm*, *mo*, each equal to 2 inches. Draw *ej*, *jk*, etc., and complete the development. Provide flaps. Cut and paste the model.

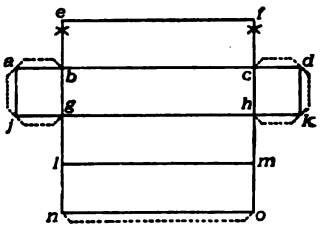


FIGURE 1
Development of the Square Prism

1. What is the area of each of the end surfaces?

2. Give the area of each side surface; of all surfaces.

3. How many inch-cubes will the model hold?

4. By measuring the edges how could you find the number of cubic inches the model will hold?

5. How many cubic inches would it hold if it were 1 inch longer? Twice as long?

6. Answer questions 1 to 4, assuming every line in the pattern twice as long as in the development.

§101. To Model a Flat Prism.

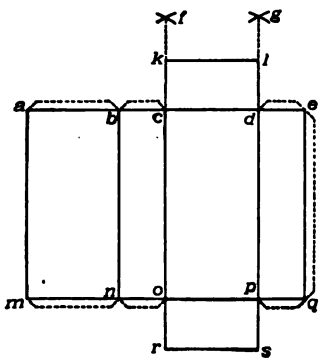


FIGURE 2
Development of Flat Prism

Draw a line *ae* (Fig. 2) 18 in. long and mark off $ed = 3$ in., $dc = 6$ in., $cb = 3$ in., and $ba = 6$ inches.

At *c* and *d* draw perpendiculars to *ae* and extend them 11 in. below *c* and *d* to *r* and *s*, and 3″ above *c* and *d* to *k* and *l*.

Make *co* and *dp* 8 in. long and draw *mq* through *o* and *p*.

Complete the development by drawing the necessary lines.

Provide it with flaps, as shown, and paste up the model, leaving the top *ckdl* open.

1. With the aid of the model of an inch-cube, find how many cubic inches would fill the flat prism.

2. To what is the product of the 3 different edges equal?

3. How could you obtain the capacity in cubic inches of such a prism by measuring the lengths of its edges?

4. How many square inches in the whole surface of the flat prism of Fig. 2, p. 167?

§102. Comparison of Prisms.

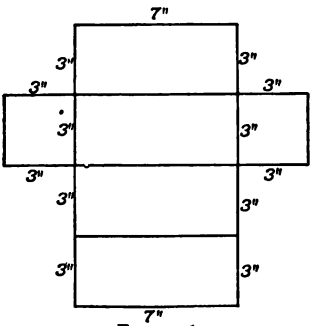


FIGURE 1
Development of Right Prism.

(a) *A right prism.*

Using the lengths of lines as shown in Fig. 1, draw a pattern on heavy paper and construct the model of a right prism, as was done in Fig. 2, p. 167. Provide flaps and paste up the model, leaving one end open, so that it may be filled with sand.

1. How many cubic inches of sand will fill the model?

2. How can you find the capacity of the model by using the lengths of the edges?

(b) *An oblique prism with parallelograms for bases.*

Similarly draw a pattern, cut, fold, and paste up a model of the oblique prism, as shown in Fig. 2, leaving an end unpasted for sand.

3. When the model of the right prism is full of sand, the sides not being bulged out, pour the sand into the model of the oblique prism. Does it fill the second model?

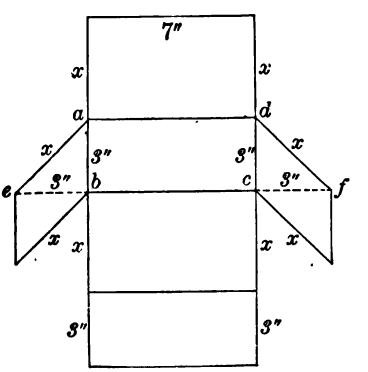


FIGURE 2
Development of Oblique Prism

4. What is the ratio of the capacities of the two models?

5. What is the ratio of the areas of the bases (ends)?

(c) *A triangular prism.*

Model a triangular prism like the one shown in Fig. 1.

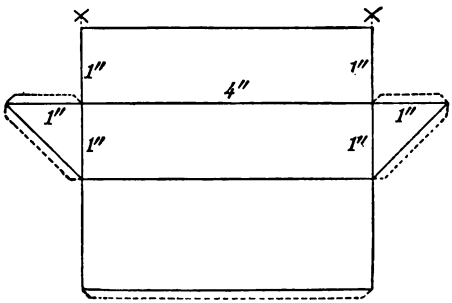


FIGURE 1
Development of Triangular Prism

6. Compare the capacity of this model with that of the model of a square prism $1'' \times 1'' \times 4''$. What is their ratio?

7. Model a triangular prism such as that of Fig. 1, but use for lengths 3″ instead of 1″ and 7″ instead of 4″. How does its capacity compare with that of the prism of Fig. 1, p. 168? Of Fig. 2, p. 168? Give the ratio in each case.

8. Compare the ratios of the bases.

§103. Volume of an Oblique Prism.

The volume of a figure is the number of cubical units in the space enclosed by its bounding surfaces.

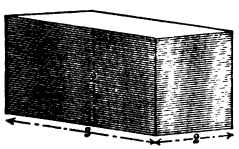


FIGURE 2
Square Pile, or Right Prism

1. How many cubic inches are there in a straight pile of visiting cards 2″ high, if each card is $2'' \times 3''$?

2. How many cubic inches would there be in the pile if it were 5″ high? 9″ high? *a* in. high?

3. Push the straight pile of problem 1 over as in Fig. 1, p. 170. How many cubic inches of paper are there in this oblique pile?

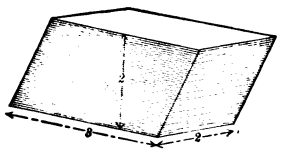


FIGURE 1
Oblique Pile, or Oblique Prism

4. Has the height of the pile been changed in Fig. 1? The area of the base? The volume?

5. How can the number of cubic units in a right prism be found from the area of its base and its height? In an oblique prism?

6. Find the volumes of square prisms having edges of the following lengths:

(1)	$3'' \times 5'' \times 6''$	(5)	$16' \times 10\frac{3}{8}' \times 20'$
(2)	$6'' \times 5'' \times 9''$	(6)	$45'' \times 16\frac{2}{9}'' \times 21''$
(3)	$15'' \times 9'' \times 8''$	(7)	$3\frac{3}{4}' \times 16\frac{1}{2}' \times 20\frac{3}{8}'$
(4)	$4\frac{2}{3}' \times 6' \times 12'$	(8)	a in. $\times b$ in. $\times c$ in.

§104. Paper-Folding. (FUNDAMENTAL GEOMETRICAL NOTIONS)

For the following exercises in paper-folding any moderately thick, glazed paper will do. Tinted or colored paper, without lines, will show the creases more clearly.

It is convenient to have the paper cut into pieces, about 4″ square Such paper is inexpensive and may be had of any stationery dealer.

EXERCISE I.—*At a chosen point on a line, make a perpendicular to the line, by folding paper.*

Fold one part of a piece of paper over upon the other and crease the paper along the fold, as at AB, by drawing the finger along the fold. Taking D to denote the chosen point, fold the paper over the point D, and bring the two parts, DA and DB, of the crease AB exactly together. Hold the paper firmly in this position and crease the paper along the line DC.

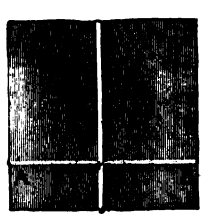


FIGURE 2

Fitting the portion of the paper between the creases DB and DC over the portion between the creases DA and DC; how do the angles, BDC and CDA, compare in size?

When two lines meet in this way making the angles at their point of meeting (intersection) equal, the lines are said to be perpendicular to each other, and each is called a perpendicular to the other.

The angles thus formed are called right angles.

EXERCISE II.—*Make a perpendicular to a line at a chosen point, not on the line, by folding.*

P stands for the point, not on the line shown by the crease, *AB*. Fold the paper back, bringing the end, *A*, down along the back side of the end, *B*. Hold carefully on the crease and slide *A* along until the crease *PD*, to be made, will go through *P*. Crease the paper as shown by *PD*. The crease *PD* then stands for the perpendicular to *AB*, through *P*.

How does the angle *PDA* compare in size with the angle *PDB*?

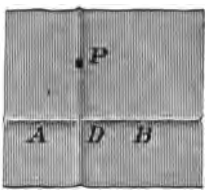


FIGURE 1

EXERCISE III.—*Through a given point make a line parallel to a given line by paper-folding.*

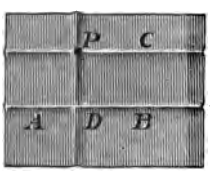


FIGURE 2

The crease, *AB*, stands for the given line, and the dot *P* stands for the point the parallel to *AB* is to go through.

Crease a perpendicular, as *PD*, through *P* to *AB* as in Exercise II.

Crease a perpendicular, as *PC*, through *P* to *PD*, as in Exercise I. The line shown by the crease *PC* is parallel to the line shown by the crease *AB*.

EXERCISE IV.—*Bisect an angle, by folding paper.*

Crease two lines, as *OA* and *OB*, lying across each other. They make the angle *AOB*.

Now fold the paper over, and bring the crease *OA* down on *OB*. Holding the creases firmly together crease the bisector *OC*.

1. How do the angles *AOC* and *BOC* compare in size? Do they fit?

2. What is the ratio of angle *AOC* to angle *BOC*? Of *AOB* to *AOC*?

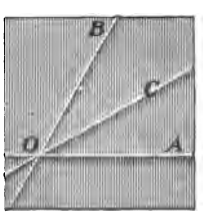


FIGURE 3

3. How could the angle AOB be divided by creases into 4 equal parts?

4. Fold a square over a diagonal and tell how the diagonal divides the square; two of the opposite angles of the square.

5. Crease both diagonals of a square. How do they divide each other?

§105. Perimeters.

The perimeter of any figure is the sum of the lines bounding the figure.

In form (6), Fig. 1, if p denote the perimeter, then,

(I) $$p = x + y + z.$$

1. What does $x + 2x + 4x$ mean? How may it be more briefly written?

2. What is the coefficient (the multiplier) of x in the answer to problem 1?

3. Write the perimeter, p, of (1), Fig. 1, in two ways.

4. Write the perimeter, p, of (13) in three ways.

5. Write the perimeter, p, of (12) in three ways.

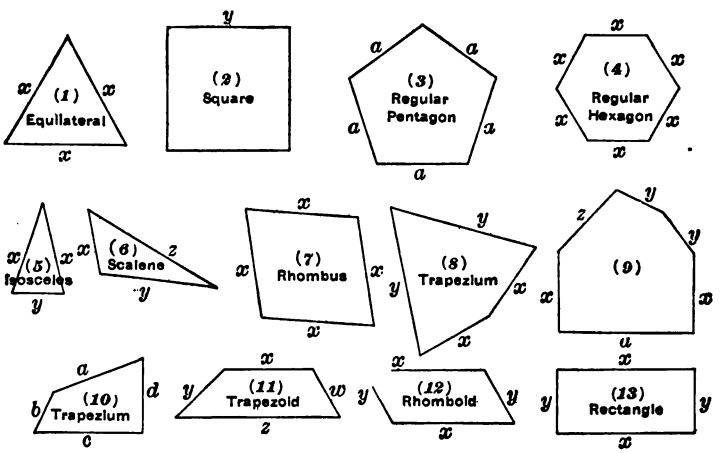


FIGURE 1

6. Write $\frac{1}{2}$ of the perimeter, p, of (13) in two ways.

7. Write an expression showing that the two answers to 6 are equal.

8. In (12), if $x = 80$ rods, and $y = 40$ rods, how many feet are there in the perimeter?

9. In (1) and (2), if x and y each $= 20$ rods and one side of the triangle rests on the square, making a new figure, omitting the common line, what is the perimeter of this new form, in feet? How many sides has the new figure thus formed?

10. Forms (1) and (13) are combined into a single figure. Write p for the new figure in two ways, supposing x the same in both.

§106. Quadrilaterals.

1. Forms (2), (7), (8), (10), (12), and (13), Fig. 1, p. 172, are different kinds of quadrilaterals. What is a *quadrilateral*?

2. What quadrilaterals have their opposite sides parallel? These figures are parallelograms. Define a *parallelogram.*

3. What parallelograms have *all* their sides *equal*? What is a *rhombus*?

4. What rhombus has *all* its angles *equal*? What is a *square*?

5. What quadrilaterals have their opposite sides equal but consecutive angles not equal? Define a *rhomboid.*

6. What parallelograms have their angles all equal? Define a *rectangle.*

7. What quadrilateral has only *one* pair of sides parallel? Define a *trapezoid.*

8. Is a trapezoid a parallelogram? Is it a quadrilateral?

9. Is a rectangle necessarily a quadrilateral? Is it a parallelogram? A square? *May* a rectangle be a square?

10. Is a square necessarily a quadrilateral? Is it a parallelogram? A rectangle? A rhombus?

§107. Perimeters of Miscellaneous Figures.

1. Denote the perimeter of each of the forms in Fig. 1, p. 172, by p and write an equation (an expression of equality) like (I) in §105, showing the value of p for each figure.

2. Omitting the lines on which the forms join, write an equation showing the value of p when (2) and (6), Fig. 1, p. 172, are joined on y, which has the same value in both.

3. In the same way join (3) and (10) on a, and write an equation showing the value of p.

4. Join (2) and (7) in which x and y are equal. If the perimeter of the figure thus formed is 240 rods, find the value of x in feet.

5. What is the length of the perimeter of a figure like (6), Fig. 1, p. 172, whose sides are 2 in., a in., and b in. long? x in., 8 in., and z in. long? Like (10), whose sides are c ft., $2c$ ft., d ft., and x ft.? Like (12), whose sides are $2x$ rd., $4y$ rd., $2x$ rd., and $4y$ rd. long?

6. Write the perimeter (p) of each of the forms of Fig. 1, p. 175.

In such forms as those from (5) to (10), Fig. 1, p. 175, some line, or lines, must be found by subtracting others. In (6) for example, note that the ends are each $x-y$. The perimeter is then $x+x+(x-y)+y+y+(x-y)=4x-y-y+y+y=4x-2y+2y=4x$.

All sides not lettered must be expressed without using other letters than those given on the figure. The perimeter means the sum of all the lines that bound the strip, or surface, of the figure.

NOTE: One-half the difference of a line m and a line n is written

$$\frac{m-n}{2} \text{ or } \tfrac{1}{2}(m-n).$$

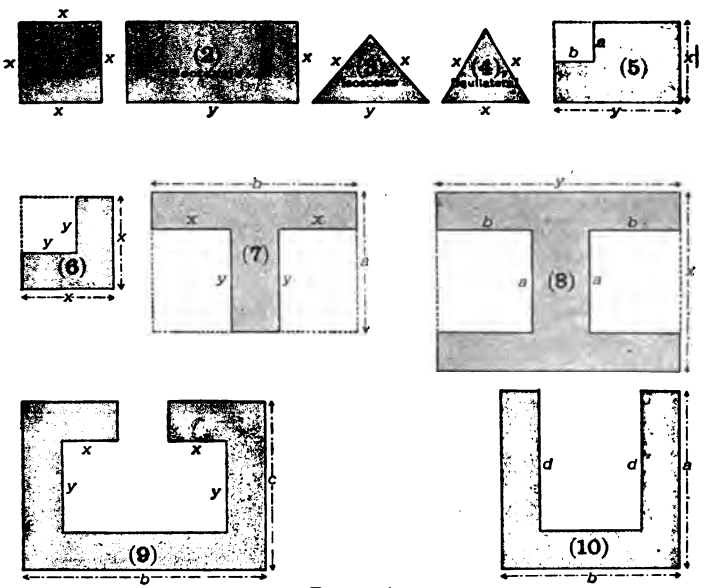


FIGURE 1

7. In (8), $a=15'$, $b=12'$, $x=25'$ and $y=30'$. Find the length of the perimeter of the figure.

8. Find the area of (8) enclosed by the solid lines.

9. Make and solve other similar problems.

§108. Measuring Angles and Arcs.

Answer all you can orally.

We may measure the amount of turning of each clock hand in either of two ways:

(1) By the length of the circular arc passed over by the tip of the rotating hand; or,

(2) By the wedge-shaped space having its point at the hand-post, A, over which the stem (A-III), of the rotating hand moves.

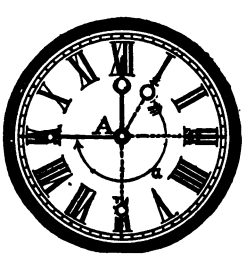


FIGURE 2

While the clock hand turns from XII to III its tip moves over 1 quadrant of arc and its stem moves over a right angle.

1. While the hand turns from XII to VI over how many quadrants does its tip move? Over how many right angles does its stem turn?

2. Answer same questions for a turn of the hand from XII to IX; from XII to XII again; from XII around through XII to III.

3. Over what part of a right angle does the stem of the hand move while the hand is passing from XII to I? From XII to II? From III to IV? From VI to VIII?

4. Over what part of a quadrant does the tip of the hand pass in each case of problem 3?

5. Over what part of a right angle does the stem of the hand move while the tip moves 1 min. along the arc? In the same case over what part of a quadrant does the tip move?

Any wedge-shaped part of the face moved over by the stem of a clock hand as it turns around the hand-post is called an angle. The curve passed over by the tip of the hand, while the stem of the hand moves over the angle, is called the arc of the angle.

ILLUSTRATIONS.—The wedge-shaped spaces, having their points at A, and included between any two positions of the hand, as A-XII and A-I, A-I and A-III, A-I and A-VI, are all angles. The space swept over by the hand, A-I, as it moves around through II, III, IV, V, VI, etc., to IX is also an angle.

As the hand moves from A-XII to A-VI, that is, so that the two positions of the hand are in the same straight line, the hand moves over a straight angle.

6. A straight angle equals how many right angles?

7. The arc of a straight angle equals how many quadrants? How many quadrants make a complete circle?

8. How many 5-min. spaces make the circumference of a complete circle? How many 1-min. spaces make the circumference of a complete circle?

9. What part of a right angle is passed over by the stem of the hand as its tip moves from one end to the other of a 1-min. space?

10. If lines were drawn from the hand-post to the ends of all the 1-min. spaces, the whole face of the clock would be divided up into how many small angles?

The instrument in common use for measuring angles and arcs is the *protractor*. (See Fig. 1.)

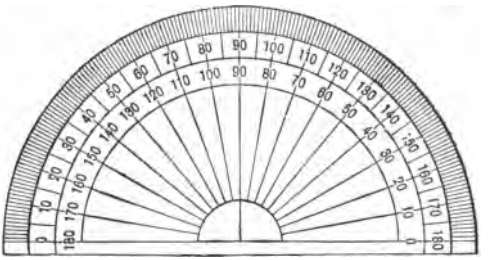


FIGURE 1

Instead of dividing the right angle up by radiating lines into 15 equal parts, the protractor divides the right angle into 90 equal parts, one of which is called the angular degree.

11. These same lines would divide the quadrant up into how many equal parts?

Each of these parts is a degree of arc.

12. How many angular degrees are there in a straight angle? How many degrees of arc in the arc of a straight angle?

13. How many angular degrees are swept over by a clock hand while moving entirely around once? How many degrees of arc in the circumference of a circle?

14. How many degrees of angle are passed over by the stem of the minute hand in two hours? In the same time

how many degrees of arc are passed over by the tip of the hand?

15. A sextant is $\frac{1}{6}$ of a circumference; how many degrees of arc are there in a sextant?

16. An octant is $\frac{1}{8}$ of a circumference: how many degrees of arc are there in an octant?

17. Study the protractors, Fig. 1, p. 177 and Fig. 1 below; notice how its marks are numbered. How many degrees of arc are there between the closest lines on the outer edge? How many degrees of angle are there between the lines which converge toward the center?

For smaller angles a shorter unit is $\frac{1}{60}$ of a degree of angle and the smaller unit is called the minute of angle. $\frac{1}{60}$ of the minute, called the second, is a still smaller unit.

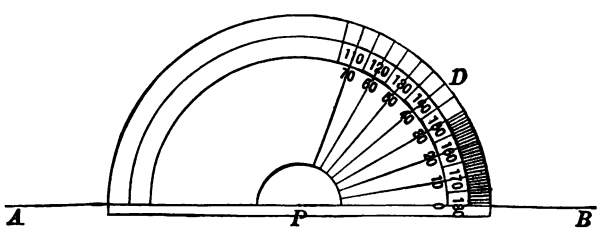


FIGURE 1

18. How many minutes of angle are there in a right angle? In a straight angle? In a complete revolution, called a *perigon*? How many seconds, in each case?

19. The arcs between the sides of the minute and the second of angle are the minute and the second *of arc*. How many minutes of arc in a quadrant? In a sextant? In an octant? In a circumference? How many seconds of arc, in each case?

NOTE: A paper protractor can be purchased at any bookstore, and each pupil should supply himself with one.

To measure an angle the center of the protractor is placed on the vertex, A, and the 0-line through A is placed along one side AD (see Fig. 1). The reading of the mark below which the other side, AF, falls, is the number of degrees in the angle, or in the arc of the protractor included between AD and AF.

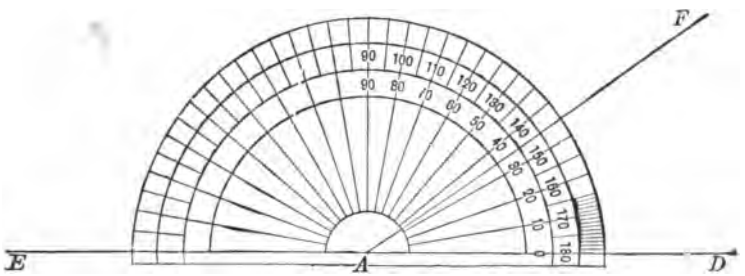


FIGURE 1

20. Draw a triangle, and, with a protractor, measure its angles. To what is the sum of all three equal?

21. With ruler draw a half a dozen triangles of different shapes, carefully measure each angle, and find the sum of the three for each triangle.

22. Measure with the protractor the number of degrees in the angle at one corner of the page of this book.

23. Draw two straight lines crossing each other as in Fig. 2. With the protractor, measure and compare angles *a* and *c*; angles *b* and *d*.

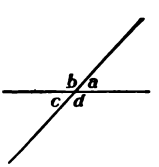


FIGURE 2

The angles a and c, or b and d, lying opposite each other, are called opposite angles, or vertical angles.

24. Draw two crossing lines in different positions, and in each case compare the measures of a pair of opposite angles. What do you find to be true?

25. Crease a pair of parallel lines, as *a* and *a*, Fig. 1, (See p. 171, Exercise III) and draw a third straight line *C* cutting across the parallels. Measure with a protractor and compare the four angles, in which 1 is written in Fig. 1. Measure and compare those in which 2 is written. What do you find to be true?

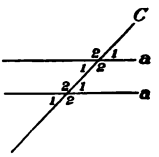


FIGURE 1

26. Crease a line perpendicular to another and measure each of the 4 angles formed. (See p. 170, Exercise I.) What do you find?

The lines which include the angle are sides of the angle. The point where the sides meet is the vertex of the angle.

27. Draw a quadrilateral of any irregular shape and measure each of its 4 angles. To what is the sum equal? Find the sum of the 4 angles of any other quadrilateral.

28. Draw a given angle at a point on a given line.

EXPLANATION: Let the given line be *ED*, the given angle 35 degrees, and the given point, *A*, Fig. 1 p. 179. Place the protractor with its center at *A*, and with the diameter of the protractor along the line *ED*. Make a dot opposite the 35-degree mark on the protractor. With the ruler draw a straight line through *A* and this dot to *F*. The angle, *DAF*, is the required angle.

29. Draw a straight line, mark a point on the line and draw the angle 75 degrees; 95 degrees; 100 degrees; 135 degrees; 55 degrees and 30 minutes.

TABLE OF UNITS OF ANGLE AND ARC MEASUREMENT

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference (or perigon)
90 degrees	=	1 quadrant (or right angle)

TABLE OF EQUIVALENTS

1,296,000″
21,600′
360° } = 1 circle = { 4 quadrants
1 perigon

§109. The Sum and Difference of Angles.

EXERCISE I.—*With ruler and compasses, draw AB and CD perpendicular to each other.*

1. Suppose a protractor supplied with a pointer as shown in Fig. 1, the center of the protractor being placed at the point, *O*, over how many degrees of arc would the tip of the pointer turn while the stem of the hands turns from line *OB* to line *OC*? From *OB* through *OC* to *OA*?

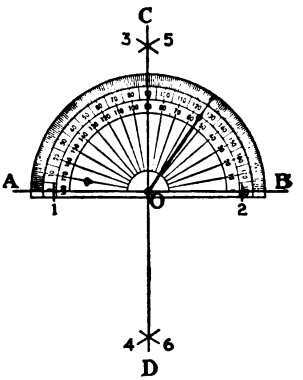
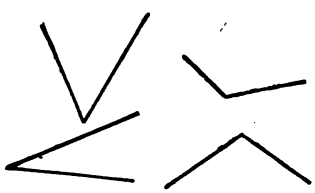


FIGURE 1

2. How many degrees are there in the angle *BOC*? In angle *BOA*? In *COD*? in *AOD*? In the angle from *OA* around through *OD* to *OB*? From *OA* around to *OC*? From *OA* entirely around to *OA* again?

3. How many degrees of angle fill the space around a point, as *O*, on one side of a straight line, as *AB*?


Acute Angles Obtuse Angles
FIGURE 2

4. How many degrees of angle fill the space around *O* on both sides of *AB*?

5. Draw a triangle having a right angle, using the square corner of a card.

6. Draw a triangle having an obtuse angle.

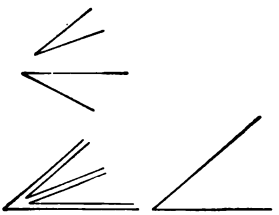
An angle that is smaller than a right angle is called an acute angle (see Fig. 2).

An angle that is larger than a right angle is called an obtuse angle.

7. Draw a triangle having three acute angles.

EXERCISE II.—*Find the sum of two angles.*

8. Draw two acute angles like those at the top of Fig. 1, carefully cut them out, and place them as in the lower part of the figure. Push their vertices and nearer sides entirely together. Draw two lines along the other sides of the angle. Remove the angles and have an angle like that on the right, which is the *sum* of the two given angles.



FIGURE

9. Calling the larger angle x, and the smaller y, what denotes the last angle, the sum?

10. Draw an angle equal to the sum of an acute and an obtuse angle.

EXERCISE III.—*Find the difference of two angles.*

11. Draw two angles like those at the top of Fig. 2, cut them out, and place them as in the lower, left-hand part of the figure. Mark along the lower side of the upper angle and cut along the mark. The lower part of the larger angle (shown at the left) equals the *difference* of the two angles.

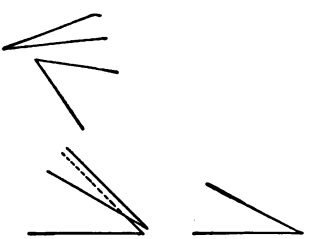


FIGURE 2

12. If the larger angle equals 52° and the smaller equals 18°, how many degrees are there in the difference?

13. If the larger angle equals 63 degrees and the smaller equals x degrees, how many degrees are there in the difference?

14. If the larger angle equals x degrees and the smaller equals y degrees, how many degrees are there in the difference?

Two angles whose sum equals a right angle or 90°, are called complemental angles (Fig. 1).

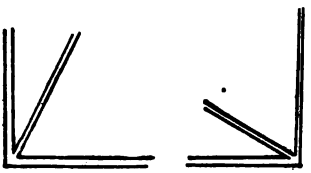


FIGURE 1

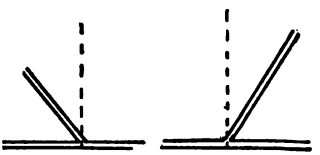


FIGURE 2

Two angles whose sum equals two right angles, or 180°, are called supplemental angles (Fig. 2).

15. The sum of all the angles that just cover the plane on one side of a straight line is equal to how many right angles (Fig. 3)? To how many degrees is this sum equal?

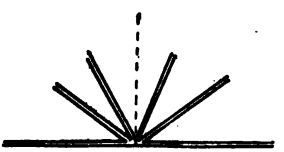


FIGURE 3

EXERCISE IV.—Draw and cut out an equilateral (equal-sided) triangle. Tear off a corner and fit it over each of the other corners in turn. How do the three angles compare in size? Tear off the other corners and place them as in Fig. 4. To what is the sum of all the three angles equal?

16. If one of the angles is x degrees, how many degrees are there in the sum of all three angles?

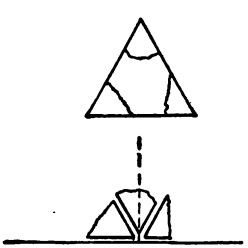


FIGURE 4

As the sum of the three angles equals both $3x$ degrees and also 180°, we may write the equation

$$3x = 180.$$

If $3x = 180$, to what is x equal? How many degrees are there in one of the angles of an equilateral triangle?

EXERCISE V.—Draw any scalene triangle (no two sides equal). Tear off the corners and place them as in Fig. 1. To how many right angles is the sum of all the angles of the triangle equal? To how many degrees?

FIGURE 1

17. Letting x, y, and z denote the numbers of degrees in the respective angles, in what other way may we write the sum of all three? What equation may we then write? Tell what the equation $x + y + z = 180°$ means.

18. Draw other scalene triangles, tear off the corners, place them as in the figure, and find whether $x + y + z = 180°$ in all cases.

19. Crease a right triangle and find whether the equation, $x + y + z = 180°$, is true for it.

20. Cut along the creases and tear off the two acute angles of a carefully creased right triangle and fit them over a carefully creased right angle. What seems to be true? If this is true what equation may you write for the sum of the two acute angles (x and y) of a right triangle?

EXERCISE VI.—Draw a quadrilateral and cut it along a straight line from one corner to the opposite corner as in Fig. 2. Such a line is called a *diagonal*.

21. The cut divides the quadrilateral into figures of what shape?

22. To what is the sum of the three angles of each part equal?

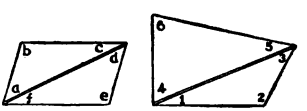


FIGURE 2

23. To what is the sum of all six angles of the two triangles equal?

24. To what is the sum of all 4 angles of the quadrilateral equal?

25. Do your answers hold true for the parallelogram of Fig. 2, p. 184?

26. Do they hold good for any shape of quadrilateral you can draw?

27. To what then is the sum of the four angles, x, y, z, and w, of any quadrilateral equal?

EXERCISE VII.—Draw a hexagon and cut it along diagonals as shown in Fig. 1.

28. Into figures of what shape is the hexagon divided?

29. Into how many such figures do the diagonal cuts divide the hexagon?

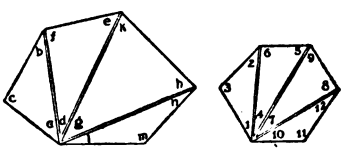


FIGURE 1

30. To how many degrees is the sum of all the angles of all the triangles equal?

31. To how many right angles is the sum of all the angles of the hexagon equal?

32. Do all your answers hold true for the regular hexagon (sides equal and angles equal) of Fig. 1 also?

33. In each of these hexagons how many less triangles than sides of the uncut hexagons are there?

34. Draw an 8-sided figure and find how many triangles the diagonal cuts from any vertex would give.

35. How many less triangles than sides were given by the quadrilaterals of Fig. 2, p. 184?

36. To how many right angles is the sum of all the angles of any 15-sided figure equal? Of an n-sided figure?

37. The angles of a regular hexagon are all equal. How many degrees does each contain?

EXERCISE VIII.—Draw a rectangle and cut it out. Cut it along a diagonal, and denote the angles by letters, as shown in Fig. 1.

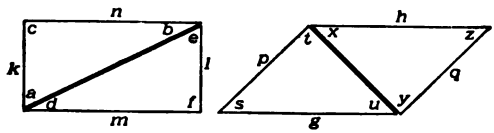


FIGURE 1

38. Turn the lower right-hand triangle around and place it on the upper piece so that e may fall at a, f at c, and d at b. Can you make the parts fit?

39. How does a diagonal divide a rectangle?

40. Show the relative length of sides m and n by an equation; of k and l; of the angles a and e; c and f; b and d; $b + e$ and $a + d$.

41. From these equations point out those which show that the opposite sides of a rectangle are equal.

42. Show from your equations how the opposite angles compare in size.

43. Write the perimeters of the parallelogram and of the rectangle of Fig. 1.

§110. Products of Sums and Differences of Lines.

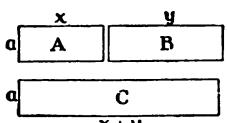


FIGURE 2

1. What is the area of A? Of B? Of C?

NOTE: The product of a and $x + y$ is written $a(x + y)$ and is read "a times the sum $x + y$."

2. Read the equation $ax + ay = a\ (x + y)$ and explain its meaning from Fig. 2, p. 186.

3. Draw a figure and show that

$$ax + ay + az = a\ (x + y + z).$$

4. What is the area of each of the parts of Fig. 1?

NOTE: $a + b$ times $x + y$ is written $(a + b)\ (x + y)$.

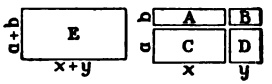


FIGURE 1

5. From Fig. 1 show the meaning of each product in the equation $(a + b)\ (x + y) = ax + ay + bx + by$. Also show from Fig. 1 why the equation is true.

6. What is the area of each part of Fig. 2?

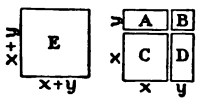


FIGURE 2

7. Write an equation showing how to multiply $x + y$ by itself, or how to square $x + y$.

8. Point out from Fig. 2 the meaning of

$$(x + y)^2 = x^2 + 2xy + y^2.$$

9. Point out from Fig. 3 the meaning of

$$b\ (a - c) = ab - bc.$$

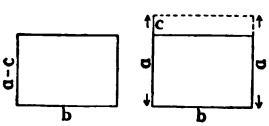


FIGURE 3

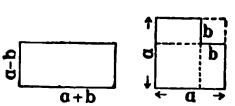


FIGURE 4

10. Draw a figure and show that

$$b\ (a + d - c) = ab + bd - bc.$$

11. Show from Fig. 4 that

$$(a + b)\ (a - b) = a^2 - b^2.$$

12. Show from Fig. 5 that

$$(a - b)\ (a - b) = a^2 - 2ab + b^2.$$

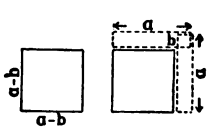


FIGURE 5

13. What is the area of the rectangle $ABCD$ (Fig. 1)? Of S?

14. What is the length of the dotted part of BC?

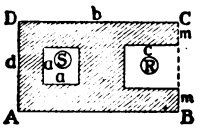


FIGURE 1

15. What is the area of R? Of the cross-ruled part?

16. Write the areas of the cross-lined figures (Fig. 2).

In each case call the difference of the two areas d, and answer with an equation.

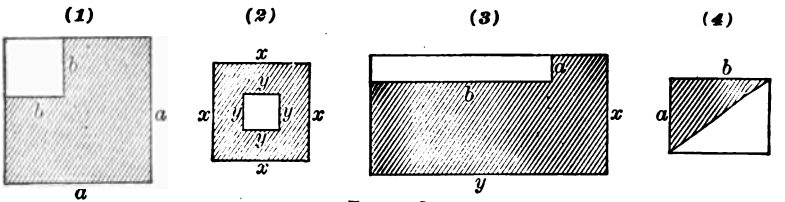


FIGURE 2

17. A lot, $50' \times 175'$, is occupied by a house which, with a front porch, covers a $40'$ square. A cement walk $3\frac{1}{2}'$ wide runs from the front line of the lot to the porch, and from the back door of the house to the rear line of the lot. The rest of the lot is covered with grass. How many square feet of grass are there?

18. A man cuts an $18''$-strip of grass entirely around a rectangular lawn, $75' \times 175'$. How many square feet of grass does he cut?

19. He then cuts another $18''$-strip around just inside of the strip mentioned in problem 18. How many square feet of grass does he cut the second time round?

20. A farmer reaps a $12'$-swath entirely round a rectangular wheat-field that is 40 rd. $\times$ 80 rd. How many square rods does he reap the first time round? ($12' = \frac{8}{11}$ rd.)

21. He again reaps a 12′-swath entirely round the field. How many square rods does he reap the second round? The third round?

22. A farmer plows an 18″-strip entirely round a rectangular piece of ground 20 rd. × 80 rd. How many sq. ft. does he plow?

23. He continues plowing round the rectangle (problem 22) until he has a plowed strip 1 rd. wide entirely round it. How many acres has he then plowed?

24. How many acres has he plowed when the plowed strip is 2 rd. wide? 4 rd. wide?

25. How many acres has he plowed when the unplowed strip of problem 22 is 4 rd. wide, the plowed strip being of the same width all round the rectangle?

26. How many acres have been plowed when the unplowed strip remaining is $3\frac{1}{2}$ rd. wide? $2\frac{1}{2}$ rd. wide?

§111. Locating and Describing Places on the Earth.

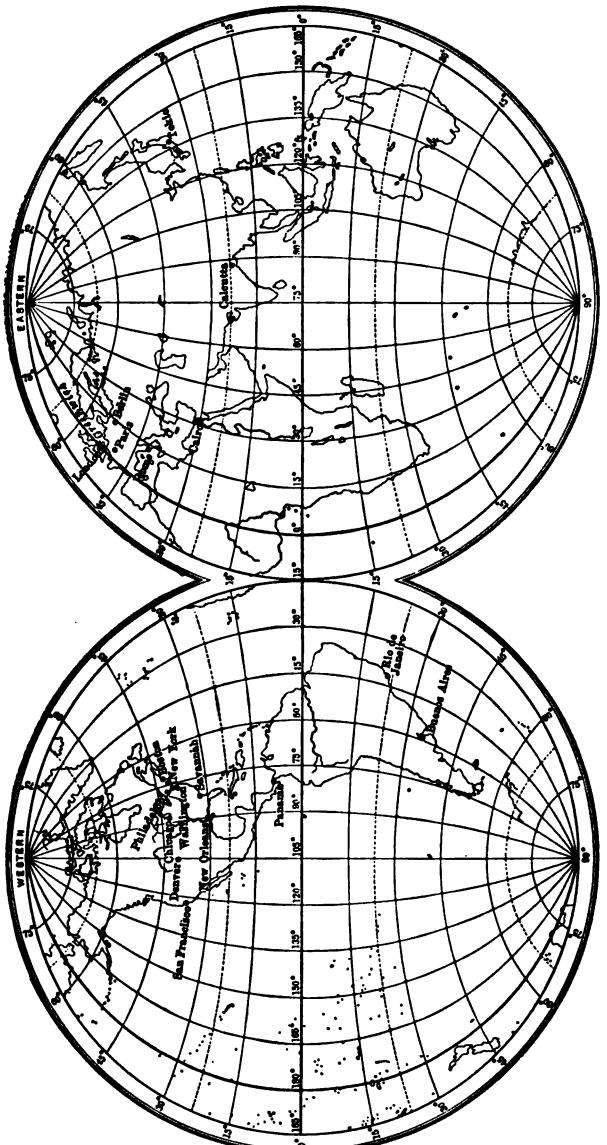
On page 190 is a map of the eastern and the western hemispheres of the earth, showing the lines that are used for locating and describing places on the earth.

An imaginary line, called the *equator*, runs entirely round the earth, dividing its surface into the northern and the southern hemispheres. This equator is shown on the map by the heavy horizontal line.

The heavy curved line going through the north and south poles, crossing the equator of the eastern half of the map at 0°, and of the western half at 180° is called the *prime meridian*.

Answer all you can orally.

1. What do the numbers 15°, 30°, 45°, etc., mean, that stand along the equator to the right and left of 0°?



MAP OF THE WORLD

2. What part of the map is north? South? East?

3. How many degrees are there between the curves that run up and down on the map through the poles?

4. How many degrees are there between the curves that run across the map from right to left?

5. Show the place on the map that is 15° west of the zero (0°) point; 30° west of 0°; 60° W.; 105° W.; 150° W.

6. Show the place that is 15° east of the 0° point; 45° E.; 60° E.; 90° E.; 120° E.; 150° E.; 165° E.

7. Point to the place that is 15° north of 0°; 30° N.; 30° south; 45° S.; 60° N.; 75° S.; 90° N.; 90° S.

8. Show the place that is 30° W. and 15° N.; 30° W. and 30° N.; 30° W. and 15° S.; 60° and 30° S.W.; 60° E. and 15° N., 60° E. and 30° S.

9. Point out the places that have the following positions:

(1) 30° W. and 75° N.
(2) 30° W. and 30° S.
(3) 45° E. and 15° S.
(4) 60° E. and 45° N.
(5) 45° W. and 45° N.
(6) 45° W. and 45° S.
(7) 45° E. and 45° S.
(8) 105° W. and 30° N.
(9) 60° W. and 60° S.
(10) 105° W. and 45° S.
(11) 90° E. and 30° N.
(12) 90° E. and 90° N.
(13) 180° W. and 75° S.
(14) 90° W. and 90° N.

Degrees measured toward the west and toward the north are written with a plus (+) sign before them. Degrees measured toward the east and toward the south are marked with a minus (−) sign before them.

10. Point out the following places, the first number being the longitudes:

(1) +15°, +30°
(2) +15°, −30°
(3) +30°, −75°
(4) −60°, −75°
(5) −75°, +60°
(6) −75°, −60°
(7) +100°, −45°
(8) +100°, +75°
(9) +160°, +80°
(10) − 40°, +65°
(11) −140°, −35°
(12) −120°, −65°

Use pencil and paper only when needed.

11. Write down about the correct distances in degrees westward or eastward, and northward or southward from 0°: of the mouth of the Amazon river; of the Mississippi river; of the Nile river; of the Ohio river; of the Niger river (Africa).

12. Write about the positions of these places, using + or − signs: Washington, D. C.; New York City; Boston; Chicago; Denver; New Orleans; London; Paris; Berlin; the south point of Florida; the south point of South America; the east point of South America.

13. About how many degrees long is South America? Africa? The Nile river?

14. About how many degrees wide is South America? Africa? The United States?

The curved lines running from west to east are called parallels; those running from north to south, meridians.

15. About how many degrees wide is the United States along the 30° parallel? Along the 45° parallel?

16. About how many degrees wide is South America along the equator? Along the −30° parallel? The −15° parallel?

17. Answer similar questions for Africa.

18. How many degrees are there between the prime meridian and the meridian of the mouth of the Nile river? Between the prime meridian and the meridian of the southeast point of Arabia? Of the southwest point of Arabia?

19. How many degrees are there between the 45th meridian and the meridian of the mouth of the Mississippi river?

20. Chicago is about how many degrees west of New

York City, that is, about how many degrees are there between a meridian through New York City and a meridian through Chicago?

21. About how many degrees is Chicago north of New Orleans? Of Panama? Of Rio de Janeiro? Of the mouth of the Amazon river?

22. About how many degrees west from New York City is Denver? San Francisco? The mouth of the Ohio river?

Distances measured westward or eastward in degrees along a parallel from the prime meridian are called longitudes.

Distances measured in degrees along a meridian northward or southward are called latitudes.

23. About what is the difference between the longitude of New York and of Chicago? Boston and Denver? Philadelphia and New Orleans? New Orleans and San Francisco?

24. Give the differences of latitude of the same pairs of places.

The latitude and longitude in degrees of some important seaports are given here:

PLACE	LATITUDE	LONGITUDE	PLACE	LATITUDE	LONGITUDE
Cape Town...	−33.93	− 18.47	Queenstown..	+51.85	+ 8.28
Hamburg	+53.55	− 9.97	San Francisco	+37.78	+122.43
Honolulu.....	+21.30	+157.85	Savannah....	+32.02	+ 81.12
Manila.....	+14.67	−121.00	Stockholm ...	+59.33	+ 18.05
Panama.....	+ 9.00	+ 79.52	Valparaiso ...	−33.00	+ 71.63
Quebec.....	+46.80	+ 71.32	Yokohama...	+35.50	−139.58

25. What is the difference of latitude of San Francisco and Honolulu? The difference of longitude?

26. What are the differences of latitude and of longitude of San Francisco and Yokohama? Of San Francisco and Valparaiso? Of Hamburg and Savannah? Of Queenstown and Quebec? Of Cape Town and Stockholm? Of

Panama and Manila? Of Panama and Valparaiso? Of Manila and Yokohama? Of Quebec and Panama?

27. Other problems may be made from the table.

28. Read the differences of latitude and of longitude of any places given on maps in your geographies.

§112. Longitude and Time.

The earth may be regarded as an immense sphere turning, like a top, round one of its diameters as an axis from west to east once every 24 hours. This carries the surface of the earth and all objects fixed upon it round through 360° in 24 hours.

Measuring one complete turn (rotation) of the earth in degrees, it may be said to be equivalent to 360°; measuring it in time, it may be said to be equivalent to 24 hours.

This gives us the following tables of equivalent measures:

360° correspond to 24 hr.;
1° corresponds to $\frac{1}{360}$ of 24 hr. $= \frac{1}{15}$ hr. $= 4$ min. of time;
1′ corresponds to $\frac{1}{60}$ of 4 min. $= \frac{1}{15}$ min. $= 4$ sec. of time;
1″ corresponds to $\frac{1}{60}$ of 4 sec. $= \frac{1}{15}$ sec. of time.

24 hr. correspond to 360°;
1 hr. corresponds to 15°;
1 min. of time corresponds to $\frac{1}{60}$ of 15° $= \frac{1}{4}°$ $= 15'$;
1 sec. of time corresponds to $\frac{1}{60}$ of 15′ $= \frac{1}{4}'$ $= 15''$.

All objects seen on the sky, as the sun, may be *regarded* as stationary, while the turning of the earth carries us past them from the west toward the east. This makes the sun *appear* to rise in the east, move over, and set in the west.

1. Will the sun pass over eastern or western places earlier? Which places have later local* times, those over which the sun passes earlier, or later? Which places have earlier local times, eastern places, or western places?

* Local sun time is obtained by setting timepieces at XII as the sun crosses the meridian.

2. What time is it at a place 45° west of Washington when it is 10 o'clock at Washington? At the same instant, what time is it at a place 30° east of Washington?

Longitude is the distance in degrees, minutes, and seconds (of arc) due eastward or westward from a chosen meridian, called the prime meridian.

Astronomers and navigators have agreed that the prime meridian shall be the meridian of the Royal Observatory at Greenwich, England.

3. The difference between the local times of two places is 4 hr. 3 min. What is the difference in longitude between them?

4. The difference of longitude between two places is 105° 45′. What is the difference of their local times?

5. It is 4:50 (4 hr. 50 min.) P.M. at a certain place and 1:48 P.M. at another. Which place is east of the other and what is the difference of their longitudes?

6. Two men met in Chicago with their watches keeping the correct local times of the places whence they came. On comparing their times one watch showed 10:47 P.M., and the other 3:18 P.M., when it was 9:26 P.M. in Chicago. From which direction did each man come?

7. If the watches (problem 6) were keeping the correct local times of their places, what were the differences of longitude between the places and Chicago?

8. Explain how it happened that the announcement of Queen Victoria's death was read in the Chicago dailies at an earlier hour than that borne by the announcement itself.

9. How may it happen that a cable message sent Wednesday forenoon may be received at a remote place Tuesday?

10. The local times of two ships at sea differ by 4 hr. 18 min. 15.6 sec. Find their difference of longitude?

11. One ship is in longitude 186° 40′ 12″ west and another is in longitude 20° 16′ 48″ west. What is the difference of their longitudes? Of their times?

12. The longitude of one ship is 3° 28′ 10″ west and that of another is 18′ 38″ east. What is the difference of their longitudes? Of their times?

13. The longitude of the Observatory of Madrid, Spain, is 3° 41′ 17″ west and that of the Berlin Observatory is 13° 23′ 43″ east. What is the difference of their longitudes? Of their times?

14. The longitudes of the Cambridge (Eng.) and of the Paris Observatories are 5′ 41.25″ east and 2° 21′ 14.55″ east respectively. What is the difference of their longitudes? Of their times?

15. When the sun is on your meridian, that is, when it is noon at your place, at what places on the earth is it forenoon? Afternoon? Night? Midnight?

The longitudes from Greenwich, Eng., of places are also often given (as below) in hours, minutes, and seconds *of time*. The plus (+) sign means that the place is west of the meridian of Greenwich, and the minus (−), that the place is east of this meridian.

Place	Longitude from Greenwich	Place	Longitude from Greenwich
	Hr. Min. Sec.		Hr. Min. Sec.
Albany	+4 54 59.99	Denver	+6 59 47.63
Algiers	−0 12 08.55	Edinburgh	+0 12 44.2
Allegheny, Penn.	+5 20 02.93	Glasgow	+0 17 10.55
Ann Arbor, Mich.	+5 34 55.19	Madison, Wis.	+5 57 37.93
Berkeley, Calif.	+8 09 02.72	Madrid	+0 14 45.12
Berlin	−0 53 34.85	Mexico	+6 36 26.73
Bombay	−4 51 15.74	New York	+4 55 53.64
Cambridge (Eng.)	−0 00 22.75	Paris	−0 09 20.97
Cambridge (Mass.)	+4 44 31.05	Philadelphia	+5 00 38.51
Cape of Good Hope	−1 13 54.76	St. Petersburg	−2 01 13.46
Chicago	+5 50 26.84	Washington	+5 08 15.78

16. Give from the table the difference of local times of Greenwich, England, and of each of the following places and state whether their time is earlier or later than Greenwich time: Albany; Algiers; Ann Arbor; Berlin; Bombay; Chicago; New York; Paris; St. Petersburg.

17. What is the longitude in degrees (°), minutes (′) and seconds (″) of arc of Albany? Of Algiers? Of Cambridge (Mass.)? Of Washington?

18. Which of each of these pairs of places has the earlier local time and how much earlier is this time?

Albany and Allegheny.
Ann Arbor and Berkeley.
Ann Arbor and Bombay.
Chicago and Cambridge, Mass.
Chicago and Paris.
Edinburgh and Madison.
Cape of Good Hope and Bombay.
Paris and Bombay.

19. Give the differences of longitude in each case of problem 18.

20. Solve other similar problems on the table.

§113. Standard Time.

For convenience of railway traffic a uniform system of time-keeping, known as Standard Time, was agreed upon in 1883 by the principal railroad companies of North America. It was decided that places within a belt of 15° extending (roughly) $7\frac{1}{2}°$ on each side of the 75th meridian west of Greenwich should use the time of the 75th meridian. All places in similar belts extending about $7\frac{1}{2}°$ on each side of the 90th, of the 105th, and of the 120th meridian should take the times of those meridians respectively, and hence should have times just 1 hr., 2 hr., and 3 hr. earlier (less) than the time of the 75th meridian *time belt*. This system has now been generally adopted by

most civilized countries. In practice, however, the dividing lines of the time belts are irregular lines running through railroad terminals. The following map shows the time belts and the names used to distinguish the times of the several belts which cover continental United States. In this system the times at all places in the United States differ only by whole hours.

FIGURE 1

1. When it is 8 o'clock A.M. (Standard Time) at Chicago, what is the time at each of the following places: New York? Pittsburg? St. Louis? Omaha? Denver? Spokane? San Francisco? (See map.)

2. When it is 9 hr. 10 min. 45 sec. P.M. at Omaha, what is the time at New Orleans? Philadelphia? Buffalo? Washington? Austin (Tex.)? Boise City? Los Angeles? El Paso? Salt Lake City? (Refer to a Geography.)

3. Answer the questions of problem 2 for 2:15 P.M. at St. Paul; for 3:25 P.M. at Dodge City, Kan.

4. Answer for other places marked on the map such questions as are asked in problems 2 and 3 for Omaha, for St. Paul, and for Dodge City.

The circle *Sq Nr* (Fig. 1) represents the earth and the curved lines represent the hourly meridians running from the equator, *qr*, and con-

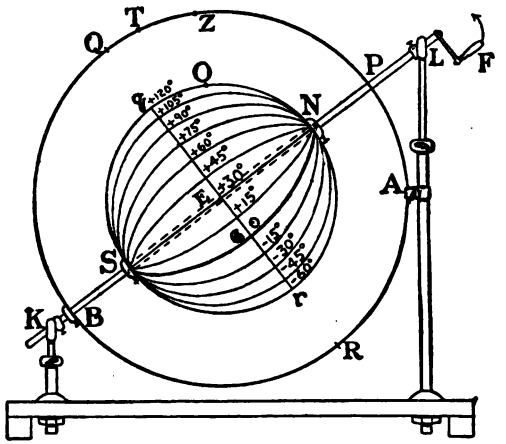


FIGURE 1

verging toward the poles *S* and *N*. If *O* represents some place in the northern hemisphere, *Sq Nr* is the meridian of the place and *BQPR* may be imagined to represent the meridian of the sky (*hour circle* of the sun), on which the sun, T, is situated. The hour circle of the sun in the apparatus is held in place at *A*. If now the crank *F* is turned so as to carry *q* forward and downward through *E* to *r*, the sun's meridian standing stationary, the motions which cause the differences of time and the changes of day and night may be understood, by recalling that only the half of the globe which is turned to the sun is light (in day). *SGN* denotes the *prime meridian*.

5. As the globe is standing in Fig. 1, what time is it at *O*? At places on the 105th meridian? On the 60th meridian? On the 15th meridian east* of Greenwich? On the 45th east?

*Remember that east is the direction toward which the globe is turning, that is eastward means from *E* toward *G* through *r* and around to *E* again.

6. If the meridian *SGN* is continued around on the other side of the globe, what will its number be?

7. Knowing that a place on the earth, as *O*, turns round through 360° in 24 hr., how long does it require the space between any two adjacent meridians of Fig. 1, p. 197 to pass under the sun's hour circle?

8. If a man should start from some place on the prime meridian (say London) on Friday noon and move westward just as fast as the globe turns eastward, what time (by the sun) would it be to him during his journey all the way round the globe? What hour and day would a Londoner call it when the traveler returned 24 hr. later?

The problem raises the question, "Where should the traveler have changed his date so that his date might agree with that of his starting place when he returns?" The answer is, "It has been agreed that the date should change at the 180th meridian." When vessels cross this meridian from the east toward the west they add a day to their reckoning. If they cross at noon on Friday, Friday noon instantly becomes Saturday noon. Crossing from the west toward the east, they repeat a day. In the case mentioned, the 24-hour period from Friday noon would "be done over again." The 180th meridian is for this reason called the *Date Line*. Trace it on the earth in a Geography.

9. The date begins on the 180th meridian of longitude and travels *westward* around the earth. Supposing the day begins at midnight, give the date and hour on the 180th meridian, when it is Wednesday, Oct. 15, 10 P.M., standard time, in Chicago; in Washington; in Denver; in San Francisco.

10. The U. S. Supreme Court has decided that *legal time* at any place is the *local time* of the place. A man in Akron, O., insured his house at 11:30 A.M. "standard" (90th meridian) time and the house took fire at that moment and was destroyed. The policy stated that the insurance would be in force at and after 12 o'clock (noon)

of the day of insuring. The difference between Akron local time and 90th meridian time is 33 minutes. Would the law require the insurance company to pay the policy? Give reason for your answer.

11. If every place on the earth has the time of that meridian of the map, p. 190, nearest to it, when it is 3:25 P.M. in England what will be the time of the following places? (1) Washington. (2) Boston. (3) New Orleans. (4) Denver. (5) Rio Janeiro. (6) Cuba. (7) Japan. (8) Philippine Islands.

MENSURATION

§114. Areas, Roofing and Brickwork.

1. Give a rule for computing from its dimensions the area of a square; a rectangle; a parallelogram; a triangle.

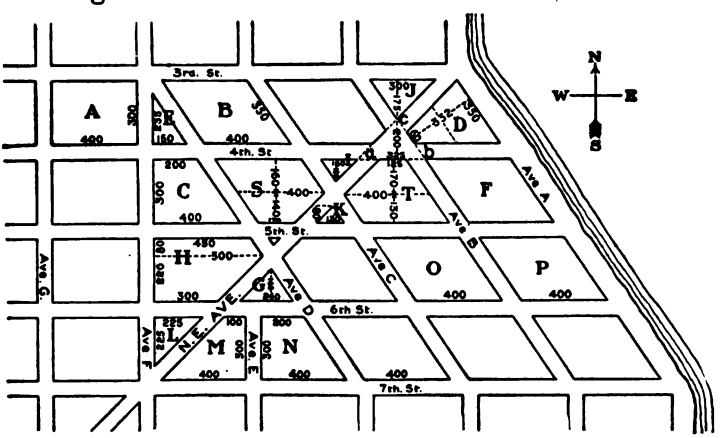


FIGURE 1

Fig. 1 represents a plot of the streets and blocks of a part of a certain city. The streets are all 75 ft. wide

between sidewalks. The numbers written on the lines indicate their lengths in feet. The dotted lines show how to divide up the areas into parts for computation.

2. If $240 per foot of frontage on both 3d, and 4th streets, was paid for block B (Fig. 1, p. 201), how much did the block cost per square foot? Per square yard?

3. Find the area in square yards of other parallelograms, such as O, P, etc., of Fig. 1, p. 201.

4. Find the area in square feet of the following triangles of Fig. 1, p. 201; E; G; I; J; K; L; *abc*.

Other areas of Fig. 1, p. 201, require a knowledge of trapezoids.

FIGURE 1

A four-sided figure (a quadrilateral) like Fig. 1, having one pair of parallel sides is called a trapezoid.

The altitude of a trapezoid is the distance square across between two parallel sides (called the bases).

5. Study the trapezoids of Fig. 2, and find how to get the length of *EF*, the line connecting the middle points of the two non-parallel sides, from the lengths of the two bases. After computing *EF*, find for each trapezoid a rectangle whose area equals the area of the trapezoid. Find the areas of the trapezoids of Fig. 2.

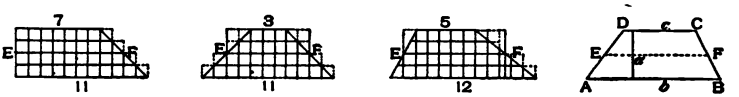


FIGURE 2

The lengths a, b, and c of the last trapezoid are called its dimensions.

NOTE.—$\frac{1}{2}$ the sum of x and y is written $\frac{1}{2}(x + y)$. n times the half sum is written $\frac{1}{2}n(x + y)$

6. How does the sum of the bases of a trapezoid compare with the base of a parallelogram having an altitude

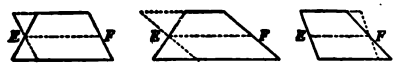


FIGURE 1

and area equal to the altitude and area of the trapezoid (Fig. 1)?

7. Supposing a, b, and c are the dimensions, in feet, of the trapezoids of Fig. 1, what are the areas of these trapezoids?

8. Find the areas in square feet of C; of D; of M; of N; of H (Fig. 1, p. 201).

9. Calling Z the area of any trapezoid, whose bases are b and c and whose altitude is a, write an equation showing how to find Z from a, b, and c.

10. Find the area in square yards of A, B, N, S, T, Fig. 1, p. 201.

A square of roofing means a 10′ square of roof surface, or 100 sq. ft. A shingle is said to be laid 4″, 4½″, or 5″ to the weather when the lower end of each course of shingles on the roof extends 4″, 4½″, or 5″ below the course next above it.

11. Draw two perpendicular center lines, and with the aid of the dimensions given in the left part of Fig. 2, complete an enlarged drawing to a convenient scale, of the development of the roof shown on the left.

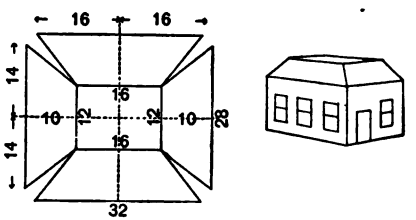


FIGURE 2

12. If 1000 shingles laid 4″ to the weather cover a square of roofing, how many shingles will be needed to cover a square, if laid 5″ to the weather? 4½″ to the weather? 3″ to the weather? 3½″?

13. The dimensions on the development (Fig. 2) being in feet, find the cost of the shingles, laid 4″ to the weather,

at \$1.10 a bunch of 250, needed to cover the four sides of the deck roof (Fig. 2, p. 203). Find the cost of enough tin to cover the 12′ × 16′ flat deck at 15¢ a 20″ × 28″ sheet, the long sides of the sheets being laid parallel to the long side of the deck, and allowing 10% loss for joints and overlap at edges.

14. When shingles are laid 4″ to the weather, 1000 shingles are estimated to cover a square (of roofing). Find the number of shingles, laid 4″ to the weather, needed to cover a roof which, with gables, is made up of the following parts:

Shapes	Altitudes	Bases	Shapes	Altitudes	Bases
2 rectangles	10′	40′	4 triangles	$8\frac{1}{2}$′	17′
2 parallelograms	12′	$26\frac{1}{2}$′	2 trapezoids	12	12′ and $20\frac{1}{2}$′
2 parallelograms	12′	14′	2 trapezoids	12′	5′ and $13\frac{1}{4}$′
2 triangles	7′	14′	2 trapezoids	12′	6′ and $14\frac{1}{4}$′

15. Find the cost of the shingles of Prob. 14 @ 90c a bunch of 250.

16. Ten years after building this house a new roof was put on it. It cost \$1.25 per M. to remove the old shingles, \$6 per M. for the new ones, and \$2.75 per M. for the labor of putting on the shingles. How much did it cost to remove the old shingles and to re-roof with new shingles?

17. If it takes 14 bricks per square foot of outside surface to lay a 2-brick wall, how many bricks will be needed to lay the side wall of the building shown in Fig. 1, deducting for 5 windows each $3\frac{1}{2}$′ × $6\frac{1}{2}$′ and for 5 each $3\frac{1}{2}$′ × $9\frac{1}{4}$′.

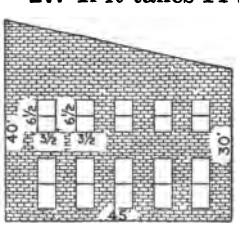


FIGURE 1

18. Make other similar problems from your own measurements or

from dimensions obtained from an architect or builder.

19. The walls of a square brick house, 24′×24′, start 6′ below the ground and rise 24′ above the ground to the eaves. From the bottom to 12′ above the ground the walls are 3 bricks thick and from this line to the eaves they are 2 bricks thick. Find the cost of laying the bricks of these walls @ $4.25 per M.

NOTE: No allowance for openings of any kind is made in figuring the *cost of laying* bricks in such walls, and outside dimensions are used, thus counting corners twice.

20. From the number of bricks found in problem 19 deduct for the following openings:

DOWNSTAIRS	UPSTAIRS
1 door 3′ 6″ × 6′ 9″	2 doors 3′ 6″ × 6′ 6″
1 door 3′ × 6′ 6″	1 window 5′ × 6′ 6″
2 windows 5′ × 6′ 6″	12 windows 3′ × 6′
9 windows 3′ × 6′	

and 1400 bricks for double counting of corners, and find the cost of the bricks @ $12\frac{1}{2}$ per M.

21. My friend's house has two octagonal (8-sided) towers, the roofs being made up of equal triangles (Fig. 1). Find the number of slates for both roofs, 240 to the square (100 sq. ft.), and cost @ $2.75 a hundred.

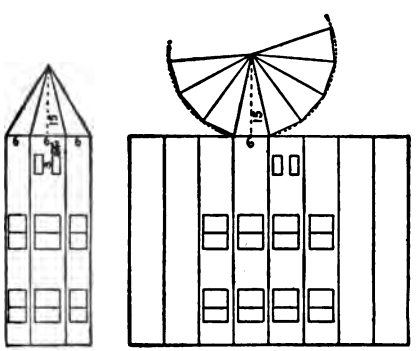


FIGURE 1

22. Draw the development of the roof and tower to a scale $\frac{1}{4}$″ to 1′, as shown in the figure.

23. From the eaves to the ground the towers are octagonal prisms, built of brick, all faces being rectangles $6' \times 40'$, and each prism has four faces exposed to the weather. How many square feet of brick surface are in the 8 exposed faces, not allowing for windows? Find the cost of laying the walls two bricks thick, @ \$6.25 per M.

24. Deduct for 16 windows, each $3'\ 6'' \times 5'\ 8''$, and for 4 windows $1'\ 6'' \times 3'$, and find the number of bricks in the two layers, and their cost @ \$14 per M.

§115. Land Measure.

Review §29, pp. 39, 40, and 41.

Since 1785 the law has required land to be marked out or surveyed in divisions of the form of squares and rectangles. In the western states the land has been surveyed under this law.

To mark out the largest squares, north and south lines, called *meridians*, are first run 24 mi. apart and marked with corner-stones, or by trees, or other permanent objects. East and west lines, called *base* lines, are then run at right angles to these meridians at distances 24 mi. apart. This would divide the land up into 24-mi. squares were it not for the convergence of the meridians toward the poles of the earth. Notice this on a map in your Geography and on the map on p. 198.

Each 24-mi. tract is then divided into 16 nearly equal squares, called townships, by running north and south, and east and west lines through the quarter points of the sides of the large tract.

1. How long is a township? How wide? How many square miles does it contain?

Certain meridians, called *principal meridians*, are run with great care, and these principal meridians govern the surveys of lands lying along them for considerable distances both toward the east and toward the west. The tiers, or rows, of townships running north and south along the principal meridians are called *ranges*. The first tier on the east is called range No. 1 east, and is written R1E; the second range is No. 2 east, written R2E, and so on.

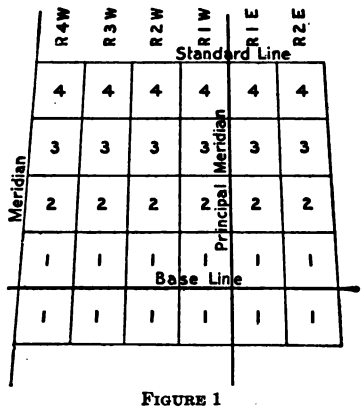


FIGURE 1

2. Point out on the drawing, Fig. 1, R1W; R2W; R3W; R2E; R4W.

Certain base lines are run with great care and are called *standard base* lines. The rows of townships running east and west are numbered with reference to these standard base lines. A township in the first row north of a base line is township No. 1 north, and is written T1N; one in the second row south is called township No. 2 south, written T2S, and so on.

3. Interpret the following symbols and point out on the drawing, Fig. 1, the townships indicated: T3N; T4N; T1S; T2N.

A township is identified by giving its number and range from some standard base and principal meridian.

Point out these townships on the drawing, Fig. 1: T1N, R2W; T3N, R1W; T4N, R2E; T1S, R3W.

The law also requires townships to be subdivided

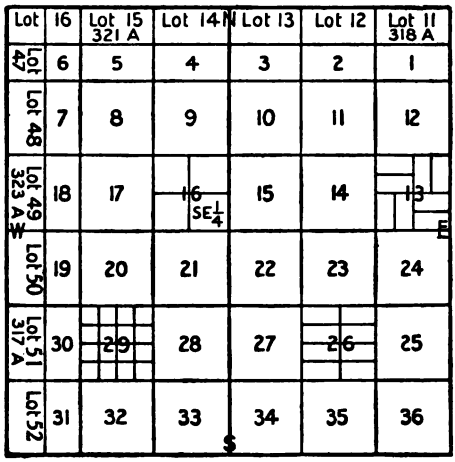


FIGURE 1

into smaller squares called *sections*. Sections are numbered, beginning at the northeast corner and running toward the west to 6, then 7 is just south of 6, and so on, as in Fig. 1.

Sections are then subdivided into quarters (see section 16), half-quarters (see sections 13 and 26), and quarter-quarters (see section 29).

4. Referring to Fig. 1, read and write the descriptions of the divisions of section 16; of section 26; of section 13; of section 29.

Whatever deviations there may be from exactly 640 A., in the sections of any township, due to convergence of meridians or other causes, are required by law to be added to, or subtracted from, the north and west rows of half-sections.

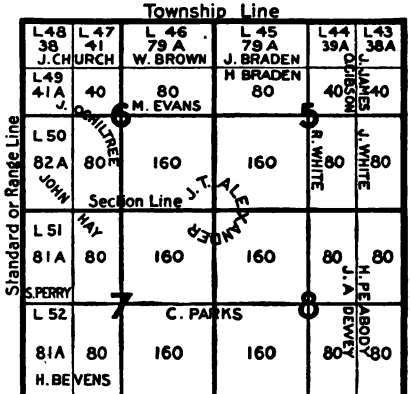


FIGURE 2

These tracts are then not called fractional sections, but are called *lots*, and are numbered in regular order as shown in Fig. 1, p. 208.

A section then always means exactly 640 A. Any fractional part of a section means the corresponding fractional part of 640 A. The lot, on the contrary, must always be measured before its area is known.

5. Following is an assessment list of farm property. Fill the blanks from Fig. 2, p. 208:

Owner	Description	No. Acres
H. Peabody	E½ SE¼ Sec. 8	.....
H. Peabody	E½ NE¼ Sec. 8	.....
J. White	E½ SE¼ Sec. 5	.....
J. James	SE¼ NE¼ Sec. 5	.....
J. James	Lot 43	.....
O. Gibson	Lot 44	.....
O. Gibson	SW¼ NE¼ Sec. 5	.....

6. Make out an assessment list for all owners in Sec. 6.

7. Make and solve a similar problem for owners in Sec. 7.

8. Correct the mistakes in this erroneous assessment list.

Owner	Description	No. Acres
S. Perry	W½ NW¼ Sec. 7	81
J. Hay	W½ SW¼ Sec. 6	82
J. Hay	W½ SW¼ Sec. 7	80
J. Ochiltree	SW¼ NW¼ Sec. 6	41
J. Ochiltree	SE¼ NE¼ Sec. 6	40
J. Ochiltree	E½ SE¼ Sec. 6	80
J. Church	NW¼ NW¼ Sec. 6	38
J. Church	NE¼ NW¼ Sec. 6	41

§116. Volumes.

The volume of any figure is the number of cubical units within its bounding surfaces.

1. Give the rule for finding the volume of a square prism (called also a *rectangular parallelopiped*).

2. Give a rule for finding the volume of an oblique parallelopiped (Fig. 1, p. 170) having the same base and the same altitude as a given rectangular parallelopiped. (See Fig. 2, p. 169).

3. Find the volumes of the forms of hollow beams, each 12 ft. long, having cross sections like Fig. 1, p. 157.

4. Give the volumes of the beams of problem 3, if the length of the beam in each case is l feet.

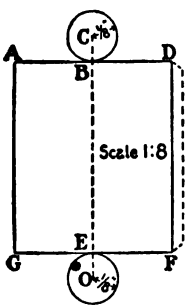


FIGURE 1

5. A model of a square prism (like Fig. 1, p. 168) having a base 2 in. square and an altitude of 8 in. will contain how many cubic inches of sand?

6. The model of a right circular cylinder, made as shown in the scale drawing of Fig. 1 and pasted along the flap, DF, and around one end with a strip of paper, was filled with sand and poured into the empty model of the square prism of problem 5. It filled the square prism a little more than $\frac{3}{4}$ full. About how many cubic inches are there in the model of the cylinder?

NOTE. It is shown in geometry that the volume of any right circular cylinder is .7854 ($=\frac{\pi}{4}$) times the volume of a square prism of the same altitude and having for one side of its base the diameter of the cylinder.

7. Find the volumes of these circular cylinders, using .7854:

(1) Diam. 4″ alt. 6″ (4) Diam. $18\frac{1}{2}$″ alt. 8″

(2) " 7″ " 7″ (5) " 6′ " 10.5′

(3) " 10″ " $8\frac{1}{2}$″ (6) " $2r'$ " a'

8. How long is AB? How many square inches in the rectangle $ADFG$ (Fig. 1, p. 210)? How many square inches are there in the entire surface of the cylinder?

9. The inside diameter of the cylindrical water tank of a street sprinkler is 3.6 ft., and its length is 10 ft.; how many liquid gallons does the tank hold?

10. The tank of the sprinkler is filled through a hose of $2\frac{1}{2}''$ inside diameter from a hydrant from which the water flows at the rate of 300 linear feet per minute. How long will it take to fill the tank?

SUGGESTION: In 1 min. a cylinder of water as large as the inside of the hose and 300′ long flows into the tank.

11. Find the weight of an iron rod $\frac{1}{2}''$ in diameter and 24′ long, if iron weighs 450 lb. per cubic foot.

12. How many cubic inches of air are there in the hollow tire of a bicycle wheel 28″ in diameter (from center line to center line of tubes), the hollow having a diameter of $1\frac{3}{4}''$?

13. How many cubic inches of rubber are there in the hollow tire of a 32″ automobile wheel if the inside diameter of the cylindrical tube is 3″ and the outside diameter is 4 inches?

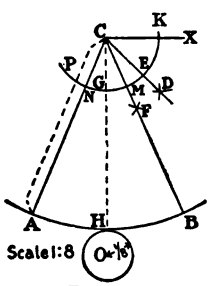


FIGURE 1

14. Fig. 1 is a scale drawing of a pattern for the paper model of a cone whose base is to be a circle 2″ in diameter and whose sloping side from the apex to the base is to be 8″. The arc AHB is just as long as the circumference of the base. Compute the length of the circumference of a circle whose radius is 2″, and of another whose radius is 8″, and find the ratio of the first to the second.

15. What part of the whole circumference (with center C) is the arc AHB? What part of 360° is the angle ACB?

16. Make a right angle, as HCX, and bisect it. CD is the bisector. Bisect angle HCD and obtain CB.

EXPLANATION: With C as a center and with a convenient radius, as CG, draw the indefinite arc, PGK. Put the pin foot on K and mark an arc at D using radius, KD. With the same radius and with the pin foot on G mark another arc at D. Connect CD. CD is then the bisector of the angle HCX.

17. Similarly bisect the angle GCE. Prolong the bisector CF to B, 8″ from C. Draw the arc BHA, making $HA = HB$. Draw CA. Prolong CH, making $HO = 1''$, and draw the lower circle. Provide the flap and paste up the model of the cone.

If such a model is carefully made and the height is measured and if the model of a cylinder has the same height and the same base, the model of the cone will be found to hold $\frac{1}{3}$ as much sand as does the model of the cylinder.

It is proved in geometry that the volume of any cone equals $\frac{1}{3}$ of the volume of a cylinder having an equal base and an equal altitude.

18. Find the volumes of these circular cones:

(1)	Diameter of base	3″,	altitude,	7″
(2)	"	"	6″	"	20″
(3)	"	"	6.8″	"	2.25′
(4)	"	"	3.95″	"	10.25″

19. How many cubic inches of water are there in a funnel-shaped vessel, if the water is 4″ deep and the diameter of the surface of the water is 4.5″? (See Fig. 1.)

FIGURE 1

20. How many cubic inches of water will it take to fill the same vessel to twice the depth, or 8″? (See Fig. 1.)

21. How many cubic inches had to be poured into the vessel to fill it to 8″ depth if the depth of the water was 4″ at the beginning?

22. A mountain peak has the shape of a circular cone whose altitude is 1.25 mi. and the diameter of whose base is 2.35 miles. Find its volume in cubic miles.

23. How much water will it take to fill a conical vase 10″ deep and 3.225″ across at the top?

24. The conical tower of a building is 24.75′ across at the base and 36.8′ high. How many cubic feet of space does it occupy?

25. If a cord be wrapped around the curved outside surface of a *half* croquet ball, as the string is wound around a top until the surface is covered, and then if the same cord be wrapped around on the flat circular base beginning at the center until the circle is covered, the former length of the cord will be found to be just twice the latter. The area of the surface of the *whole* ball is then how many times the area of the circular section of the ball? What is the length of the radius of this circular section?

It is proved in geometry that the area of the surface of a sphere equals 4 times the area of a circular section going through the center of the sphere.

26. The radius of a ball is $1\frac{1}{2}$″. What is the area of the greatest circular section of the ball? What is the area of the surface of the ball?

27. Measure the circumference of a baseball and find how many square inches of leather there are in the cover of the ball.

28. How many square inches of paint will it take to cover the surface of a globe of 10″ radius? Of 10″ diameter?

29. The average diameter of the earth is 7918 mi. How many square miles are there in its surface?

30. Calling s the surface and r the radius of a sphere, write an equation showing the relation between s and r.

It is seen on p. 169 that the volume of a right triangular prism equals $\frac{1}{2}$ the volume of a square prism whose base and altitude are equal to the base and the altitude of the square prism.

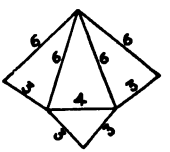


FIGURE 1

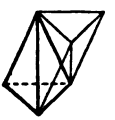


FIGURE 2

31. Find the volume of a right triangular prism whose altitude is 18″ and whose base is a triangle having a base of 8″ and an altitude of 5″ inches.

If a triangular pyramid be carefully modeled (Fig. 1) and filled with sand it will be found that just 3 times the volume of the pyramid is equal to the volume of the model of a triangular prism (see Fig. 2) of equal base and equal altitude.

It is proved in geometry that the volume of any pyramid equals $\frac{1}{3}$ of the volume of a prism having an equal base and an equal altitude. Notice in Fig. 2 how a triangular prism may be completed on a triangular pyramid having the same base and the same altitude as the prism.

32. Calling V the volume, B the area of the base, and a the altitude of a triangular pyramid, write an equation, showing the way V would be computed from B and a.

33. Find the volume of a pyramid whose base contains 16 sq. in. and whose altitude is 12 inches.

34. The Great Pyramid of Egypt is 481 ft. high and its base is a 756′ square. If it were solid and had smooth faces, how many cubic feet of masonry would it contain?

35. At the close of the nineteenth century the United States had 195,887 mi. of railroad. How many times would these railroads, if placed end to end, encircle the earth? (Use $\pi = 3\frac{1}{7}$, and the radius of the earth = 3959 miles.)

36. The ties used for these roads would contain wood enough to make a pyramid 1395′ high with a 2192′ square for its base. How many cubic feet of wood were used for the railroad ties?

37. It has been computed that the materials used for the road beds for these railroads would make a solid pyramid 2470 ft. high and having a 3870 ft. square for its base. How many cubic feet would this make?

If the entire surface of a globe, or sphere, were divided up into small triangles and rectangles, and the sphere were cut up by planes cutting along the curved sides of these figures and passing through the center, O, the volume of the sphere would be divided up *approximately* into small pyramids, having their vertices at the center (see Fig. 1). The volume of each pyramid would be the product of its base by $\frac{1}{3}$ of the radius of the sphere. The sum of the areas of the bases of all these pyramids would equal the surface of the whole sphere and the sum of the volumes of all the pyramids would equal the surface of the whole sphere multiplied by $\frac{1}{3}$ of the radius of the sphere.

FIGURE 1

38. Calling V the volume and r the radius of a sphere, give the meaning of the formula:

$$V = \frac{r}{3} \times 4\pi r^2 = \frac{4}{3}\pi r^3.$$

39. Find the number of cubic inches in a sphere of 2″ radius.

40. How many cubic inches in a croquet ball 4″ in diameter? In a tennis ball 1.75″ in diameter? In a base-ball 2.9″ in diameter? In a globe 10.15″ in diameter?

41. Calling the sun, the moon, and the planets all spheres with diameters in miles as in the following table, compute two or three of their circumferences in miles, their surfaces in square miles and their volumes in cubic miles:

Moon..... 2160	Mars..... 4230	Uranus 31,900
Mercury..... 3030	Jupiter.....86,500	Neptune.... 34,800
Venus..... 7700	Saturn.....73,000	Sun.....866,400
Earth..... 7918		

42. What is the ratio of the surface (including the ends) of a cylinder having an altitude 12 inches and a radius of 6 inches, to the surface of a sphere 12 inches in diameter? What is the ratio excluding the end surfaces?

43. What is the ratio of the areas of the curved surfaces of a right cone and a right cylinder, both having an altitude 12 inches and radius of base of 6 inches?

44. Find the ratio of the volume of a sphere 12 inches in diameter to the volume of a right cylinder 12 inches high and 12 inches in diameter.

45. Find the ratio of volumes of a right cone 12 inches high and having a circular base 12 inches in diameter, and a sphere 12 inches in diameter.

46. Find the ratio of volumes of a cone and a cylinder each 12 inches high and each having a circular base 12 inches in diameter.

§117. The Right Triangle.

A right triangle is a triangle that has a right angle.

EXERCISE I.—*To find the relation of the squares of the sides of an isosceles right triangle.*

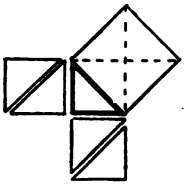


FIGURE 1

Construct an isosceles right triangle and on each of its three sides draw a square. Draw the dotted lines and cut the side squares as shown in Fig. 1. Fit the pieces over the large square on the hypothenuse. If the area of each side square were 9 sq. in. what would be the area of the large square?

EXERCISE II.—*To find the relation of the squares of the sides of a triangle, having one of the shorter sides twice as long as the other.*

Draw a right angle and mark off sides a and $2a$, one twice as long as the other. Draw the hypothenuse, h.

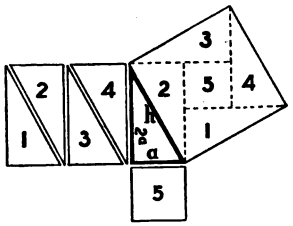


FIGURE 2

With a square-cornered card draw squares on each of the three sides. Cut off the side squares. Cut the larger side-square first into two equal rectangles, and then cut each rectangle along diagonals, as shown. Place the several pieces as the numbers in Fig. 2 indicate. It is plain that the area of the square on the hypothenuse equals the sum of the areas on the side-squares, as above.

EXERCISE III.—*To find the relation of the squares of the sides of any right triangle.*

Draw any right triangle and then draw squares on each of the sides.

Then find the middle point of the larger side-square, by marking a point where the diagonals intersect. Through this middle point draw a line parallel to the hypothenuse of the given triangle. Through the same point draw also a perpendicular to the parallel. Cut as shown, and place the pieces on the largest square as shown by the numbers in Fig. 1.

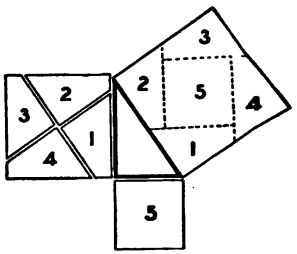


FIGURE 1

Thus, it is clear that with any right triangle the sum of the squares on the sides equals the square on the hypothenuse.

1. Denoting the length of either short side of Fig. 1, p. 217, by a, what denotes the area of the square drawn upon this side?

2. Denoting the length of the hypothenuse of Fig. 1, p. 217, by h, what denotes the area of the square drawn on the hypothenuse?

3. Write an equation from Fig. 2, p. 217, showing the relation between a^2 and h^2.

4. Denote the lengths of the three sides of any right triangle (Fig. 1) by a, b, and h (h being the hypothenuse). What will denote the areas of each of the squares on the three sides?

5. Write an equation showing the relation of the squares of the sides of any right triangle.

6. The sides of a right triangle are $3''$ and $4''$. What is the length of the hypothenuse?

SUGGESTION: $h^2 = 3^2 + 4^2 = 9 + 16 = 25$. What is the value of h?

7. The hypothenuse of a right triangle is 10″, and one of the sides is 6″. What is the other side?

SUGGESTION: $10^2 = a^2 + 6^2$. Find the value of a.

8. The altitude and base of a right triangle are denoted by a, b, and the hypothenuse by h. Find the unknown side in each of the following right triangles:

(1) $a = 9, b = 12$ (4) $a = 32, b = 24$

(2) $a = 12, h = 20$ (5) $b = 21, h = 35$

(3) $b = 15, h = 25$ (6) $b = 27, h = 45$

Before the hypothenuse of a right triangle whose sides are 34 and 26, can be computed, it is necessary to know how to find the square roots of numbers.

§118. Squares and Square Roots.

The product obtained by using any number twice as a factor is called the square of that number. Thus, 36 is the square of 6, because 6, used twice as a factor gives 36 ($6 \times 6 = 36$).

The square of a number as 6 is often written thus, 6^2. What does the small 2 show?

Table of squares of numbers to be learned:

SQUARES OF UNITS	SQUARES OF TENS	SQUARES OF HUNDREDS
$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$
$2^2 = 4$	$20^2 = 400$	$200^2 = 40000$
$3^2 = 9$	$30^2 = 900$	$300^2 = 90000$
$4^2 = 16$	$40^2 = 1600$	$400^2 = 160000$
$5^2 = 25$	$50^2 = 2500$	$500^2 = 250000$
$6^2 = 36$	$60^2 = 3600$	$600^2 = 360000$
$7^2 = 49$	$70^2 = 4900$	$700^2 = 490000$
$8^2 = 64$	$80^2 = 6400$	$800^2 = 640000$
$9^2 = 81$	$90^2 = 8100$	$900^2 = 810000$

1. Write all the pairs of numbers which, multiplied together, give the product 36; the product 16; the product 64; the product 49.

NOTE: Write the pairs of factors of 36 thus: 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6. Proceed similarly with the rest.

The square root of a number is one of the two equal factors of it. The sign of square root is $\surd$, called the radical sign. Thus, $\surd 25$ means the square root of 25, which is 5.

2. Give the square roots of the following numbers:

9; 16; 49; 64; 81; 400; 2500; 3600; 160000; 490000; 810000.

To find the square root of a number not in the table p. 219), it is necessary to learn how the square of a number is formed from the number.

From the table answer the following questions:

3. How many digits are there in the square of any number of units? Of tens? Of hundreds?

4. Find the square of each of the following decimals: .1, .3, .5, .7, .9, .01, .03, .04, .06, .07, .09, .001, .003, .005, .006, .007, .009.

5. How many decimal places are there in the square of any number of tenths? Of hundredths? Of thousandths?

6. The square of any number of units then (as 5) contains only units and tens. What places of *any* number that is a square (as 576) must contain the square of the units (4) of its square root (24)?

7. What places must contain the square of the tens? Of the hundreds? Of the tenths? Of the hundredths? Of the thousandths?

8. If, then, we separate a number, whose square root is desired, into two-digit groups, beginning at the decimal point and proceeding both toward the left and toward the right, what one of these groups must contain the square of the units of the square root? The square of the tens? Of the hundreds? Of the tenths? Of the hundredths?

9. Find the square of 46.

Common Method	and Meaning
46	$46 = 40 + 6.$
46	$46^2 = (40 + 6)^2 =$
276	$(40 + 6)\ (40 + 6) =$
184	$40 \times 40 + 40 \times 6 +$
2116	$6 \times 40 + 6 \times 6 = 40^2$
	$+ 2 \times (40 \times 6) + 6^2$
$46^2 = 2116$	$= 1600 + 480 + 36$

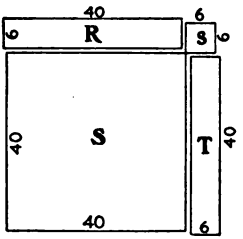


FIGURE 1

Show the meaning of the parts of this sum in Fig. 1.

Thus it is seen that the square of a two-figure number is the sum of (1) the square of the tens, (2) twice the product of the tens by the units, and (3) the square of the units.

This shows that if any two-figure number be denoted by $t + u$, where t denotes the tens and u the units, the square is formed thus:

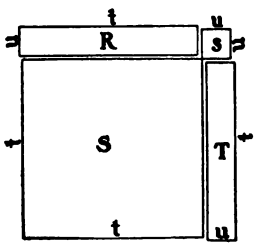


FIGURE 2

$40+6$	$t+u$
$40+6$	$t+u$
$240+36$	$tu+u^2$
$1600+240$	$t^2+\ tu$
$1600+480+36$	$t^2+2tu+u^2=(t+u)^2$

Show the meaning of the equation by Fig. 2.

10. Find the square root of 2116.

Convenient Form

$46 =$ required root.

$\overline{2116}$ denoted by $t^2+2tu+u^2$.

$1600 = t^2$, greatest square (of the table) in 2116.

$2t=80$	516	contains $2tu+u^2$, where $t = 40$.
$2t+u=86$	516	denoted by $2tu+u^2$, where $u = 6$.

Check: $46 \times 46 = 2116$.

Check by casting out the 9's:

Excess of 46 = 1. Excess of 2116 = 1.

Excess of 46 multiplied by excess of 46 (1×1) = excess of 2116 (or 1).

SHORTENED FORM

21'16 | 46 = required root.
1600
80 | 516
6
86 | 516

11. Find the square roots of the following numbers:

(1) 484 (3) 1156 (5) 1296 (7) 7569 (9) 9604
(2) 729 (4) 1225 (6) 5625 (8) 9409 (10) 110224

12. Find the square root of 2079.36.

CONVENIENT FORM

20'79.36' | 45.6
16'00.
80 | 4'79.
5
85 | 4'25.
90. | 54.36
.6
90.6 | 54.36

Ans. $\sqrt{2079.36}=45.6$

Check: (1) $45.6\times45.6=2079.36$
(2) Excess 45.6 = 6; exc. 2079.36 = 0; $6\times6=36$: Exc.$(45.6)^2$=Exc.2079.36

EXPLANATION: Separate the number into two-digit groups. Beginning on the extreme left, subtract the greatest square of tens in 20 hundreds, viz., 16 hundreds ($=40^2$), and write 4 tens on the right as the first root digit. Double the 4 tens and use the result 80 as a *trial* divisor. 80 is contained in the remainder, 479, 5 times. Write 5 as the 2d root digit and also add it to the 80, giving 85 as the *complete* divisor. Double the part of the root found, 45, giving 90 for the next *trial* divisor and complete the steps as before.

13. Find the square root of 1578 to 3 decimal places.

CONVENIENT FORM

15'78.00'00'00' | 39.724+
9
60 | 678.
9
69 | 621.
78.0 | 57.00
.7
78.7 | 55.09
79.40 | 1.9100
.02
79.42 | 1.5884
79.440 | .321600
.004
79.444 | .317776

.003824, remainder

EXPLANATION.—Annex zeros and proceed as before.

In practice the decimal point is used only in the root.

Check:

(1) $39.724\times39.724=1577.996176$
rem. = .003824
1578.

(2) Exc. 39.724=7; exc. 1578=3; exc. .003824 = 8; $7\times7=49$; exc. 49=4. Exc. $(39.724)^2$ + exc. .003824=exc.1578

14. Find the square roots of the following numbers:

(1) 1900.96	(4) 61.1524	(7) .5476
(2) 4719.69	(5) 75.8641	(8) .458329
(3) 5055.21	(6) 79.9236	(9) 1.216609

15. Find to the nearest .001 the square roots of the following numbers:

(1) 5	(4) .85	(7) 1683	(10) 1.85
(2) 7	(5) .25	(8) 6875	(11) 26.79
(3) 15	(6) 125	(9) 7328	(12) 64.893

16. Find, by multiplication, the values of the following expressions:

(1) $(\frac{1}{2})^2$	(3) $(\frac{4}{5})^2$	(5) $(\frac{8}{15})^2$	(7) $(\frac{a}{b})^2$
(2) $(\frac{2}{3})^2$	(4) $(\frac{12}{13})^2$	(6) $(\frac{21}{32})^2$	(8) $(\frac{x}{y})^2$

17. Make a rule for finding the square root of a common fraction.

18. Find, without reducing the common fractions to decimals, the values of the following expressions and prove them by multiplication:

(1) $\sqrt{\frac{1}{4}}$	(4) $\sqrt{\frac{144}{169}}$	(7) $\sqrt{\frac{81}{289}}$
(2) $\sqrt{\frac{4}{9}}$	(5) $\sqrt{\frac{64}{121}}$	(8) $\sqrt{\frac{400}{729}}$
(3) $\sqrt{\frac{16}{25}}$	(6) $\sqrt{\frac{49}{144}}$	(9) $\sqrt{\frac{a^2}{b^2}}$

19. The sides of a right triangle are respectively 34″ and 26″ long. How long is the hypothenuse?

20. The center pole of a circus tent is 35′ high, and a guy rope is stretched from the top of the pole to a stake 56′ from the bottom. How long is the rope, supposing the ground level and the rope straight, allowing 4′ for tying?

21. 50′ of the top of a tree standing on level ground is broken by the wind and remains fastened to the stump.

If the top strikes the ground 30′ from the stump, how high was the tree?

22. A horse is staked out by a rope 40′ long to the top of a stake 5′ high. Over what area can the horse graze?

23. The vertical mast of a hoisting derrick 35′ high is held in position by four guy ropes staked to the ground at the four corner points of a square. The stakes are 75′ from the bottom of the mast. How much will the rope for the 4 guys cost at 2.5¢ a foot, 20′ being allowed for knots?

24. What is the area of the square whose corners are at the stakes?

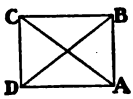


FIGURE 1

25. Find the length of the diagonals of a rectangle 48′ × 64′.

26. A single-track railroad runs diagonally across a rectangular farm 120 rd. × 160 rd. If each line of rails is as long as a diagonal of the farm, how many yards are there in the rails?

27. The center pole of a circus tent is held by 6 ropes tied to stakes at the corners of a regular hexagon (see Fig. 3). The ground is level, the pole is 48′ high and a side of the hexagon is 64′. How many feet of rope are needed, if the knots take 180′?

28. The sides of an equilateral triangle are 20′ long. A straight line drawn from any vertex to the middle of the opposite side is perpendicular to this side and bisects it. What is the area?

FIGURE 2

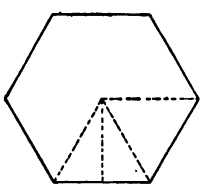


FIGURE 3

29. One side of a regular hexagon is 36′. Find the area.

30. A straight line drawn from the vertex of an isosceles triangle to the middle of the base is perpendicular to the base. Find the area of a regular pentagon whose sides are 24″, if the radius of the circle drawn around the pentagon is 18″.

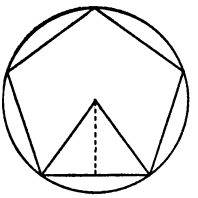


FIGURE 1

§119. Cubes and Cube Roots.

1. What is the volume of a cube whose edge is 5″? 6″? 7″? 11″? 18″? 21″? 24″?

The cube of a number is the product obtained by using the number 3 times as a factor.

Thus the cube of 4 is $4 \times 4 \times 4 = 4^3 = 64$.

Table of cubes of numbers to be learned:

$1^3 = 1$	$10^3 = 1000$
$2^3 = 8$	$20^3 = 8000$
$3^3 = 27$	$30^3 = 27000$
$4^3 = 64$	$40^3 = 64000$
$5^3 = 125$	$50^3 = 125000$
$6^3 = 216$	$60^3 = 216000$
$7^3 = 343$	$70^3 = 343000$
$8^3 = 512$	$80^3 = 512000$
$9^3 = 729$	$90^3 = 729000$

The cube root of a number is one of its 3 equal factors. Cube root is indicated by the sign $\sqrt[3]{\ }$, called a radical sign.

Thus, $\sqrt[3]{729}$ means one of the three equal factors of 729, which is 9.

2. Between what two whole numbers are the following cube roots?

$\sqrt[3]{16}$ $\sqrt[3]{35}$ $\sqrt[3]{78}$ $\sqrt[3]{450}$ $\sqrt[3]{675}$

$\sqrt[3]{895}$ $\sqrt[3]{58000}$ $\sqrt[3]{480000}$

3. Find by multiplication the values of the following cubes:

$(2.5)^3$ $(6.7)^3$ $(12.5)^3$ $(\frac{1}{2})^3$ $(\frac{2}{3})^3$ $(33\frac{1}{3})^3$ $(18\frac{3}{4})^3$

4. Make a rule for finding the cube of any common fraction.

5. Prove the following relations by multiplication:

$\sqrt[3]{2197} = 13$ $\sqrt[3]{4096} = 16$ $\sqrt[3]{\frac{125}{512}} = \frac{5}{8}$

$\sqrt[3]{\frac{729}{1231}} = \frac{9}{11}$ $\sqrt[3]{\frac{3375}{4913}} = \frac{15}{17}$; $\sqrt[3]{27a^3} = 3a$

6. Make a rule for finding the cube root of any common fraction.

7. Find the cube roots of the following:

$\frac{1}{8}$ $\frac{27}{64}$ $\frac{125}{343}$ $\frac{729}{8000}$ $\frac{343}{1000}$ $\frac{729}{125000}$

8. Find the cube root of 57512.456.

SOLUTION

57512.456 | 38.6 cube root.

27

$3\times30^2=$ 2700	30512	2700 goes in 30512 about 8 times.
$3\times30\times\ 8=$ 720		
$8^2=$ 64		
3484	27872	$=8\times3484$
$3\times380^2=433200$	2640.456	
$3\times380\times6=$ 6840		
$6^2=$ 36		
440076	2640.456	$=6\times440.076$

Hence, $38.6=\sqrt[3]{57512.456}$.

Check: (1) $(38.6)^3=57512.456$.

(2) Excess of 38.6=8; excess of $8\times8\times8$, or 512=8.
Excess of 57512.456=8.
Hence, excess of $(38.6)^3$=excess of 57512.456.

9. Find the cube roots of the following:

(1) 1906624 (2) 12487168 (3) 80621.568 (4) $\frac{1728}{4913}$

10. Find the length of the edge of a cube whose volume is 1953.125 cubic feet.

11. Find the length of the edge of a cube whose volume is $\frac{15625}{32768}$ cubic inches.

§120. Triangles Having the Same Shape (Similar Triangles).

1. Write the numerical values of the following ratios from Fig. 1:

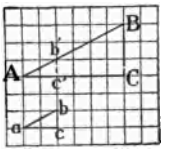


FIGURE 1

(1) $\frac{ac}{AC}$ (2) $\frac{bc}{BC}$ (3) $\frac{ab}{AB}$ (4) $\frac{bc}{ac}$ (5) $\frac{BC}{AC}$

(6) $\frac{ab^*}{ac}$ (7) $\frac{AB}{AC}$ (8) $\frac{bc}{ab}$ (9) $\frac{BC}{AB}$

*The length of ab is $\sqrt{4+1}=\sqrt{5}$. Why? See Ex. III, page 217.

2. In Fig. 2 the triangles have the same shape. Write the values of the ratios;

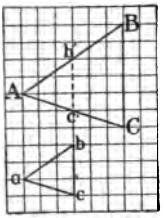


FIGURE 2

(1) $\frac{bc}{BC}$ (2) $\frac{ab}{AB}$ (3) $\frac{ac}{AC}$

3. In Fig. 3 the triangles have the same shape, and $AC = 7 \times ac$. Find the sides of the triangle ABC, if $ac = \frac{7}{40}''$, $ab = \frac{1}{4}''$, and $bc = \frac{11}{40}''$.

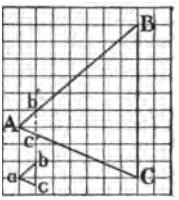


FIGURE 3

4. Supposing that ac $(= \frac{1}{7} AC)$ represents 7′ (Fig. 3), ab, 10′ and bc, 11′, what lengths do AC, AB, and BC represent?

5. In Figs. 4 and 5, $AB = 4ab$. If AB represents 1 mi., BC, 3 mi., and AC, $3\frac{1}{2}$ mi., what distances do ab, bc, and ac represent?

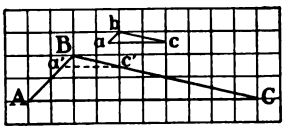


FIGURE 4

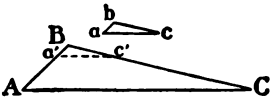


FIGURE 5

6. Draw a triangle having sides of 1″, $\frac{3}{4}''$, and $\frac{1}{2}''$, and

another having sides of 3″, $2\frac{1}{4}$″, and $1\frac{1}{2}$″. Call the angles opposite the sides 1″, $\frac{3}{4}$″, and $\frac{1}{2}$″, *a*, *b*, and *c*, respectively, and the angles opposite the sides 3″, $2\frac{1}{4}$″, and $1\frac{1}{2}$″, *A*, *B*, and *C*, respectively. Do these triangles have the same shape? Carefully cut out the triangles and place angle *a* over angle *A*; then angle *b* over angle *B*; and, last, angle *c* over angle *C*. What do you find to be true in each case?

In triangles having the same shape, angles lying opposite proportional sides in different triangles are called corresponding angles.

7. Read the pairs of corresponding angles in Figs. 1, 2, 3, and 4, p. 227.

8. In triangles having the same shape, how do corresponding angles compare in size (see problem 6)?

9. In Fig. 4, p. 227, find the ratio of the side lying opposite the angle *B* in triangle *ABC* to the side lying opposite the corresponding angle in triangle *abc*. Find a similar ratio between any other pair of such sides. How do the ratios compare?

10. Answer similar questions for the triangles of Figs. 2, 3 and 4, p. 227.

In triangles having the same shape, sides, as AB and ab (Fig. 4, p. 227) or BC and bc, that lie opposite the equal angles are called corresponding sides.

11. In triangles having the same shape state a law of relation of corresponding sides.

12. Triangles *ABC* and *ade* (Fig. 1) have the same shape. Compare their corresponding angles. What relation holds for their corresponding sides, i. e., how do these ratios compare: *AB*:*ad*? *BC*:*de*? *AC*:*ae*? With a protractor measure each pair of corresponding angles. How do they compare in size?

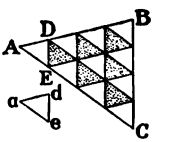


FIGURE 1

13. What is the ratio of the areas of triangles ABC and ade (Fig. 1, p. 228)? How may this ratio be found from the ratio of a pair of corresponding sides, as AB and ad?

14. Draw a pair of triangles, one having sides of $1''$, $1\frac{1}{2}''$, and $1\frac{3}{4}''$, and the other having sides of $4''$, $6''$, and $7''$. Give the ratio of each pair of corresponding sides; the ratio of the areas of the triangles. How may the ratio of the areas be computed from the ratio of a pair of corresponding sides?

15. State a law connecting the ratio of the areas of triangles having the same shape with the ratio of any pair of corresponding sides.

§121. Uses of the Triangles.

In Fig. 1 $MNRQ$ represents an $18'' \times 20''$ board, provided with a moving arm, OC, and a protractor. The arm is made of light wood. A slot, ES, is cut away and a thread is stretched on its under side and fastened at the ends. The ends of the thread are sunk into the wood so as to permit the thread to move smoothly over the protractor as the arm CO is turned round O. A common pin stuck vertically at C and at O completes the apparatus. The pin at O should be

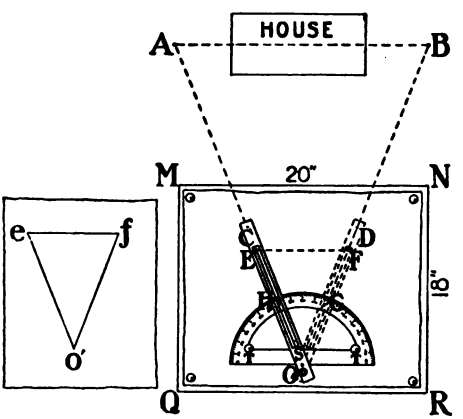


FIGURE 1

driven through far enough to fasten the arm to the board at O as a pivot. The protractor is held by thumb tacks at T, T. The board may be placed on a window sill, or on a post in a fixed position, while measurements are being made. It is desired to measure the distance from A to B, A and B being on opposite sides of a large building.

Use the proportion form of statement in all problems, calling the length of the unknown line x. Then find the value of x. Solve some problems like this from your own measures. Buildings, steeples, hills, trees, furnish good problems.

1. The board was placed on a window sill and the arm set in such position, FO, that when the eye sighted along the pins, O and D, these pins were in line with B. The mark on the protractor at G was 52°. The board being held firmly in place, the arm was swung around O until the pins at O and at C were sighted into line with A and the reading on the protractor at H was 114°. How many degrees were there in the angle EOF?

2. The distances from A and from B to the window at O were measured and found to be 484′ and 520′. A triangle $O'ef$ (shown in Fig. 1, p. 229) was drawn having angle $eO'f = 62°$, and $O'e = 4.84''$, $O'f = 5.20''$. The side ef was then drawn, measured, and found to be 5.18″. How long is the line AB?

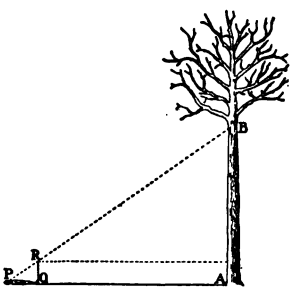


FIGURE 1

3. A woodman desires to fell only such trees as will furnish two 10-ft. cuts between the stump and the first limb. He wishes to allow 4 ft. for height of stump and waste in cutting at the bottom and top ends. To test a standing tree, he places

a 4-ft. stick QR vertically in the ground 33 ft. from the tree, lies down on his back with his feet against the stake, and sights over the top of the stake to the first limb B. The distance from his eye, P, to the soles of his feet Q being $5\frac{1}{2}$ feet, should he fell the tree?

4. In Fig. 1, p. 230, if $PQ = 6$ ft., $RQ = 4\frac{1}{2}$ ft., and $QA = 60$ ft., how long is AB?

5. How long is AB if PQ and RQ are the length, as in problem 4, and $QA = 90$ feet?

6. A woodman steps off a distance of 9 steps from a tree, faces the tree, and holds his axe-handle at arm's length in front of his eye, as the ruler is held in Fig. 2, p. 78. His arm is 28 in. long, and he finds that 2 ft. of the axe-handle just cover the distance from the ground to the first limb of the tree. If his step is 3 ft. long, how high is the first limb?

7. If $AB = 35$ rd., $AK = 80$ rd., and $BC = 56$ rd., how long is KD (Fig. 1)?

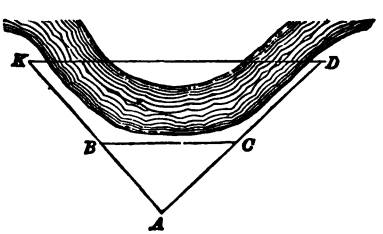


FIGURE 1

8. Make measures on objects in your vicinity and solve such problems as are suggested by those given.

FIGURE 2

9. How long is x (Fig. 2)?

10. The thumb is held 2 in. from the end of the pencil, and the pencil is held 2 ft. in front of the eye and parallel

to the line to be measured. When the end of the pencil is sighted into line with the corner *b* of the table, the end

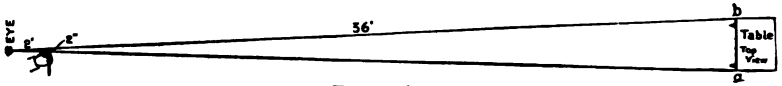


FIGURE 1

of the thumb is in line with the corner *a*. How long is the end, *ab*, of the table, if the eye is 36 ft. from the table?

11. Ten pounds of potatoes contain 7.83 lb. of water and 2.17 lb. of solid matter. How much water and solid matter would a bushel of 60 lb. contain? A peck?

12. Of the solid part of the potatoes, 1.8 lb. is starch; .22 lb. is proteid (muscle-food); .1 lb, ash; .04 lb. crude fiber, and .01 lb. fat. How much of each food-substance does 1 bu. of potatoes contain? A peck?

13. What percent of the whole quantity of potato is the starch? The proteid? The ash? The crude fiber? The fat?

14. What percent of the solid portion is the starch? The proteid? The ash? The crude fiber? The fat?

15. In 12 oz. of wheat flour there is 1.37 oz. of water; 1.66 oz. of proteid; 8.63 oz. of carbohydrate, and 0.23 oz. of fat. The rest is unfit for food. How much water is there in 25 lb. of wheat flour? How much proteid? How much carbohydrate? How much fat?

16. 8 oz. of corn meal contain 1 oz. of water; .736 oz. of proteid; 6.032 oz. carbohydrate, and .152 oz. of fat, and .08 oz. of waste matter. How much water is there in a 50-lb. sack of cornmeal? How much proteid? How much carbohydrate? Fat? Waste matter?

17. One pound of milk furnishes .033 lb. of proteid, .04 lb. of fat, and .05 lb. carbohydrates. What would it cost

to buy milk enough at 2¢ a lb. to furnish 1 lb. of proteid? At 3.5¢ a pound? How much fat would be bought at the same time? How much carbohydrates?

APPLICATIONS OF PERCENTAGE

§122. Insurance.

To insure one's person or property, against loss, or injury is to make a written agreement to pay a company a stated sum at stated times for the company's promise to pay a specified sum of money in case of loss of life, or to make good a loss or injury to one's person or property within a stated time.

The person who insures is called the insured.

The written agreement is the policy.

The sum of money to be paid by the company in case of loss, or injury, is called the face of the policy. The face of the policy may also be said to be the amount for which the property is insured.

The sum paid for the insurance by the insured is the premium.

The most common kinds of insurance are the following:

Life insurance, or insurance against loss of life;

Fire insurance, or insurance against loss of property by fire;

Marine insurance, or insurance against loss at sea;

Accident insurance, or insurance against loss of life, or injury to the person, or loss or injury of property, from accidents.

Insurance rates are usually stated at a certain price per $100, or $1000, payments of the premium to be made annually, semi-annually, quarterly, etc. Insurance problems are, however, often treated in the language of rate percent.

1. What will it cost to insure $680 worth of household furniture, at $1.75 per $100?

2. An art gallery, valued at $500,000, is insured at the rate of $12.50 per $1000. What is the premium?

3. A vessel, valued at $16,000, was insured for $\frac{7}{8}$ of its value. Find the face of the policy.

4. A growing crop was insured at $5 per $100. The premium was $140. What was the face of the policy?

5. If it costs $420 to insure a house for $\frac{3}{4}$ of its value, or $3.50 per $100, $3\frac{1}{2}\%$, what is the house worth?

6. A stock of hardware was insured for $7000, insurance costing $110.75. What was the rate of insurance?

7. If a shipment of grain is worth $840, and the premium amounts to $17.60, what is the rate of insurance?

8. A machinist insured his tools, valued at $360, for $\frac{7}{8}$ of their value, paying $8.19. What rate did he pay?

9. A farmer paid a premium of $10.50 for insuring his stock at $1\frac{1}{2}\%$, or at $1.50 per $100. For what amount was the stock insured?

10. A man paid $67.50 for the insurance of a steam launch at $1\frac{1}{2}\%$. For what amount was the launch insured?

11. What is the rate paid for insuring a bridge, valued at $15,000, for $\frac{2}{3}$ of its value, the premium being $240?

12. A library, worth $28,000, was insured for $\frac{6}{7}$ of its value, the premium being $720. What was the rate of insurance?

13. The contents of a grain elevator were insured at a rate of $\frac{3}{8}\%$. What was the amount of the policy, the premium paid being $272.40?

14. The premium paid for insuring a quantity of lumber, for two-thirds of its entire value, at 3%, was $36. What was the value of the lumber?

15. A piece of property, valued at $27,200, was insured

for $\frac{5}{8}$ of its value. The premium was \$212.50. What was the rate?

16. A tank of oil, holding 2592 gal., worth 18¢ per gallon, was insured at 4%. Find the premium.

17. A man insured a cargo worth \$2400 at $3\frac{1}{2}\%$. $\frac{5}{8}$ of the cargo was lost at sea. What amount of insurance should the man obtain? What premium had he paid?

18. What is the rate of insurance paid for insuring 26,000 bu. of grain worth 77¢, for $\frac{5}{8}$ of its value, if the premium is \$500.50?

19. What sums must be paid for policies to insure a factory, valued at \$21,000, at $1\frac{2}{3}\%$; a house, worth \$28,000, at $\frac{1}{4}\%$; and a barn, valued at \$5600, at $1\frac{3}{8}\%$?

20. A steamboat had a cargo, valued at \$2000, which was destroyed by fire. The insurance was \$1500. What percent of the value of the cargo was covered by insurance? What was the premium, at $4\frac{1}{2}\%$?

§123. Taxes.

The national government needs money for the support of the army and navy, for pensions, salaries of officials, etc.

The state government must care for the blind, the deaf and dumb, the insane, and for criminals; must pay salaries, provide for education, etc.

The county must build bridges, court-houses, etc., must pay salaries of county officers, support education, etc.

Cities and villages need police and fire protection, schools, street-paving, lighting, etc.

Most of these expenses are met by taxes.

A tax is a sum of money levied by the proper officers to defray the expenses of national, state, county, and city governments, and for public schools and public improvements.

There are two kinds of property tax: tax on real estate (as houses, lands, etc.), and tax on personal (or movable) property. In addition to these there is also a poll tax. This is a sum of money, ranging from $.50 to $3.00 levied on each male citizen from 21 to 50 years of age.

The tax rate is usually stated as a certain number of mills on $1, or as a certain number of cents on $100 of assessed valuation.

Assessed valuation means the estimated value of the property that is assessed.

1. If the tax rate is 12 mills on the $1, what is the tax on a piece of property assessed at a valuation of $4500?

2. If the tax rate is 75 cents on $100, what should be the tax on a factory valued at $480,000, and assessed at $\frac{4}{5}$ of its valuation?

3. The assessed valuation of a town is $2,000,000. A tax levy of $49,000 is made on personal property and real estate, and a collector's commission of 2% is paid. What is the tax rate?

4. What tax should be paid on property and real estate assessed at $6000 as in problem 3?

5. A town wishes to raise $23,500 to build a public hall. If the collector's commission is $2\frac{1}{2}\%$, what is the total amount to be collected?

6. The taxable property of a town is valued at $923,846 and the rate of taxation is $3\frac{2}{3}\%$. What is the whole amount of the tax?

7. The taxable property of a town is $869,472, and the rate is $8\frac{1}{2}$ mills on the dollar. What is the amount of the tax?

8. What amount of money must be collected to raise a net tax of $643,000, allowing 6% for collection?

9. The tax in a city is $49,682; the rate is 15 mills on the dollar. What is the assessed valuation?

10. What is the rate of taxation when property assessed at $8940 pays a tax of $160.92?

11. The net amount collected as a tax was $2896.42. The collector's commission was $3\frac{1}{2}\%$. What was the whole amount collected?

12. The assessed valuation of the taxable property of a town was $1,218,694. The tax to be raised was $18,280.41. How many mills on the dollar was the rate of taxation?

13. A tax collector received $175.18 as his 2% commission on a certain sum collected. The assessed valuation of the taxable property was $922,000. What was the tax of a man, whose property was valued at $18,000?

14. The real property of a certain town is valued at $4,560,800, and the personal property at $945,900. If $12,000 is to be raised by taxation, how much must a man pay, whose property is valued at $9840?

15. A net tax of $8965 is to be raised in a certain city. The assessed valuation of the taxable property is $5,689,243. The collector's commission is $1\frac{1}{2}\%$. What will be the tax of a man, whose property is valued at $56,000?

16. Property on a city street is assessed 2% on its valuation. How much more will a man pay who owns 100 front feet, valued at $125 per foot, than one who owns property valued at $7500?

17. If the assessed valuation of the taxable property in a certain city is $1,069,210, and the whole amount of tax

collected is $13,899.73, what is the rate of taxation and the net amount after deducting the collector's commission of $1\frac{7}{8}\%$?

18. The taxable property in a town is $872,990. The rate of taxation is .015. What is the net amount collected after deducting the collector's commission of $2\frac{1}{2}\%$?

19. The assessed valuation of the taxable property in a certain city is $4,968,390. The tax collected amounts to $94,399.41. How many mills on the dollar was the rate of taxation?

20. A certain town wishes to raise by taxes $20,500 to build a schoolhouse. What tax must be levied to cover this and the cost of collection at 4%?

§124. Trade Discount.

Many manufacturers and wholesale dealers publish expensive catalogues, giving descriptions and prices of the merchandise in which they deal. When prices fall, rather than publish new catalogues, they print or write after the original printed prices certain discounts which are allowed from the printed prices.

Additional discounts are often allowed for payment within 10, 20, 30, or 60 days.

The following problems will illustrate the meaning and use of such discounts, called *trade*, or *commercial discounts*.

1. A hardware dealer bought the goods named in the following bill:

(1) 2 doz. braces, @ 45¢ each
(2) 50 lb. $\frac{3}{4}''$ bolts, @ $3\frac{1}{2}$¢
(3) 12 bales barb wire, @ $2.15

(4) 25 kegs nails, @ $2.50
(5) 8 cooking stoves, @ $28.75
(6) 12 baseburners, @ $32.50
(7) 150 lb. screws, @ $4.75 per cwt.

The bill was subject to discounts of 10%, 7%, and also 5% 30 da., or 6% 10 da. What was the total cost if the bill was paid in 30 da.? In 10 days?

2. Find the amount needed to settle the following bill, subject to the successive discounts, 10%, 8%, and 5% off for 30 da., or 6% for cash, the bill being paid in cash:

(1) 2 chests carpenter's tools, @ $48.50
(2) 2 doz. augers, @ 25¢ each
(3) 2 doz. carpenter's rules, @ $1.80
(4) 480 lb. hinges, @ $4.75 per C.
(5) 3 doz. tin buckets, @ 22¢ each
(6) 14 scoop shovels, @ 85¢
(7) 6 wash boilers, @ 78¢
(8) 12 washtubs, @ 52¢

3. What amount will be needed to settle the bill of problem 2 in 30 days?

4. What amount will settle the following bill of house furnishings, the bill being subject to the discounts, 20%, 10%, 8%, and 5% 30 da., 6% cash, the bill being paid in 30 da.?

(1) 8 dining tables @ $30
(2) 12 sets dining-room chairs @ $18.50 a set
(3) 10 bookcases @ $12.50
(4) 8 rocking chairs @ $12.75
(5) 15 center tables @ $15
(6) 12 Wilton rugs @ $25
(7) 250 yd. ingrain carpet @ 45¢.

5. What amount paid in cash will be needed to settle the bill?

6. What amount in cash will settle this bill of plumber's supplies?

(1) 40′ lead pipe @ 15¢, discounts 20%, 10%, 7%, 2% cash

(2) 100′ iron tubing @ $3\frac{1}{2}$¢, discounts 20%, 10%, 7%, 2% cash

(3) 8 bathtubs @ $20, discounts 20%, 7%, 2% cash

(4) 8 lavatory outfits @ $10, discounts 20%, 7%, 2% cash

(5) 6 water meters @ $5, discounts 20%, 12%, 2% cash

(6) 4 pumps @ $25, discounts 20%, 7%, 2% cash

(7) 5 pump valves @ $2.85, discounts 20%, 7%, 2% cash

§125. Stocks and Bonds.

A stock is a written agreement (called also a stock certificate), made by a company to pay the holder a certain part of the earnings of the company.

The sum paid is reckoned at a certain number of dollars per *share*, or at a certain rate percent of the face value of a share. A share usually has a face value of $100, $500, $50, $1000, etc. A share of mining stock often has a much smaller face value.

When a stock company pays to the holders of its stock $2 on every $100 of its capital stock, or 2% on its stock, the company is said to be paying a $2 *dividend*, or a 2% *dividend*.

When stock is paying a high rate of dividend it may sell in the market *above par*, or for more than its face value. If the rate of dividend is low, the stock may sell for less than its face value, or *below par*. When it sells for its face value it is sold *at par*.

A bond is a written agreement made by a national, state, or city government, or by a company, to pay the holder a stated sum of money with interest on it at a stated rate. The sum of money to be repaid is called the face of the bond.

Stocks and bonds are usually purchased through an agent, called a *broker*, who makes a business of buying and selling stocks and bonds. He charges a certain percent (usually $\frac{1}{8}\%$) of the face value for buying, and an equal rate for selling. This charge is called *brokerage*.

In the following problems, regard the face value of the stock or bond as \$100 and the brokerage as $\frac{1}{8}\%$, unless otherwise stated.

Following are newspaper quotations on stocks and bonds:

STOCKS

	Open	High	Low	Noon
Amal. Copper	65⅛	66⅛	65⅛	65⅛
American Sugar	129¼	130	129⅛	129¼
B. & O. com	101¼	101¼	100⅜	101
B. & O. pfd	95½	95½	95½	95½
Chi. & Alton com	36¼	37	36¼	36¼
Chi. & Alton pfd	71⅛	72¼	71⅛	72
Ill. Central R. R.	148	148⅛	148	148⅛
Peoples Gas	105⅛	105½	105½	105½
Pressed Steel com.	65½	65½	65½	65½
Pressed Steel pfd.	94½	94½	94½	94½
Rubber Goods com.	24½	24½	24½	24½
U. S. Steel com	37⅛	37⅛	37⅛	37⅛
U. S. Steel pfd.	87⅜	87½	87⅛	87⅛
St. L. and S. F. com.	78¼	80⅜	78	80⅛
St. L. and S. F. pfd.	79⅛	80½	79⅞	80½

BONDS

Closing bid and asked prices for government bonds were as follows:

	Bid	Asked
New 2s	108⅛	109¼
Coupons	108⅛	109¼
New 3s	108½	109
New 3s coupon	108⅜	109
New 3s small	108	109
Registered 4s	112½	113
Coupon 4s	112½	113
Registered 4s new	139½	139⅜
Coupon 4s new	139½	139⅜
Registered 5s	107⅛	107⅜
Coupon 5s	107⅛	107⅜

BOND TRANSACTIONS

No. Sales		
27000	Atchison Gen. 4s	100⅛@100¼
12000	B. and O. gold 4s	101 @101⅛
1000	B. and O. 3½s	94½
1000	C. B. and Q. 4s	106⅜
9000	C. and A. 3½s	76½@ 76¾
25000	U. P. 4s	102¼

No. Sales		
5000	U. S. Leather 4s	85¼
3000	B. & O. coupon 4s	106⅛@106⅜
82000	Cen. Pac. R. R. 3½s	88⅛@ 88¼
10000	Wabash R. R. 5s	115⅛@115½
10000	Manhattan 4s	105½

1. A certain express company has a capital stock of $500,000, divided among its stockholders in shares of the par value of $100 each. In 6 mo. the net profits amount to $10,500, which the company distributes among its stockholders as a dividend. What is the rate of dividend? How much does a holder of 200 shares receive as dividend?

2. What must a man have paid for 400 shares Amal. Copper stock on the date of the table, including brokerage, if he bought at the opening price? At the highest price? At the lowest? At the noon price?

3. Answer similar questions for U. S. Steel pfd. (preferred).

Preferred stocks are stocks which pay a fixed dividend (say of 7%) before any dividends are paid on common stocks.

Common stocks pay dividends dependent on the net earnings of the company after expenses and dividends on preferred stock have been paid.

4. How much did a man gain or lose by buying 2000 Am. Sugar at the opening price and selling at the highest price, paying brokerage of $\frac{1}{8}\%$ for buying and also for selling?

5. Answer similar questions for B. & O. com.; for B. & O. pfd.

6. How much does a man make or lose who buys 2500 shares Ill. Cen. R. R. stock at the lowest quotation of the table and sells at the highest, paying brokerage of $\frac{1}{8}$ for buying and $\frac{1}{8}$ for selling?

7. If a man invests in St. L. and S. F. pfd. stock, paying 7% annual dividend, at the lowest quotation of the table, what interest does he receive on his investment?

SUGGESTION.—The investor must pay $79\frac{7}{8}$ + $\frac{1}{8}$ per share (of $100), and he receives $7 per year as interest.

8. Answer a similar question on C. & A. pfd.

9. What interest does a man receive on an investment in Government New 2s if he invests at the price "bid," brokerage $\frac{1}{8}$? At the price "asked"?

NOTE.—Government 2s, 3s, etc., are government bonds paying 2%, 3%, etc., annually.

10. Answer similar questions (1) for New 3s; (2) for New 3s coupon; (3) for Registered 4s; (4) for Registered 4s new.

11. What rate of interest does the investor who bought the 1000 B. & O. $3\frac{1}{2}$s receive?

12. Answer similar questions for the investor who bought the 9000 C. & A. $3\frac{1}{2}$s, if he paid the lowest quoted price; the highest.

13. Which pays the higher rate of interest on the investment, and by how much, U. P. 4s as quoted, or Wabash R. R. 5s at the lowest quotation? At the highest?

14. Which is the more profitable investment, and by how much, B. & O. coupon 4s at the lowest quotation, or a straight loan at 4%?

15. An investor in Chi. & Alton com. stock at the highest quotation received two 2% and one 3% dividend, and sold the stock 18 mo. later at $38\frac{1}{2}$. What rate of interest did he receive on his investment? How much profit did he make if he bought 1000 shares?

16. Answer similar questions on the table.

§126. Compound Interest.

With certain classes of notes, if the interest is not paid when due, it is added to the principal and this amount becomes a new principal, which draws interest. Savings banks add the interest on savings deposits at each interest-paying period and pay interest on the entire amount.

Interest on interest is called compound interest.

Bankers also collect their interest on loans and then reloan the interest, and in this way virtually receive compound interest on their money.

Compound interest is usually payable annually, or semi-annually.

1. Find the compound interest on a note of $200 at 6% for 5 yr., interest payable annually.

SOLUTION:

Interest on $200 for 1 yr. at 6% $= 200 \times \$ .06 = \$ 12.00$
Amount of $200 for 1 yr. at 6% $= 200 \times \$ 1.06 = \$212.00$
Amount of $212 for 1 yr. at 6% $= 200 \times \$(1.06)^2 = \$224.72$
Amount of $224.72 for 1 yr. at 6% $= 200 \times \$(1.06)^3 = \$238.23$
Amount of $238.23 for 1 yr. at 6% $= 200 \times \$(1.06)^4 = \$252.52$
Amount of $252.52 for 1 yr. at 6% $= 200 \times \$(1.06)^5 = \$267.68$

The compound interest = $267.68 − $200 = $67.68.

NOTE.—The $267.68 is called the *compound amount*. The expressions $(1.06)^3$, $(1.06)^4$, and $(1.06)^5$ mean $1.06 \times 1.06 \times 1.06$, $1.06 \times 1.06 \times 1.06 \times 1.06$, and $1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06$, and are read "1.06 cube," "1.06 to the 4th power," and "1.06 to the 5th power."

2. A man paid compound interest at 6%, interest compounded annually, on a note of $150 for 7 yr. How much was required to pay the note?

3. A note for $500, bearing 6% interest, has 3 interest coupons attached, the coupons, when due, bearing 7% interest, payable annually. If the note runs 4 yr., no coupons being paid in the meantime, how much money will be required to pay the entire debt at the end of this time?

NOTE.—Many notes have coupon notes attached for the amount of interest due at each interest-paying period. These coupon notes usually bear a higher rate of interest than does the note itself.

4. A note of $450, bearing interest at 7%, interest payable semi-annually, coupons bearing 8% interest, amounts

to what sum at the end of $4\frac{1}{2}$ yr., no coupons being paid until the end of the time?

§127. Savings Deposits.

Savings banks receive, use, and pay interest on the savings of persons who deposit their money and leave it in the bank to the end of an interest term.

The *interest term* is usually 6 months, and the rate of interest paid by the bank for the privilege of using the deposit is usually 3%, or 4%. *Interest days* are the days on which an interest term ends and the next term begins.

If, at the end of the interest term, the interest is not withdrawn, it is added to the deposit. The interest for the next period is then paid on the whole amount of the former deposit and the accrued interest. In this way savings banks pay *compound interest.*

Interest is usually calculated only on the whole number of dollars on deposits. Thus, if \$255.45 is the deposit during the interest term, the interest is calculated on \$255, say at 3% for $\frac{1}{2}$ year, giving \$3.83. This is added to the \$255.45, however, giving \$259.28, and \$259 draws interest for the next term.

When the account is drawn upon during the interest term the sum of the withdrawal is subtracted from the first deposit, or from the sum of the first and second deposits, etc., and interest is paid on this difference and on all later deposits for the whole number of months to the end of the term.

Interest terms are usually 6 months, beginning Jan. 1 and July 1; or 3 months, beginning on the first of Jan., Apr., July, and October.

In the following problems interest terms are taken as 6 months.

1. A man puts $450 into a savings bank, paying 3%, and leaves it through 2 interest terms of 6 months. He then draws out both the deposit and the accrued interest. How much money does he draw out?

2. Find the withdrawal value of the following savings deposits, the rate and the number of interest terms being as shown:

	Deposit	Number Interest Terms	Rate Paid
(1)	$75	2	3 %
(2)	$150	3	3 %
(3)	$365	3	4 %
(4)	$1275	2	4 %
(5)	$2560	3	3½%
(6)	$4060	3	3 %
(7)	$6575	2	4 %
(8)	$5672.90	2	3½%

3. A man puts $30 a month in a savings bank, paying 3%, for 12 months, and leaves it all until the last deposit has been in a full interest term. What is then the withdrawal value of his account?

4. A man opens a savings account with a bank paying 3% on deposits, and reserves the privilege of withdrawing money from the account at any time. The deposits and withdrawals, with dates, are shown here:

Date	Deposits	Withdrawals	Balance
Jan. 1	$280		
Feb. 26		$135	
Mar. 20	75		
May 2		90	
May 20	150		
June 15		80	
June 20	110		

Find for each date of withdrawal, the total deposits, the total withdrawals, and the balance. The interest term running from Jan. 1 to July 1, find the withdrawal value of this account on July 1.

5. A savings deposit of $1200 remains in a bank paying 3%, over 5 interest terms. What amount is due on the account at the end of the 5th term? Carry calculations to the 3d decimal place and shorten each amount to the nearest .01.

§128. Partial Payments.

For calculating the balance due on a note or bond on which partial payments have been made, nearly all the states have adopted the following rule, known as "The United States Rule of Partial Payments," which has been made the legal rule by a decision of the Supreme Court of the United States:

RULE.—*Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal and proceed as before.*

ILLUSTRATIVE EXAMPLE

1. Find the balance due on the following note at maturity :

$1250. Chicago, Ill., May 21, 1900.

On or before two years after date I promise to pay to the Order of P. A. Hopper, Twelve Hundred Fifty and $\frac{0}{100}$ Dollars at the Corn Exchange National Bank, for value received, with interest at 6 per cent per annum. JOHN P. MILLER.

Due May 21, 1902.

The indorsements on the back of this note were as follows:

Nov. 21, 1900..... $80.00

P. A. Hopper

Feb. 21, 1901..... $10.00

P. A. Hopper

May 21, 1901.....$150.00

P. A. Hopper

Feb. 21, 1902.....$500.00

P. A. Hopper

SOLUTION:

Principal on May 21, 1900		$1250.00
Interest for 6 mo. on $1		.03
Interest due Nov. 21, 1900		$37.50
Amount Nov. 21, 1900 (date of first payment on $80)		$1287.50
First payment		80.00
New principal Nov. 21, 1900		$1207.50
Interest for 3 mo. on $1		.015
Interest to Feb. 21, 1901 (date of second payment of $10)		$ 18.11
Payment being less than interest no settlement is made,		$1207.50
Interest for 6 mo. on $1		.03
Interest to May 21, 1901 (date of third payment of $150)		$ 36.23
Amount May 21, 1901		$1243.73
Sum of second and third payments ($10+$150)		160.00
New principal May 21, 1901		$1083.73
Interest for 9 mo. on $1		.045
Interest to Feb. 21, 1902 (date of fourth payment of $500)		$ 48.77
Amount Feb. 21, 1902		$1132.50
Fourth payment		500.00
New principal Feb. 21, 1902		$ 632.50
Interest for 3 mo. on $1		.015
Interest to date of maturity, May 21, 1902		$ 9.49
Balance due at maturity, May 21, 1902		$ 641.99
Answer		$ 641.99

2. The following payments were made on a \$650 note, bearing 7% interest and dated April 20, 1901:

July 30, 1901	\$75.00
Jan. 15, 1902	\$15.00
Aug. 12, 1902	\$175.00
Jan. 1, 1903	\$50.00

Find the amount due April 20, 1903.

3. A note of \$2800, dated Feb. 23, 1900, and bearing 7% interest, carried the following indorsements:

Feb. 23, 1901	\$100.00
July 16, 1901	\$50.00
Jan. 1, 1902	\$800.00
July 15, 1902	\$85.00
Nov. 28, 1902	\$380.00

Find the amount due Feb. 23, 1903.

USE OF LETTERS TO REPRESENT NUMBERS

§129. Problems.

1. Joseph had 45¢ and he earned 15¢ more selling oranges. How many cents had he then? (Answer by indicating the operation you perform, thus: 45¢ + 15¢.)

2. William had x marbles and Harold gave him y marbles. How many marbles had he then?

3. A cow gave x lb. of milk at the morning milking and y lb. at the evening milking. How many pounds did she give at both milkings?

4. James earned 80¢ selling papers on Saturday and spent 45¢ of his earnings. How many cents did he save?

5. During July a boy earned m dollars and spent s dollars. How many dollars did he save?

6. Helen bought 15 pencils at 3¢ apiece. How much did she pay for all?

7. Elizabeth took x music lessons during February at y dollars per lesson. What was the cost of her lessons for the month?

8. A thermometer rose $A°$ on one day and 5 times as much the day following. How much did it rise on the following day?

9. The area of a rectangular lot is 63 sq. rd. and the length of one side is 9 rd. How long is the other side?

10. The area of a rectangle is x square inches and the length of one side is y inches. How long is the other side?

11. How many lots each of b ft. frontage can be made from a frontage of a feet?

12. An orange boy earns a cents on Wednesday, three times as many cents on Thursday, and as many cents on Friday as on Wednesday and Thursday together. On all three days he earns 80¢. How much does he earn on Thursday? On Friday?

NOTE.—In all such problems use the equation. We have $a + 3a + 4a = 80$, or $8a = 80$. If $8a = 80$, $a = 10$. $3a = 30$, Thursday earnings, and $4a = 40$, Friday earnings.

13. The altitude of a rectangle is x in. and the base is $3x$ in. How long is the perimeter? What is the area of the rectangle?

14. $4a$ means $4 \times a$. Compare the values of $a \times 4 \times a$ and $4 \times a \times a$. How is $a \times a$ written? How is $4 \times a \times a$ written? *Ans.* $4a^2$.

15. The mercury stood at x degrees at 2 P.M. and fell 3° during the next hour. The reading at 3 P.M. was 25°. What was the reading at 2 P.M.?

16. The mercury stood at 12° at 8 A.M. During the next two hours it rose x degrees. What was the thermometer reading at 10 A.M.? If it had fallen y degrees, what would have been the reading at 10 A.M.?

17. The thermometer read 28°, the mercury fell x degrees, then $3x$ degrees, and then rose $6x$ degrees, when the reading was 32°. What was the number of degrees in each of the three changes?

18. James paid \$$x$ for a hat and twice as much for a coat. He paid \$4.50 for both. What did he pay for each?

19. I paid \$30 for a bicycle and sold it for \$$x$. How much did I gain?

20. I paid $\frac{1}{2}$ of what I gained for a coat. How much did I pay for the coat?

21. Louis had x apples and ate y of them. How many did he have left?

22. Henry had $7a$ papers and sold $4a$ of them. How many did he have left?

23. I walked x miles due south one day and y miles due north the next day. How far was I then from my starting point?

24. A man rows a boat downstream at a rate that would carry his boat a miles per hour through still water, and the current alone would carry him down b miles per hour. How far will he go in 1 hour?

25. A man walks from rear to front through a railway coach 3 mi. per hour, and the coach is running at the same time a mi. per hour. How fast does the man pass the telegraph poles along the track?

26. How fast does he pass them if he walks from front to rear?

27. Mary had a pencils and sold them at $5¢$ apiece. How many cents did she receive for them?

28. James sold x oranges at a cents apiece. How many cents did he receive for them?

29. A dealer sold a wagons for $40a$ dollars. What was the price per wagon?

30. A farmer paid x dollars for 15 A. of land. How much did he pay per acre?

31. The area of a rectangle is m sq. rd. and it is l rd. long. How wide is the rectangle?

32. A township is x m. square and contains 36 sq. mi. What is the value of x? What does x represent?

33. Helen bought x dolls at $10¢$ apiece and y yd. of muslin at $8¢$ a yard. How many cents did she pay for both?

34. James had m cents and earned c cents more. He invested all his money in papers at $3¢$ apiece. How many papers did he buy?

35. He sold his papers at $5¢$ apiece. How much did he receive for the papers? How much did he gain?

36. The base of a triangle is x ft. and the altitude is y ft. What is the area in square inches?

37. The area of a triangle is x sq. ft. and the base is 6 ft. long. What is the altitude?

38. A parallelogram has a base $(x + y)$ in. long, and is 6 in. high. What is the area? Express this answer in two ways and make an equation by writing the two expressions equal. Why are the two expressions equal?

39. A parallelogram having an altitude of x in. and a base of $2x$ in. has an area of 32 square inches. Find the lengths of the altitude and base.

§130. Uses of the Equation.

1. If each brick weighs 7 lb. and there are 10 bricks in the bucket (Fig. 1), with how many pounds of force does the bucket pull downward on the rope, the bucket itself weighing 5 pounds?

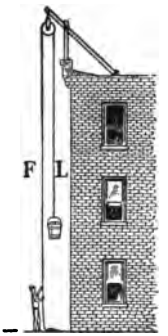


FIGURE 1

2. If we denote the number of pounds of force with which the man pulls downward on the rope to balance the bucket by p, write an equation showing the number of pounds in p, the 5-lb. bucket being loaded with 10 bricks.

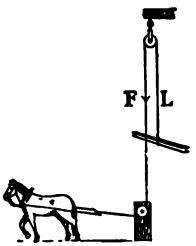


FIGURE 2

3. A horse is raising a 10-ft. steel I-beam weighing 30 lb. for each foot of length. (See Fig. 2.) If the force the horse must exert to hold the beam suspended in the air be denoted by F, write an equation showing the number of pounds in F.

4. A wagon weighing 1800 lb. is loaded with 3 T. of coal. When it is being drawn over ordinary pavement it pulls backward on the traces of the team with a force of $\frac{1}{50}$ as many pounds as there are pounds in the entire weight of the coal and wagon. If the force exerted by the moving team is F lb., write an equation showing the number of pounds in F.

5. Write an equation showing how many pounds each horse draws if both draw with equal force.

These problems show how the equation may be used in solving simple problems of mechanics; and we need to learn the laws upon which the use of the equation is based.

§131. Principles for Using the Equation.

1. Two weights, one of x lb. and the other of y lb., are tied to strings which pass over pulleys at A and B Fig. 1. The strings are knotted at C. If x is greater than y, how will C move? Under what condition will C remain at rest? What relation is shown to exist between x and y by a movement of C toward the right?

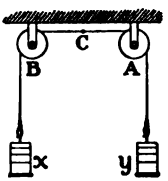


FIGURE 1

C's standing stationary indicates a *balance in value* between the forces y lb., drawing toward the right, and x lb., drawing toward the left. This fact is expressed in symbols by writing x lb. $= y$ lb., or, simply $x = y$.

2. If a 10 lb. weight be hung to the weight y, how many 5-lb. weights must be hung to x to secure a *balance of value*?

NOTE...This is shown by writing $y + 10 = x + 5 + 5$, read "y plus 10 equals x plus 5 plus 5."

3. How many pounds in all were added to x?

4. If 50 lb. were added on the right, how many 5-lb. weights must be added on the left before the sign of equality (=) can be written between the numbers? If $7a$ lb. were added on the right, how many a-lb. weights would be needed on the left for balance?

5. How many pounds would be needed on the right if b lb. are added on the left? $b + c$ lb.? $p + 35$ pounds?

6. Write the equation form of statement for each case of Problems 4 and 5.

7. If a greater number of pounds were added to y lb. than to x lb., say 15 lb. to y lb. and 10 lb. to x lb., what would occur in the apparatus shown in Fig. 1?

This *destroying the balance*, or *equality*, is stated briefly in signs by writing $y + 15 > x + 10$, which is read "y plus 15 is *greater than* x plus 10."

If the balance, or equality, is destroyed by adding a heavier weight to x than to y, the fact is stated in *symbols* thus: $y + 10 < x + 15$; read "y plus 10 is *less than* x plus 15."

Notice in each case that the *vertex* of the horizontal V *always points toward the smaller number.*

An expression in which the sign $<$ or $>$ stands between two numbers is called an expression of inequality.

We now have signs for writing briefly the three possible relations which may exist between any two numbers, as a and b. They are called *relation signs.*

8. Read and give the meanings of (1) $a > b$, (2) $a = b$, (3) $a < b$, (4) $x + 20 < x + 15 + 10$.

9. If $y = x$, and a lb. are added to y lb. and b lb. to x lb., how will the knot C move for case (1) Problem 8? For case (2)? For case (3)? State in symbols the fact shown by the knot C in each case after adding the weights, using the correct relation signs.

10. If any given weight, a lb., be added to y, what must be true of the weight b lb. if, when added to x, we may write $y + a = x + b$?

11. If z lb. be added to both the y lb. and the x lb., when $y = x$, what equation may we write?

12. If 25 lb. be added to both the y lb. and the x lb., in the equation $y = x$, what equation is obtained?

PRINCIPLE I (FOR ADDITION OF EQUATIONS).—*If the same number, or equal numbers, be added to both sides of an equation, the sums are equal.*

ILLUSTRATION.—If $y = x$, and $a = b$, then we may write

$$y + a = x + a,$$
and $y + b = x + b,$
and $y + a = x + b,$
and $y + b = x + a.$

13. If 8 lb. be removed from the left (Fig. 1, p. 254), how many 4-lb. weights must be removed from the right to *restore the balance*?

NOTE.—This is written in signs $y - 8 = x - 4 - 4$, and is read "$y - 8$ equals x minus 4 minus 4."

14. If $y = x$, and if a lb. be removed from the left, and b from the right, to what must the value of b be equal to enable us to write $y - a = x - b$?

PRINCIPLE II (FOR THE SUBTRACTION OF EQUATIONS). —*If the same number, or equal numbers, are subtracted from equal numbers, the differences are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then we may write:

$$y - a = x - a,$$
and $y - b = x - b,$
and $y - a = x - b,$
and $y - b = x - a.$

15. If in Fig. 1, p. 254, y be doubled, what corresponding change in x will restore the balance?

NOTE.—The double of y is written $2y$ and read "two y."

16. If $y = x$, and if 4 weights, each equal to y, are put on the right of the apparatus in Fig. 1, p. 254, how many weights, each equal to x, must be put on the left to keep the balance?

17. If a weights, each equal to y, are on the right, how many weights, each equal to x, must go on the left to secure balance?

NOTE.—The equation is $ay = ax$.

18. If $a = b$, and a weights, each equal to y, are put on the right, and b weights, each equal to x, are put on

the left, what will be shown by the scales if $y = x$? (Answer with an equation.)

PRINCIPLE III (FOR THE MULTIPLICATION OF EQUATIONS).—*If equal numbers are multiplied by the same number, or by equal numbers, the products are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then

$$ay = ax,$$
and $by = bx,$
and $ay = bx,$
and $ax = by.$

19. If $y = x$, and half of the weight on the left be taken off, what fractional part of the weight on the right must be taken off to restore the balance?

20. If only $\frac{1}{3}$ of y be kept on the right (Fig. 1, p. 254), what fractional part of x must remain on the left?

21. If only $\frac{y}{a}$ lb. be kept on the right (Fig. 1, p. 254), what fractional part of x lb. must remain on the left for balance?

22. Suppose $y = x$ and $a = b$, and that $\frac{y}{a}$ lb. are on the right, and $\frac{x}{b}$ lb. are on the left, what equation would be shown to be true by the apparatus?

NOTE.—The equation is $\frac{y}{a} = \frac{x}{b}$, read "$y$ divided by a equals x divided by b."

PRINCIPLE IV (FOR THE DIVISION OF EQUATIONS.)—*If equal numbers are divided by the same numbers, or by equal numbers, the quotients are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then

$$\frac{y}{a} = \frac{x}{a}, \text{ and } \frac{y}{b} = \frac{x}{b}, \text{ and } \frac{y}{a} = \frac{x}{b}, \text{ and } \frac{y}{b} = \frac{x}{a}.$$

In Fig. 2, p. 186, $a\ (x + y)$ represents the area of the whole rectangle, while ax and ay represent the areas of its two parts. As the two parts of the large rectangle, taken together, must equal the whole rectangle, we may write:

$$a(x + y) = ax + ay$$

In Fig. 1, p. 187, the area of the whole rectangle is given by $(a + b)\ (x + y)$, while the areas of the several parts are ax, ay, bx, and by. Since the parts, taken together, make up the whole rectangle, we may write;

$$(a + b)\ (x + y) = ax + ay + bx + by$$

These two equations illustrate the meaning of a fifth principle of the equation, viz.:

PRINCIPLE V.—*Any whole equals the sum of all its parts.*

These five fundamental principles or laws, exemplified by the pulley device (Fig. 1, p. 254), *must not be violated* in using the equation.

§132. Problems.

1. Find the value of the letter x, y, or z, in each of these problems:

(1) $3x = 15$ lb. (4) $9z = 36¢$ (7) $11x = 55$

(2) $8x = 24$ ft. (5) $\frac{3}{4}x = \$3$ (8) $\frac{7}{8}y = 21$

(3) $7y = 18$ mi. (6) $\frac{2}{3}y = 14$ sq. in. (9) $ax = 3a$

2. If $3x = 9$, to what is $2x$ equal? $7x$? $\frac{1}{2}x$?

3. If $7x = 21$, to what is $5x$ equal? $\frac{2}{3}x$? $6\frac{1}{3}x$?

4. If $x + 3 = 8$, what is x? $3x$? $5x$? x^2?

5. If $2x + 7 = 15$, what is x? $3x$? $7x$? x^2? $\sqrt{x}$?

6. If $x - 3 = 6$, what is x? $3x$? x^2? $\sqrt{x}$?

7. $2x + 3x + 6x = 33$. Find the value of x.

8. $4x - 2x + x = 64$. Find the value of x.

9. $7x - x + 2x = 32$. Find the value of x.

10. $xy = 16$, and $y = 8$. What is x? If $x = 8$, what is y?

11. $ax = 35$, and $a = 5$. What is x? If $x = 5$, what is a?

12. $ab = 10$; what is b if $a = 1$? 2? 3? 4? 10? 20?

13. By what principle may we write

$$c(x + y + z) = cx + cy + cz,$$

x, y, and z denoting the bases of 3 rectangles whose altitudes are each equal to c? (Answer by sketching the proper figure and pointing out the rectangles whose areas represent each of the products in the equation.)

14. If $a = 8$ and $b = 5$, find the values of the following expressions and tell what ones are equal:

(1) $(a+b)$
(2) $(a-b)$
(3) $2ab$
(4) $(a+b)^2$
(5) $(a-b)^2$
(6) a^2+b^2
(7) $a^2-2ab+b^2$
(8) $a^2+2ab+b^2$
(9) $(a+b)(a-b)$
(10) a^2-b^2
(11) $a(a^2+b^2)$
(12) $b(a-b)$

15. If $x = 9$ and $y = 4$, tell which of the following express true relations and write the correct *relation sign* in other cases: (See p. 255.)

(1) $x>y$
(2) $x=y$
(3) $x<y$
(4) $y<x$
(5) $x+y=13$
(6) $x-y=5$
(7) $(x+y)^2=169$
(8) $(x-y)^2=25$
(9) $x^2-y^2>25$
(10) $x^2+y^2=169$
(11) $(x+y)^2=(x^2+y)^2$
(12) $(x-y)^2=x^2-y^2$
(13) $(x+y)^2=x^2+2xy+y^2$
(14) $(x-y)^2=x^2-2xy+y^2$
(15) $(x-y)^2=x^2+2xy+y^2$
(16) $(x+y)(x-y)<x^2-y^2$
(17) $(x+y)(x-y)=x^2+2xy+y^2$
(18) $(x-y)(x+y)=(x^2+y^2)$
(19) $(x-y)(x+y)<(x^2-y^2)$
(20) $x(x+y)=x^2+xy$

§133. Statements in Words and in Symbols.

1. Write the symbolic statements for these verbal phrases and statements. Let x stand for the number when there is but one number to be symbolized in the problem:

(1) A certain number increased by 15.

(2) Twice a number diminished by 8.

(3) Seven times a number increased by three times the number.

(4) The square of a number, divided by 8.

(5) The sum of the square and the first power of a number.

(6) Eight times a number, divided by three.

(7) One-third of 10 times a number.

(8) Three times a certain number, diminished by one, equals 20.

(9) Eighteen times the square of a number equals 72.

(10) Twenty-five times a number, increased by 5, equals 30 times the number, diminished by 15.

(11) One-eighth of the sum of a certain number and 18.

(12) Six times the difference between a certain number and 3 equals 18.

(13) The product of the sum and the difference of x and 3 equals 18 (x being greater than 3).

(14) The difference between x and 18 is greater than 18; is less than 25; is equal to 20 ($x > 18$ in each case).

2. State in words what these expressions mean. For example, (1) means "double a certain number, diminished by 9":

(1) $2x-9$	(4) x^2+x	(7) $x(x-4)$
(2) $16x+x$	(5) x^2-x	(8) $3(9-x)$
(3) $28x+17$	(6) $(x+1)(x+1)$	(9) $12(x^2-1)$

(10) $(x-1)(x-1)$

(11) $7x-2+16$

(12) $5x+7=42$

(13) $(x-4)(x+4)=20$

(14) $(a+b)(a-b)=a^2-b^2$

(15) $(a+b)^2=a^2+2ab+b^2$

3. Translate into symbols these verbal phrases and statements, using a and b, or x and y, for the two numbers:

(1) The sum of two numbers equals 25.

(2) The difference of two numbers equals 15.

(3) The sum of the squares of two numbers is less than 27.

(4) The square of the sum of two numbers equals 100.

(5) The difference of the squares of two numbers equals 9.

(6) The sum of the squares of two numbers equals seven times the difference of the numbers.

(7) The product of two numbers equals their sum.

(8) The quotient of two numbers equals their difference.

(9) A certain number increased by 1 equals another number diminished by 3.

4. Translate into words these symbolic expressions. For example, (1) means "one-ninth of the difference between 6 times a certain number and its square":

(1) $\frac{6x-x^2}{9}$

(2) $\frac{x+3}{7}$

(3) $\frac{3-x}{x}$

(4) $\frac{a+b}{a-b}$

(5) $\frac{x^2}{x+2}$

(6) $\frac{8x-5}{3}=\frac{7y+4}{5}$

(7) $\frac{x}{y}+1=\frac{x+1}{y+1}$

(8) $\frac{a^2-b^2}{a-b}=a+b$

(9) $\frac{x}{y}=\frac{y}{x}$

(10) $\frac{x^3}{y^3}=\left(\frac{x}{y}\right)^3$

(11) $\sqrt{mn}=\sqrt{m}\sqrt{n}$

(12) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x-y}}=1$

(13) $a^2b^2=(ab)^2$

(14) $\frac{a^2}{b^2}+1=\left(\frac{a}{b}\right)^2+1$

(15) $\sqrt{a}+1=\sqrt{a+1}$

5. Find the number which may be put in place of the letter in each of these equations to furnish true equations:

(1) $2x + 3 = 7$.

SOLUTION:
$$2x + 3 - 7$$
$$3 - 3$$
$$2x - 4, \text{ Principle II,}$$
$$\frac{2x}{2} - \frac{4}{2}$$
$$x - 2, \text{ Principle IV.}$$

Check: $2x + 3 - 2 \times 2 + 3 - 4 + 3 - 7$.

(2) $3x - 1 = 5$ (4) $\frac{1}{5}x + 1 = 2$ (6) $\frac{6x}{7} + \frac{2}{7} = \frac{14}{7}$

(3) $8x - 6 = 18$ (5) $\frac{3x}{4} - 2 = 1$ (7) $\frac{6x}{7} - \frac{2}{3} = \frac{15}{3}$

NOTE.—First multiply both sides of (7) by 21.

6. Find the numbers of problem 1 (8), (9), and (10).

§134. Problems. FOR EITHER ARITHMETIC OR ALGEBRA.

1. The mercury column in a thermometer rose a certain number of degrees one day, and 3 times as many degrees the next day. It rose 12° during the 2 days. How many degrees did it rise each day?

ARITHMETICAL SOLUTIONS:

A certain number denotes the rise the first day.
3 times this number denotes the rise the second day.
Hence, 4 times a certain number denotes the rise in two days.
4 times a certain number equals 12° (by the given problem).
Once the number equals 3°, the rise the first day (Principle IV).
3 times the number equals 9°, the rise the second day (Principle III).
Check: 3° + 9° = 12°, the rise in two days.

ALGEBRAIC SOLUTION:

Let x denote the first day's rise.
Then, $3x$ denotes the second day's rise.
$x + 3x$ denotes the rise in 2 days.
$4x = 12°$.
$x = 3°$, the first day's rise (Principle IV).
$3x = 9°$, the second day's rise (Principle III).
Check: 3° + 9° = 12°.

2. A man bought 4 times as many hogs as cows, and after selling 5 hogs he had 23 hogs left. How many cows did he buy?

ARITHMETICAL SOLUTION:

A certain number represents the number of cows bought.
4 times this number represents the number of hogs bought.
4 times this number minus 5 denotes the number of hogs left.
Then 4 times this number, minus 5, equals 23 (by the problem).
4 times this number equals 23 plus 5 (Principle I).
4 times this number = 28.
This number = 7, the number of cows (Principle IV).
Check: $4 \times 7 - 5 = 23$.

ALGEBRAIC SOLUTION:

Let x denote the number of cows bought.
Then, $4x$ denotes the number of hogs bought.
$4x - 5$ denotes the number of hogs left.
Then $4x - 5 = 23$ (by the problem).
$4x = 28$ (Principle I).
$x = 7$ (Principle IV). *Ans.* 7 cows.
Check: $4 \times 7 - 5 = 23$.

3. Two masses were placed on one scale pan of a balance and found to weigh 18 lb. One of the masses was then placed in each pan, and it required 4 lb. *additional* on the light pan to balance the scales. What was the weight of each mass?

ARITHMETICAL SOLUTION:

A certain number of pounds denotes the weight of the heavier mass.
Another number of pounds denotes the weight of the lighter mass.
The first number plus the second number denotes the combined weight (18 pounds).
The first number minus the second denotes the difference of the weights, or the additional weight, which equals 4 pounds.
2 times the first number equals 18 plus 4 equals 22.
The first number equals 11.
11 plus the second number = 18. (Principle IV).
The second number = 7. (Principle II).
The weights are, then, 7 lb. and 11 lb.
Check: 11 lb. + 7 lb. = 18 lb., and 11 lb. − 7 lb. = 4 lb.

ALGEBRAIC SOLUTION:

Let x denote the number of pounds in the weight of the heavier mass.

Let y denote the number of pounds in the weight of the lighter mass.

Then, $x + y$ denotes the number of pounds in the combined weight, $= 18$.

$x - y$ denotes the number of pounds in the additional weight, $= 4$.

$$\begin{aligned} x + y &= 18 \\ x - y &= 4 \\ \hline 2x &= 22 \quad \text{(Principle I).} \\ x &= 11 \quad \text{(Principle IV).} \\ 11 + y &= 18 \\ y &= 18 - 11 = 7 \quad \text{(Principle II).} \end{aligned}$$

Check: $11 + 7 = 18$

$11 - 7 = 4$

Solve the following problems by both the algebraic and the arithmetical method, and state which of the solutions is the shorter:

4. A man cut for me 3 times as many ash trees as oak trees, and as many hickory trees as ash trees and oak trees together. In all he cut 32 trees. How many of each kind did he cut?

5. A man bought 4 times as many 2¢ stamps as 5¢ stamps, and $\frac{1}{2}$ as many 10¢ stamps as 2¢ stamps. For all he paid $4.95. How many of each kind did he buy?

NOTE.—Let x denote the number of 5¢ stamps purchased.

6. The side of a rectangular field is twice its breadth and the distance around the field is 240 rd. How many acres does the field contain?

7. I bought a number of books one day, 4 times as many the next day, and 3 books the third day. In all I bought 33 books. How many did I buy each day?

8. A newsboy sold a certain number of papers on Wednesday, twice as many on Thursday, and 3 more on

Friday than on Thursday. In all he sold 48 papers. How many did he sell each day?

NOTE.—Let x denote the number of papers sold on Wednesday.

9. Texas lacks 17,470 sq. mi. of being large enough to make 5 states the size of Illinois. How large is Illinois if Texas contains 265,780 square miles?

10. Maude has 78¢, which lacks 18¢ of being 6 times as much money as James has. How many cents has James?

11. It is twice as far from Chicago to Niles by a certain route as from Niles to Albion, and from Chicago to Albion is 190 mi. How far it is from Chicago to Niles?

12. Rochester, N. Y., is 17 mi. more than twice as far from Chicago as is Detroit. Rochester is 587 mi. from Chicago. How far is Detroit from Chicago?

13. Niagara Falls is 8 mi. less than 9 times as far from Chicago as is Michigan City. Niagara Falls is 514 mi. from Chicago. How far is it from Chicago to Michigan City?

14. It is 4 times as far, less 16 mi., from Chicago to N. Y. City as from Chicago to Ann Arbor. If it is 976 mi. from Chicago to N. Y. City, how far is it to Ann Arbor?

15. Champaign is 5 mi. more than $\frac{1}{3}$ as far from Chicago as is Cairo. If it is 128 mi. from Chicago to Champaign, how far is it from Chicago to Cairo?

16. A father and his son together earn $75 a month, and the father earns 4 times as much as the son. How much does each earn?

17. William has 3 times as many marbles as Joseph, and both together have 24. How many marbles has each?

18. A line 32 in. long was divided into 2 parts, one of which is 8 in. longer than the other. How long is each part?

19. A tree 75 ft. high was broken by the wind so that the part standing was 15 ft. longer than the part broken off. How long was each part?

20. The area of a rectangular field is 6400 sq. rd. and its length is 160 rd. What is its width?

21. The area of a rectangular sidewalk is 120 sq. rd. and the width is $1\frac{1}{2}$ yd. How long is the walk?

22. A man received \$180 for 72 days' work. What was his daily wage?

23. Find the side of a square field equal in area to a rectangle 80 rd. long by 20 rd. wide.

24. A man received \$3000 for 80 A. of land. How much did he receive per acre?

25. The circumference of a circle is 2200 rd. What is the radius? (Equation: $6\frac{2}{7}r = 2200$.)

26. The area of a circle is $201\frac{1}{7}$ m.^{2}. How long is the radius? The circumference?

27. Find the radius and the circumference of a circle whose area is 1 square inch.

§135. Formal Work.

Find the value of the unknown quantity, x or y, in each of the following:

1. $6x+4=22$

2. $3x-10=14$

3. $4y-2y+2=16$

4. $8y+y+3=21$

5. $\frac{3}{4}x-2=16$

6. $\frac{5}{6}x+5=35$

7. $6(x+2)=72$

8. $9x+3=30$

9. $\frac{x+4}{3}=8$

NOTE.—First multiply both members by 3.

10. $\frac{2x-14}{5}=2$

11. $\frac{30}{x}=15$

NOTE. — First multiply both members by x.

12. $\frac{34}{2y}=17$

13. $\frac{7x+18}{3}=13$

14. $\frac{25}{2x}=5$

15. $\frac{x+3x+12}{11}=4$

16. $\frac{7x-2x+x}{5}=18$

17. $21-x=33-2x$

NOTE.—First add $2x$ to both members, then subtract 21 from both members.

18. $12x+\frac{x}{4}=x+90$

19. $7x-\frac{2x}{3}=2x+39$

20. $\frac{12x+27}{21}=3$

21. $\frac{36-5x}{3}=7$

22. In the following equations W denotes the weight of the rope, or cable, in lb. per yd.; L the *working load* in tons; S, the *breaking load* in tons, and D, the *diameter* of the rope, or cable, in inches.

For hemp cable, $W = .577D^2$; $L = .109D^2$; $S = .654D^2$.

For tarred hemp cable, $W = 1.036D^2$; $L = .247D^2$; $S = 1.480D^2$.

For manila rope, $W = .765D^2$; $L = .329D^2$; $S = 1.877D^2$.

For iron wire rope, $W = 3.847D^2$; $L = 2.862D^2$; $S = 17.012D^2$.

For steel wire rope, $W = 3.946D^2$; $L = 4.441D^2$; $S = 27.630D^2$.

Find the weight per yd., the working load and the breaking load of a hemp cable of 1″ diameter; of $2\frac{1}{2}$″ diameter.

23. Solve similar problems for tarred hemp rope; for manila rope.

24. Find W, L, and S for an iron wire rope for which $D = 1\frac{1}{4}''$; $D = 2\frac{1}{8}''$.

25. For the same values of D, find W, L, and S for steel wire rope.

§136. Equations Containing Two Unknown Numbers.

In these problems two numbers will be unknown. The facts of each problem will furnish two equations. From these facts write both equations. Then change one or both equations, or combine them, in accordance with Principles I-V, pp. 255-258.

1. The sum of two numbers is 10 and their difference is 4. Find the numbers.

SOLUTION: Let x stand for the larger number.
Let y stand for the smaller number.
Then, (1) $x + y = 10$ (why?)
and, (2) $x - y = 4$ (why?)

Add Equations (1) and (2) member to member and write
(3) $2x = 14$. By which Principle?
Then, (4) $x = 7$.

Which Principle enables us to get Equation (4) from Equation (3)?

Putting this value, 7, of x, in place of x in Equation (1), we have
(5) $7 + y = 10$

Subtracting 7 from both sides of Equation (5), we get
(6) $y = 3$

By which Principle do we have the right to subtract 7 from both sides of Equation (5) and then write the two differences equal?

The two desired numbers are then 7 and 3.

N.B.—The problem shows how to find any two numbers when their sum and their difference are known.

2. The sum of two numbers is 18 and their difference is 6. Find the numbers.

3. A piece of wire 8 ft. long is cut into two pieces, one of which is $2\frac{1}{2}$ ft. longer than the other. How long is each piece?

4. A 16-oz. mixture of acid and water contains 9 oz. more water than acid. What is the weight of each part of the mixture?

5. In a school of 40 children there are 12 more girls than boys. How many girls are in the school? How many boys?

6. The side of a rectangular lot is 75 ft. longer than the end and it is 350 ft. around the lot. Find the length, width and area of the lot.

7. The sum of two weights is 40 lb. and their difference is 8 lb. Find the weights.

8. I am thinking of two numbers. Their sum is 23 and their difference is 8. What are the numbers?

9. The top end of a May-pole 20 ft. high broke so that the part left standing was 5 ft. longer than the part broken off. How long was each piece?

10. A mixture of 18 oz. of vinegar and water contained twice as many ounces of vinegar as water. How many ounces of each liquid was in the mixture?

SOLUTION: Equation (1) $x + y = 18$. Explain how this equation is obtained.

Equation (2) $x = 2y$. How obtained?

Write $2y$ in place of x in Equation (1), and have $3y = 18$. Find y. From Equation (2) then find x.

Put the numbers found for x and y in Equation (1) in place of x and y and notice whether the result is a true equation. This is called a "check."

11. A mixture of water and acid weighing 28 oz. contains 3 times as much water as acid. How many ounces of each does the mixture contain?

12. A silver dollar weighs 412.5 grains and it contains 9 times as much pure silver as alloy. Find the weight of pure silver and of alloy in a silver dollar.

13. A piece of bronze weighing 15 lb. is made of copper and tin. It contains 4 times as much copper as tin. How much of each metal is there in the piece?

14. A bronze gun, weighing 25 tons, is made of copper and tin in the proportion of 9 times as much copper as tin. How much of each metal is there in the gun?

15. How many cubic inches of pure acid and of water are needed to make a gallon (231 cu. in.) of acid solution $12\frac{1}{2}\%$ pure, i.e., containing $12\frac{1}{2}\%$ of pure acid and $87\frac{1}{2}\%$ of water?

16. The sum of one number and twice another is 11. The difference of the two numbers is 5. Find the numbers.

17. The sum of two numbers is 13. Twice the first number minus 3 times the second is 1. Find the numbers.

18. The sum of a number and half another number is $8\frac{1}{2}$. Twice the first number minus 3 times the second is 5. Find the numbers.

19. Divide 24 in. into two parts, one of which is 6 in. greater than the other.

20. Divide 32 into two parts, one of which is 3 times the other.

21. Divide 28 into two parts, one of which is $\frac{3}{4}$ of the other.

22. Two numbers are to each other as 3 to 5 and their sum is 64. Find the numbers.

23. If 1 be added to the numerator of a certain fraction it becomes $\frac{1}{2}$; but if 1 be added to the denominator, the fraction becomes $\frac{1}{3}$. Find the fraction.

24. Three times one number added to 2 times another number equals 23. Also 2 times the first number minus the second number equals 6. Find the numbers.

SOLUTION: The equations are $3x + 2y = 23$

and $2x - y = 6$

Multiply both sides of the second equation by 2 and get $4x - 2y = 12$.

Now add this to the first equation and have $7x = 35$.

From this find x, then substitute the value of x in the second equation and find y.

Check with the first equation.

25. Eight times a number minus another number equals 4. Three times the first number plus 5 times the second equals 66. Find the numbers.

SUGGESTION.—Multiply the first equation through by 5 and add the second equation.

26. Two numbers are as 2 to 7 and their difference is 25. Find the numbers.

27. One man said to another, "Give me one of your sheep and I'll then have as many as you." The other replied, "No; you give me one of yours and I'll have twice as many as you." How many sheep had each?

Find the numbers the letters stand for in the following and check your work:

28. $\begin{cases} x + y = 17 \\ x - y = 7 \end{cases}$

29. $\begin{cases} x + y = 17 \\ x - 2y = 8 \end{cases}$

30. $\begin{cases} 2x + y = 25 \\ x + y = 15 \end{cases}$

31. $\begin{cases} 2x + 3y = 30 \\ 2x - y = 10 \end{cases}$

32. $\begin{cases} 2x - y = 18 \\ 3x - 2y = 20 \end{cases}$

33. $\begin{cases} 6x + y = 8 \\ x - y = 3 \end{cases}$

34. $\begin{cases} 8a + 2b = 37 \\ 6a + 2b = 17 \end{cases}$

35. $\begin{cases} 3c + d = 21 \\ c - 2d = 2 \end{cases}$

APPLICATIONS OF THE EQUATION

§137. Thermometers.

Fig. 1, p. 272, shows the way the thermometer tube is marked off and numbered on the three thermometers most used by civilized countries. The Fahrenheit, or common, scale is on the left; the Centigrade scale, used everywhere in scientific work, is on the right; and the Reaumur (Ro'mur) scale, much used in Germany, is between the other two.

On all scales the fundamental points are the *freezing point* where the top of the mercury column stands when the bulb is in water containing melting ice, and the *boiling point*, where the top of the mercury column stands when the bulb is in water commencing to boil. For any temperature the *reading* is the number which belongs to the mark on the scale at the top of the mercury column.

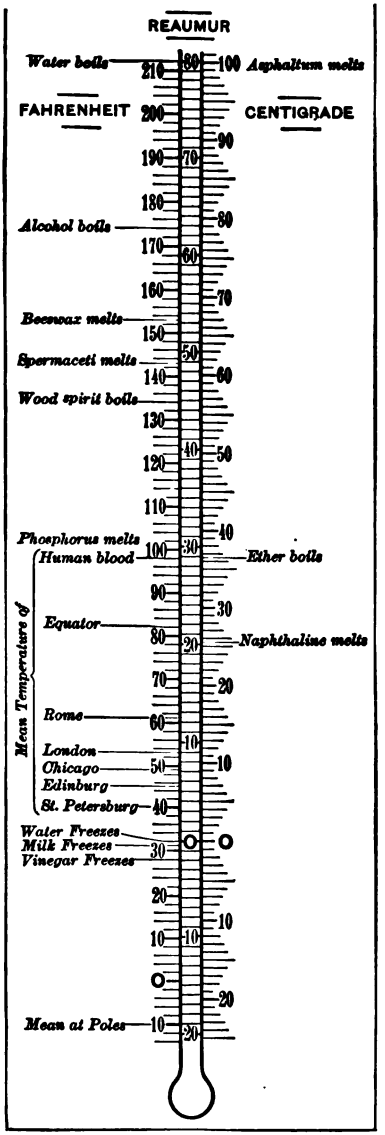


FIGURE 1

1. What is the reading for "Water boils" on the Fahrenheit scale? on the Centigrade scale? on the Reaumur scale?

2. What are the three scale readings for "Water freezes"?

3. Into how many equal parts is the space between the boiling point and the freezing point divided on the Reaumur scale? On the Centigrade? On the Fahrenheit?

4. The lengths of these small spaces are marked off on the tubes above the boiling point and below the freezing point and numbered, as shown in Fig. 1. Notice how the numbers run on each of the scales between the freezing and the boiling points; below the freezing point. How should the numbering continue above the boiling point?

5. What do we call a

change of temperature sufficient to expand, or to contract, the mercury column an amount equal to one of the short spaces on the Fahrenheit scale? On the Centigrade scale? On the Reaumur scale?

These are written 1°F., 1°C., and 1°R., and are read "1 degree Fahrenheit," "1 degree Centigrade," and "1 degree Reaumur."

6. If the readings above 0° are written +1°, +2°, +3°, +5°, etc., how should we write the readings below 0°?

7. What do the + and − then tell us about the readings?

8. Point out these readings in the figure: +2°F.; +36°C.; +29°R.; −14°C.; −20°R.; −38°F.; −10°F.

9. Give the readings on each scale for the mean temperature at the Equator; at Rome; London; Chicago; Edinburgh; St. Petersburg; at the Poles.

10. Which degree is the longest, the 1°F., the 1°C., or the 1°R.? Which is the shortest?

11. How many Fahrenheit degrees are equal to 80°R.? How many Centigrade degrees equal 180°F.? How many Reaumur degrees equal 100°C.?

12. On a sunny day find how many Fahrenheit degrees warmer it is in the sun than in the shade? Give the *C*- and *R*-readings corresponding to the two *F*-readings and also the *C*- and *R*-differences.

13. Give the Centigrade and Reaumur readings that correspond to the following Fahrenheit temperatures:

(1) + 25°	(5) + 72°
(2) + 80°	(6) + 212°
(3) − 10°	(7) − 12°
(4) + 32°	(8) + 98°

§138. Applied Algebra (Graduation of Thermometers).

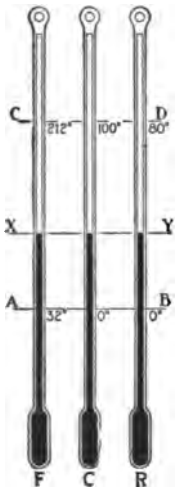


FIGURE 1

The problems of §137, pp. 272-73, show how the use of the equation simplifies some kinds of arithmetic problems. The work given here will illustrate how some problems that would be quite difficult by arithmetic can be easily handled by means of the equation.

EXPERIMENTS.—*To convert thermometer readings from one scale to another.*

Three thermometer tubes, exactly alike, were filled with mercury to the same height. Their bulbs were immersed in a vessel containing melting ice. Points were marked on the tubes at the tops of the mercury columns. Beside the point on the first tube was written 32, and beside those on the second and third tubes 0 was written.

All three of the bulbs were then immersed in water commencing to boil, and the tops of the mercury columns were marked. Beside the mark on the first 212 was written. Beside that on the second, 100, and beside the mark on the third, 80. Let AB of Fig. 1 denote the straight line passing through the tops of the three mercury columns when the bulbs are in melting ice. Let CD denote the line passing through the tops of these columns when the bulbs are in boiling water.

Suppose the space between the line AB and the line CD on the first tube to be divided into 180 equal parts and call each part one degree Fahrenheit (written 1°F.). Denote its length (in inches or in centimeters) by f.

Suppose this same space on the second tube between the lines AB and CD to be divided into 100 equal spaces, and call one of these smaller spaces one degree Centigrade (written 1°C.). Denote its length (in in. or in cm.) by c.

Divide the space on the third tube between the lines AB and CD into 80 equal spaces, and call one of these spaces one degree Reaumur (written 1°R.). Denote the length of this space (in in. or in cm.) by r.

Notice carefully that no two of the lengths f, c, and r are equal. Denote the length of the space from AB to CD by S.

1. S equals how many times f? c? r?

2. Since 180 f equals S, and 100 c equals S, how must 180 f compare with 100 c? (Answer this by writing an equation.)

This problem exemplifies a new principle of much importance in using equations:

PRINCIPLE VI.—*Numbers that are equal to the same number are equal to each other.*

3. Since 180 f equals S, and 80 r also equals S, how must 180 f compare with 80 r?

4. If $180\,f = 100\,c$, f equals how many times c?

5. If $180\,f = 80\,r$, f equals how many times r?

6. From these two equations, giving the value of f, write another equation by the aid of Principle VI.

7. Explain the meaning of the following equations:

(1) $f = \frac{5}{9}c$ (2) $f = \frac{4}{9}r$ (3) $c = \frac{4}{5}r$

Now suppose spaces equal to f marked off on the first tube above 212 and below 32, spaces equal to c marked off on the second tube above 100 and below 0, and spaces equal to r marked off on the third tube above 80 and below 0. The first tube is then graduated to the Fahrenheit scale, the second, to the Centigrade scale, and the third, to the Reaumur scale.

§139. Equivalent Readings on the Three Thermometers.

On all thermometers, when the tops of the mercury columns are above 0, the readings are called *positive*, and are marked +; when below zero they are called *negative*, and are marked −.

If the thermometers are exposed to the same temperature between that of melting ice and of boiling water, the tops of the mercury columns will stand in a straight line, as XY. Let the mark that stands by this line at the top of the column on the first be called F; on the second top, C; and on the third, R.

1. Denoting by S the distance from the line AB to the line XY show by Fig. 1, p. 274, that (1) $S = f$ (F − 32); (2) $S = Cc$; and (3) $S = Rr$.

2. Show by Principle VI that (4) $f\ (F - 32) = cC$; (5) $f\ (F - 32) = rR$; and (6) $cC = rR$.

3. Show from equations (1), (2), and (3), problem 7, §138, that (7) $c = \frac{9}{5}f$; (8) $r = \frac{9}{4}f$ and (9) $r = \frac{5}{4}c$, by using Principles III and IV.

4. In equation (4) problem 2 substitute $c = \frac{9}{5}f$ from (7) problem 3 and by Principles IV and II show that (I) $F = \frac{9}{5}C + 32$.

5. In a similar way show that the following equations are true:

(II) $F = \frac{9}{4}R + 32$ (V) $R = \frac{4}{9}\ (F - 32)$

(III) $C = \frac{5}{9}\ (F - 32)$ (VI) $R = \frac{4}{5}C$

(IV) $C = \frac{5}{4}R$

NOTE.—F, C, and R may be regarded as standing respectively for any corresponding readings on the Fahrenheit, Centigrade, and Reaumur thermometers.

6. From equations (1) to (VI) calculate F and C, if (1) R = +36°; (2) R = +72°; (3) R = +44°.

§140. Problems.

1. Convert the following into their F. and R, equivalents by the aid of the proper equations (I) to (VI).

MELTING TEMPERATURES ON CENTIGRADE SCALE

Ice	0.0°	Zinc	412.0°
Benzol	+ 4.4°	Antimony	432.0°
Tallow	43.0°	Silver	1000.0°
Paraffin	46.0°	Copper	1100.0°
Wax	62.0°	Gold	1200.0°
Sulphur	115.0°	Cast iron	1200.0°
Tin	230.0°	Cast steel	1375.0°
Bismuth	250.0°	Wrought iron	1600.0°
Cadmium	320.0°	Platinum	1775.0°
Lead	326.0°	Iridium	1950.0°

2. Convert these Fahrenheit boiling temperatures into their C. and R. equivalents:

Ether	+95°	Alcohol	+172.4°
Carbon Disulphide	+114.8°	Water	+212°
Sulphuric Acid	+14°	Mercury	+674.6°
Chloroform	+141.8°	Zinc	+1904°

3. Convert any observed Fahrenheit readings to their Centigrade and Reaumur equivalent readings.

4. The heat of the body of a healthy man is 37.2°C. What are the Fahrenheit and Reaumur equivalents?

5. Solid carbonic acid dissolves in ether and the solution is a liquid of −90°C. What is the Fahrenheit equivalent?

SOLUTION: In equation (I) if $C = -90°$, $\frac{9}{5}C = -162°$. Then 162° measured downward and 32° measured upward, or, $-162° + 32° = -130°$.

6. For every vapor and gas there is a so-called *critical*

temperature above which it remains in the gaseous condition, no matter how high the pressure upon it. The following are the critical temperatures of some vapors and gases: Ether vapor, +196°C.; carbonic acid, +31°C.; etheline, +9°C.; oxygen, −118°C.; nitrogen, −145°C., and hydrogen, −174°C. What are the Fahrenheit equivalents?

§141. Laws of the Thermometer Represented Graphically.

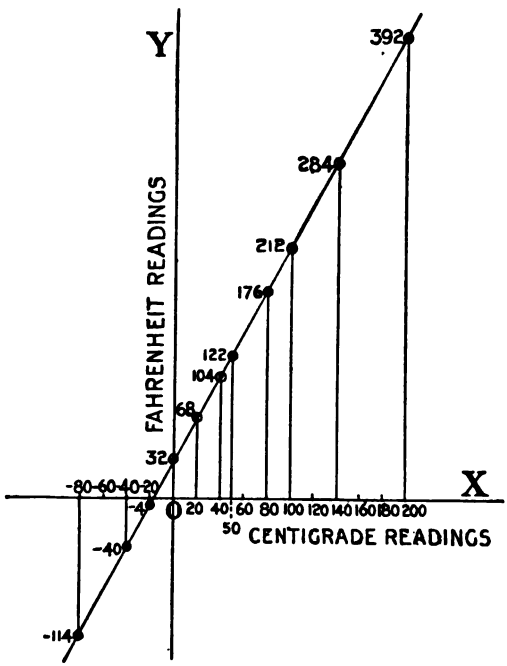


FIGURE 1

When it is necessary to change a great many readings from one scale to another it is convenient to plot the equations. To illustrate this, take the equation $F = \frac{9}{5}C + 32$. Draw two perpendicular lines, OX and OY, Fig. 1.

Let all Centigrade readings be measured off horizontally and the corresponding Fahrenheit readings vertically to any convenient scales.

The equation shows that if $C = 0°$, $F = 32°$; if $C = 20°$, $F = 68°$; and so on. In this way we fill out the following table:

C.	F.	C.	F.
0°	32°	100°	212°
29°	68°	200°	392°
40°	104°	-20°	-4°
50°	122°	-40°	-40°
75°	167°	-80°	-112°

NOTE.—The numbers preceded by the sign − are readings below zero, and such numbers for C, must be measured off from 0 toward the left, and for F. they must be measured off downward.

Study the points on Fig. 1, p. 278, and note whether they seem to lie on a straight line. With a ruler draw a straight line through these points.

For minus (−) values of C, as for $C = -20$, proceed thus: $F = \frac{9}{5} \times (-20) + 32 = 9 \times (-\frac{20}{5}) + 32 = 9 \times (-4) + 32 = -36 + 32$. But 36° measured downward and then 32° measured upward is the same as 4° measured downward, or $F = -4°$, if $C = -20°$; and similarly for other minus (−) values of C.

1. Measure off the value C = 60°, then measure vertically upward to the line drawn through the points, thus obtaining the corresponding Fahrenheit reading. What is F. for C. = 60°?

2. Similarly, find from the drawing the values of F. corresponding to these values of C.: 120°; 160°; 180°; 220°; −60°; −100°; 110°.

3. Make a similar table and plot of one or more of the other five equations of problem 5, p. 276.

METHODS OF SHORTENING AND CHECKING CALCULATIONS

§142. Illustrations.

I. Additions, subtractions, multiplications, and divisions with both integers and decimals are conveniently checked by casting out the 9's.

II. To check against gross errors (blunders), first *think* through the problem, making rough mental calculations with numbers that are approximately correct, and decide *about what the result must be.*

III. Check by performing reverse operations. That is, check addition by adding columns in reverse order; check subtraction by adding the subtrahend to the remainder; check division or square and cube roots by multiplication, and so forth.

The second rule may be illustrated by a few problems:

1. Find the value of $15\frac{4}{5}$ A. of land at $\$87\frac{1}{2}$.

Think thus: 16 A. @ \$90 would be worth \$1440; at \$85, 16 A. would be worth \$80 less, or \$1360. At $\$87\frac{1}{2}$, $15\frac{4}{5}$ A. of land is worth $\frac{1}{5}$ of \$85 less than \$1440 $-$ $\frac{1}{2}$ of \$80 (=\$1400), or about \$1383. The exact computation gives \$1382.50.

Or thus: $87\frac{1}{2} = \frac{7}{8}$ of 100. Therefore, $15\frac{4}{5} \times 87\frac{1}{2} = 1580 \times \frac{7}{8} = 7 \times 197.5 =$ \$1382.5. *Ans.* \$1382.5.

2. On June 13, 1903, wheat was quoted in Chicago at $76\frac{3}{8}$¢ per bushel. Find the cost of 150 bu. at this price.

3. 250 shares of Illinois Central R. R. stock sold at $\$135\frac{1}{4}$ a share. For how much did they sell?

4. The diameter of a circular rod is $1\frac{3}{4}$″. How many inches in the circumference of a right section of the rod?

How many square inches are there in the area of a right section of the rod? (For an approximation use $\pi = 3\frac{1}{7}$.)

5. The outside diameter of a circular hollow iron tube is $2\frac{1}{2}''$ and the inside diameter is $2''$. How many cubic inches are there in a $12'$ length of the tube?

6. A distance was measured with a chain 98.75 ft. long and was found to contain 38.75 lengths of the chain. The chain was supposed to be $100'$ long. How great an error was made in measuring the line by using the supposed length?

§143. Shortening and Checking Addition.

1. The noon temperatures on 7 successive days were 66°, 54°, 44°, 62°, 66°, 79°, 88°. Find the average for the week.

```
      66
      54
      44   164
226   62
      66   292
371   79
      88
   7)459
     65.6°   Ans.
```

MENTAL WORK. 70 + 50 + 44 = 164; 164 + 60 = 224; 224 + 2 = 226; 226 + 70 = 296; 296 − 4 = 292; 292 + 80 = 372; 372 − 1 = 371; 371 + 90 = 461; 461 − 2 = 459.

This is called two-column addition. A little practice will make this method useful for 1, 2, or 3 columns of figures. It may be used to advantage to check additions made in the ordinary way.

Another method in use by expert accountants is to group the figures into sums of 10, 20, 30, and so on. Thus, in the given problem, 8 + 2, 6 + 4, 6 + 4, and 9 makes 39. Then 7 + 3, 8 + 6 + 6, 6 + 4, 5, are 45. Sum, 459.

2. Write a few two and three column addition problems and practice these methods until you can use them rapidly.

§144. Making-up Method of Subtraction.

1. A paying-teller in a bank had $5485 in his cash drawer in the morning, and during the day he paid out

the following amounts: $37.50; $165.75; $10.25; $3.50; $2.88; $1.76; $65.17; $968.23; $3.67. How much money remained in the drawer?

CONVENIENT FORM

Total,	$5485.00
	37.50
	165.75
	10.25
	3.50
	2.88
	1.76
	65.17
	968.23
	3.67
	$4226.29, balance

MENTAL WORK.—Add the first column, thinking thus: 10, 23, 31, 41. 41 and 9 make the next larger number than 41 ending in 0 (the first figure in the total). Write the 9 in the result and add the 5 into the second column. Then 11, 21, 34, 48. 48 + 2 = 50, the next number larger than 48 which ends in 0 (the second figure of the total). Write 2 in the result and add 5 into the thrid column. Then, 21, 31, 39. 39 + 6 = 45. Write 6 and add 4 into fourth column. Then, 10, 17, 26. 26 + 2 = 28. Write 2 and add 2 to next column. Then, 12. 12 + 2 = 14, and finally, 1 + 4 = 5. Write the 4.

The advantage of this method is that it foots all the numbers and subtracts their sum, at once, as the numbers stand in the account book, from the total, giving the balance directly.

2. A bank customer's deposit at the beginning of the month was $398.75. During the month he drew out the following amounts: $16.75; $1.75; $5.25; $12.87; $128.32; $40.45; $2.18; $9.16; $1.57; $11.38; $12.62. Find the customer's balance at the end of the month.

§145. Shortened Multiplication.

1. Multiply 73 by 67.

$73=70+3$
$67=70-3$

$73\times 67=(70+3)\,(70-3)=70^2+70\times 3-3\times 70-3^2=70^2-3^2=4900-9=4891.$

Algebraic form: $(a+b)\,(a-b)=a^2-b^2$.

This applies to finding the product of any two numbers the sum of whose units is 10 and whose tens digits differ by 1.

RULE.—*Find the difference between the square of the tens and the square of the units in the larger number.*

2. Find these products by the rule:

(1) 34×26	(4) 58×42	(7) 71×69
(2) 22×18	(5) 65×55	(8) 99×81
(3) 43×37	(6) 98×82	(9) 95×85

3. Find the square of 38.

$38^2=(30+8)^2=30^2+2\times8\times30+8^2=1444$

Algebraic form: $(a+b)^2=a^2+2ab+b^2$.

Show the correctness of the following rule:

RULE.—*Square the tens, double the product of the tens by the units and square the units, then add the three results. The sum is the square of the number.*

4. Square these numbers by the rule:

(1) 36	(3) 64	(5) 19	(7) 125
(2) 47	(4) 58	(6) 95	(8) 148

NOTE.—Call the 12 in (7) 12 tens; also call the 14 in (8) 14 tens.

36 might be written $40 - 4$. Then, $36^2 = (40 - 4)^2$, $(40 - 4)^2 = 40^2 - 2 \times 4 \times 40 + 4^2$. Algebraic form: $(a - b)^2 = a^2 - 2\ ab + b^2$. Make a rule for squaring 36 in this form.

5. To find mentally such products as $35\frac{2}{5} \times 35\frac{3}{5}$; and 40.4×40.6.

$35\frac{2}{5} \times 35\frac{3}{5} = 35 \times 35 + (\frac{2}{5} + \frac{3}{5})\,35 + \frac{2}{5} \times \frac{3}{5} = 35 \times 36 + \frac{6}{25} = 1260\frac{6}{25}$; $40.4 \times 40.6 = 40 \times 40 + (.4 + .6)\,40 + .4 \times .6 = 1640.24$.

RULE.—*When the integers are the same and the sum of the fractions is 1, multiply one integer by the other, increased by 1, and add the product of the fractional parts.*

6. Find, by the rule, the following products:

(1) $5\frac{1}{2} \times 5\frac{1}{2}$	(4) $33\frac{1}{3} \times 33\frac{2}{3}$	(7) 10.25×10.75
(2) $16\frac{1}{2} \times 16\frac{1}{2}$	(5) 30.7×30.3	(8) $70\frac{1}{9} \times 70\frac{8}{9}$
(3) $20\frac{1}{4} \times 20\frac{3}{4}$	(6) 50.2×50.8	(9) $90\frac{3}{4} \times 90\frac{1}{4}$

When long decimals are to be multiplied and the product is required to only a few decimal places, contracted multiplication is of great advantage. The next problem illustrates this sort of multiplication.

7. The radius of a circle is 238.36 ft. What is the length of the circumference to the second decimal place, or to the nearest .01 foot?

NOTE.—The length of the diameter is 476.72 feet.

COMMON FORM		SHORTENED FORM	
476.72		476.72	
3.1416		6141.3	digits reversed
28	6032	1430.16	
47	672	47.67	
1906	88	19.07	
4767	2	48	
143016		29	
1497.66	3552	1497.67	

EXPLANATION.—In the shortened form the digits of the multiplier are written in reverse order, and the units digit is always written under that decimal place in the multiplicand which is to be the last one retained in the product.

Multiply by units digit first, then by tens, and so on; in each case begin the multiplication by any digit with the digit just above it in the multiplicand. Begin the writing of each partial product in the same vertical line on the right.

NOTE.—It is necessary on beginning to multiply by any digit to glance at the product by the preceding digit of the multiplicand to see how many units are to be added into the product by the digit just above. Thus, the multiplication by 4 would begin with 6, but 4 times the preceding digit (7) is 28, and this being nearly 3, the product 4×6 would be increased by 3, giving 27.

Expert computers use the shortened form altogether.

8. Find these products to the nearest .01 by the method of shortened multiplication:

(1) 36.428×3.1416 (3) 7.8843×1.0863

(2) 186.086×108.336 (4) 168.7431×28.329

9. Find the following products to the nearest .001 by shortened multiplication:

(1) 36.1872×6.8734 (3) 629.3865×3.1416

(2) 128.63×3.8629 (4) 1284.683×3.1416

§146. Shortened Division.

1. Divide 648.7863 by 68.372 to the nearest .01.

CONVENIENT FORM

```
   9.49—Quotient
  ..
68.37)648.7863
      615 33
       33 45
       27 35
        6 10
        6 15
```

EXPLANATION.—Find the units digit of the quotient in the usual way. Then cut off one digit from the right of the divisor and find the next digit of the quotient, then cut off another digit from the divisor, etc. A dot is sometimes placed over each digit in the divisor as it is set aside.

2. Find quotients for the following to the nearest .01:

(1) $1786.786 \div 3.1416 = ?$

(2) $632.068 \div 8.6249 = ?$

(3) $1206.3862 \div 28.3762 = ?$

(4) $865.28476 \div 361.2946 = ?$

§147. Shortened Square Root: Square Root by Subtraction.

1. Find the square root of 68432.93628 correct to the third decimal figure.

```
   261
  . . . . .
  68432.93628
  4
 46|284
   |276
521|  832
   |  521
 522)311.9(597
     2610
      509
      470
       39
       37
```

EXPLANATION.—By placing a dot over every alternate digit in the given number, counting both ways from the decimal point, we obtain as many dots as there are to be figures in the square root. Find half the figures by the ordinary method of square root, and the rest by contracted division as shown. The result will very nearly equal the square.

Square root = 261.597, nearly.

2. Find, by the shortened method, to two decimals the square roots of the following:

(1) 6432.1864 (2) 38,629.72468 (3) 8764.932651

Square root may be found by subtraction, after noticing that $1 = 1^2$; $1 + 3 = 2^2$; $1 + 3 + 5 = 3^2$; $1 + 3 + 5 + 7 = 4^2$; and so on.

It is seen that the square root of the sum of the odd numbers in order from 1 upward is equal to the number of odd numbers added. An example will illustrate the use of this principle:

3. Extract the square root of 104976.

```
 104976(324=square root
  1
 ---
  9
  3
 ---
  6
  5     3 subtractions
61|149
  |61
 ---
  88
  63
641|2576  2 subtractions
  |641
 ----
 1935
  643
 ----
 1292
  645
 ----
  647
  647   4 subtractions
```

EXPLANATION.—Place a dot over each alternate digit beginning on the right, thus separating the number into groups of 2 digits each. From the first group on the left subtract 1, then 3, then 5, and so on as shown, until the remainder is less than the next odd number. The number of subtractions is the first root digit. In the present case it is 3.

Bring down the next two-digit group. Double the root digit found and annex 1 to it, and subtract, then replace the 1 by 3 and subtract, etc. The number of subtractions indicates the next root digit.

Study the remaining steps and learn how to proceed further. This method gives the exact square root and may be used to check results found in the ordinary way. Some computing machines are based on this method of obtaining the square root.

4. Find the square roots, by subtraction, of the following:

(1) 1156	(4) 174.24	(7) 54,756
(2) 44,944	(5) 49,284	(8) 289,444
(3) 5776	(6) 77841	(9) 107,584

§148. Miscellaneous Problems for Review.

1. If one breathes 18 times a minute and at each breath consumes 2 cubic inches of oxygen more when moving about than when sitting, how many gallons less oxygen does one get in 8 hours if he sits than if he is active?

2. If the total tonnage of merchandise shipped through the Detroit river in 1907 was 67,296,477 tons and it was carried in 23,721 vessels, what was the average tonnage of one vessel?

3. If the total tonnage of merchandise shipped through the Detroit river in 1906 was 60,577,732 tons and it was carried in 24,077 vessels, what was the average tonnage of one vessel?

4. What change is there in the average tonnage of a vessel from 1906 to 1907?

5. Duluth, Superior, and Two Harbors form a single community. From the three ports there were shipped in 1906, 47,726,778 bushels of wheat and 43,531,540 bushels of corn, and in 1907, 63,349,585 bushels of wheat and 44,355,990 bushels of corn. What were the total shipments of wheat and corn together in 1906? In 1907? In both years?

6. What was the difference of the total shipments of wheat and corn together in 1906 and 1907 from this community?

7. The freight sent through the Soo was 1,567,741 tons in 1881; 4,527,759 tons in 1886; 8,888,759 tons in 1891; 16,239,000 tons in 1896; 28,403,065 tons in 1901; 44,270,680 tons in 1905; 51,751,080 tons in 1906; and 58,217,214 tons in 1907. What was the increase in tons from 1891 to 1896? From 1896 to 1901? From 1901 to 1905? From 1905 to 1906? From 1906 to 1907? From 1881 to 1907?

8. The total tonnage of all the commerce of the Great Lakes for 1907 was 83,387,919 tons, which was 20% larger than the tonnage for 1905. What was the tonnage for 1905?

9. The total tonnage of the commerce of the Great Lakes for 1907 was 10% larger than for 1906. What was the total tonnage for 1906? See prob. 8.

The following table is an extract from the milk record of a herd of milk-cows:

WEIGHT OF MORNING (A.M.) AND EVENING (P.M.) MILKINGS IN POUNDS

1901 Nov.	Queen		Beauty		Beech-wood		Myrtle		Spot		Rose		Total
	A. M.	P. M.	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	A.M.	P.M.	
1	6.5	6.5	11.0	10.4	12.5	10.2	7.3	5.7	12.0	11.5	11.0	9.6	
2	5.6	6.0	11.0	8.4	11.0	11.0	7.2	5.6	11.5	10.5	11.3	8.8	
3	6.4	6.0	10.8	9.5	11.7	11.5	7.3	5.7	12.0	10.4	11.2	9.8	
4	6.0	6.5	12.5	10.2	12.6	13.0	7.2	6.0	11.5	11.5	11.0	9.8	
5	6.0	7.5	11.4	10.2	12.0	11.4	6.7	5.8	11.6	11.3	11.0	10.2	
6	6.6	6.8	12.0	10.0	8.0	11.6	8.0	5.0	12.5	11.8	11.7	10.0	
7	7.4	8.5	10.8	12.4	12.0	11.7	7.0	7.2	12.0	12.5	11.6	12.0	
8	7.8	7.3	11.2	11.6	13.2	12.2	7.2	5.7	11.5	11.3	11.5	10.1	
9	7.5	6.4	11.6	10.4	12.0	11.6	6.6	5.4	12.0	12.2	10.7	10.0	
10	7.8	7.2	11.0	10.4	12.0	12.0	6.7	4.7	11.5	8.5	11.8	10.6	
11	6.8	7.3	12.2	11.0	13.6	11.5	7.7	5.3	9.0	10.0	12.3	10.0	
12	7.8	7.7	12.5	12.0	13.0	11.6	8.0	5.5	12.0	12.6	12.2	10.6	
13	7.6	8.0	13.4	11.5	13.5	13.0	7.2	5.8	11.1	13.0	11.8	11.2	
14	7.0	8.0	12.0	11.8	14.0	12.7	7.7	5.7	12.5	12.7	11.5	10.3	
15	8.3	7.5	13.0	10.7	15.0	12.0	7.6	5.0	14.0	12.4	12.3	9.0	
16	9.0	7.0	12.5	11.0	15.2	13.7	7.0	5.5	12.7	12.3	14.1	8.5	
17	8.0	7.0	12.5	10.5	14.0	13.0	6.1	6.4	10.0	9.0	12.0	10.5	
18	8.1	7.7	13.4	10.9	14.0	11.5	6.6	5.0	12.5	11.5	11.0	9.1	
19	7.0	7.0	11.5	11.4	13.6	11.0	6.2	5.3	12.2	11.7	11.1	10.0	
20	7.5	6.5	12.6	10.5	12.7	12.0	6.5	4.9	11.8	11.4	11.5	9.5	
21	7.0	7.6	12.0	10.2	13.7	11.3	6.1	5.2	13.0	11.0	10.6	9.6	
22	8.0	7.5	12.2	11.9	14.7	11.2	6.7	6.0	12.4	10.7	11.3	9.5	
23	7.5	9.0	12.2	11.0	14.4	12.0	7.0	5.5	12.8	10.8	11.3	10.4	
24	7.0	7.1	13.5	10.6	13.8	11.5	7.2	5.1	13.0	10.7	12.0	10.4	
25	7.4	8.0	12.5	11.2	14.0	10.5	7.4	5.7	12.7	11.7	12.0	11.0	
26	6.5	8.4	12.0	11.8	12.0	12.0	6.1	5.8	13.2	11.5	11.5	10.5	
27	7.5	8.5	13.7	11.8	14.0	13.0	6.1	5.8	13.0	12.7	11.4	10.5	
28	7.2	9.2	12.0	12.0	13.6	13.8	6.6	4.9	13.1	12.5	11.7	10.5	
29	6.5	9.0	13.5	12.6	15.0	13.5	6.7	5.0	10.2	12.6	12.5	12.6	
30	7.2	8.0	11.7	12.7	14.2	12.2	6.0	4.3	12.4	11.5	12.7	10.3	

For the next 5 problems see how quickly you can get results correctly.

10. How many pounds of milk did Queen give during Nov., 1901, at the morning milkings? At the evening milkings?

11. Answer similar questions for Beauty; for each of the other cows.

12. Without rewriting, find the total quantities of milk for each day of the month. Write totals in a column on a separate slip. Time yourself.

13. Foot the totals for each cow for the whole month. Time yourself.

14. Foot the totals for each day, and compare the result with that of problem 13.

15. The total tonnage of the commerce of the Great Lakes in 1907 was 83,387,919 tons, 40,727,972 tons of which was iron ore. How many tons was not iron ore?

16. The south-bound traffic through the Detroit river in 1907 amounted to 46,967,166 tons, and the north-bound traffic amounted to 20,329,311 tons. Find the total traffic. Find the difference in tons of the north-bound and the south-bound traffic.

17. Find the differences in the following 9 exercises, subtracting from left toward the right, and noting the time taken:

(1)	8645	(4)	86432	(7)	503002
	4382		37549		384965
(2)	68325	(5)	97004	(8)	1001003
	39162		68231		146897
(3)	87461	(6)	140601	(9)	1000098
	19385		98731		899179

18. The United States Government paid pensions amounting in 1893 to $158,155,342; in 1898, to $145,748,865, and in 1905, to $142,099,286. Find to the nearest .01, (1) the percent of decrease in amount paid for pensions from 1893 to 1898; (2) the percent of decrease from 1898 to 1905.

19. In the year 1897, 558 tons of rose-wood was imported into Great Britain. One half of it sold at 5¢ and the rest at 10¢ a pound. Find the value in Great Britain of this rosewood.

20. How much cork will a tree yield in 150 years at the rate of 2000 lb. every 8 years?

21. The area of the United States is 3,600,000 sq. mi. and it contains 450,000,000 acres of woodland. What part of the area of the United States is woodland?

22. Find the total number of pounds and the total value of these purchases:

650 lb. cut loaf sugar @ $5.74 per hundredweight
825 lb. granulated sugar @ $4.80 per hundredweight
400 lb. powdered sugar @ $5.24 per hundredweight
700 lb. confectioner's sugar @ $4.89 per hundredweight
350 lb. extra white sugar @ $4.78 per hundredweight.

23. Find the weight and total value of this shipment

5400 lb. plain beeves @ $5.70 per hundredweight
6375 lb. choice beeves @ $5.80 per hundredweight
8450 lb. fair beeves @ $4.90 per hundredweight
8625 lb. medium beeves @ $4.60 per hundredweight
40,700 lb. veal calves @ $6.50 per hundredweight
43,000 lb. western steers @ $8.50 per hundredweight
8450 lb. Texas steers @ $3.70 per hundredweight
25,200 lb. beef cows @ $2.85 per hundredweight.

24. How many tons did the purchases of problem 22 weigh? The shipment of problem 23?

25. A troy ounce of pure gold is worth $20.67. How much is an avoirdupois pound of pure gold worth?

26. A speed of 60 miles an hour is equivalent to how many feet per minute? Per second?

27. Find the cost of plastering the walls and ceiling of a room 27 ft. long by 24 ft. wide by 13 ft. high at 27 cents per square yard, making no allowance for openings.

28. In 1901 the total imports of tea into the United States were 83,395,924 lb. and of coffee, 511,041,459 lb. If the average cost of tea was 37c per lb., and of coffee, 18c per lb., what was the total cost of both?

29. What will it cost to plaster the four walls and the ceiling of a hall 60 ft. long, 36 ft. wide, and 18 ft. high, at 22½¢ per square yard, after deducting 486 sq. ft. for door and window openings?

30. A man husks two rows of corn, each row 80 rd. long, and the rows 44 inches apart. Find the yield per acre if the two rows yield 12 bushels.

31. Two vertical chimneys stand on level ground and 76 ft. apart. One chimney is 48 ft. tall and the other is 105 ft. tall. Find the distance between their tops.

32. From a field which is 80 rods square there was sold 80 square rods. Find the value of the remainder at $120 per acre.

33. Mr. A. is 308 yd. north of a gate. Mr. B. is 277 yd. south of it. Both start at the same time to walk toward the gate, Mr. A. at the rate of 2⅓ yd. per second, and Mr. B. at 2 yd. per second. In how many seconds will they be equally distant from the gate?

34. Find the cost of 15 scantlings 3 in. by 4 in. and 20 ft. long, at $20 per M.

35. What is the shortest distance a fly might fly from a corner of a room 14 ft. high by 18 ft. wide, by 24 ft. long to the diagonally opposite corner?

36. The entire population of the United States in 1880 was 50,155,783; in 1890, 62,802,432 and in 1900, 75,994,575. The area of the United States is 2,970,230 square miles. Find the average population per square mile of the United States on each of these 3 dates.

37. A rectangular pile of books is 40 ft. $\frac{1}{2}$ in. long, 7 ft. $9\frac{1}{2}$ in. wide and 5 ft. 6 in. high. The books are done up in packages $7\frac{3}{4}$ inches long, $5\frac{1}{2}$ in. wide, and 6 in. high, with 6 books in a package. How many books are there in the rectangular pile?

38. The books of the last problem are to be shipped at an expense of 9 cents a package. What will be the cost of shipping the whole pile?

39. A speed of 44 ft. per second is equivalent to how many miles per minute? Per hour?

40. The total area (including all openings) of the walls and ceiling of a room 20 ft. long by 18 ft. wide is 1082 square feet. How high is the room?

SUGGESTION: Call the height x.

41. The product of two numbers is 46,122,714, and one of the numbers is 6382. Find the other number.

42. The sum of two numbers is 18 and their difference is 4. Find the numbers.

43. How many posts, set 1 rod apart, are needed to fence the perimeter of a square whose area is 1764 square rods.

44. Find the G. C. D. of 559, 689, and 871.

45. Find the G. C. D. of $\frac{5}{16}$ and $\frac{17}{32}$.

46. Reduce $\frac{3}{3}\frac{5}{9}\frac{6}{9}\frac{8}{6}$ to its lowest terms.

47. Find the L. C. M. of 87, 970 and 7035.

48. Add $\frac{7}{13}$, $\frac{15}{23}$, and $\frac{18}{43}$.

49. Subtract $\frac{15}{31}$ from $\frac{21}{32}$.

50. The numerators of two fractions are 12 and 127, and the denominators are 4 and 12. What is the product of the fractions?

51. Add 3 to both the numerator and the denominator of $\frac{7}{8}$ and subtract $\frac{7}{8}$ from the result.

52. Subtract 1 from both the numerator and the denominator of $\frac{5}{8}$ and subtract $\frac{5}{8}$ from the result.

53. Add 1 to both the numerator and the denominator of $\frac{12}{13}$ and subtract $\frac{12}{13}$ from the result.

54. Subtract 5 from both terms of the fraction $\frac{20}{21}$ and subtract the result from $\frac{20}{21}$.

55. Prove by an example of your own choosing that the value of a fraction is changed by adding the same number to both of its terms.

56. Prove by an example of your own choosing that the value of a fraction is changed by subtracting the same number from both of its terms.

57. Prove by an example of your own that the value of a fraction is *not* changed by multiplying both of its terms by the same number.

58. Prove that the value of a fraction is not changed by dividing both terms of a fraction by the same number.

59. How many yards of cloth @ $\$2\frac{3}{4}$ can be bought for $\$30\frac{1}{4}$? At \$d can be bought for \$D?

60. A piece of wire $72\frac{3}{4}$ in. long was cut into pieces each of which was $9\frac{3}{32}$ in. long. How many pieces were there?

61. A rectangular piece of copper plate contained $\frac{1}{6}$ sq. in. and it was $\frac{5}{9}$ in. long. How wide was it?

62. A rectangular sheet of paper contains $\frac{737}{1152}$ sq. ft. and is $8\frac{3}{8}$ in. wide. How long is it?

63. Change to decimals the following:

$$\frac{3}{4} \quad \frac{17}{25} \quad \frac{49}{75} \quad \frac{135}{525} \quad \frac{1}{1200}$$

64. Change to common fractions:

$$.33\frac{1}{3} \quad .37\frac{1}{2} \quad .62\frac{1}{2} \quad .44\frac{4}{9} \quad .55\frac{5}{9} \quad .11\frac{1}{9} \quad .42\frac{6}{7}$$

65. Cube $\frac{22}{35}$; find the *cube root* of 34.328125.

66. Simplify $\frac{2.16}{32.5} \times \frac{6.5}{1.18} \div \frac{.708}{6.25}$.

67. Find the sum of 328, 24.85, .73, 4161, and 700.603.

68. The minuend is 600.0031 and the subtrahend 78.7869. Find the difference.

69. Find the sum and the product of 87.5, $38\frac{5}{16}$, and 3.89.

70. The sum of two numbers is 1 and one of the numbers is .18379. Find the other number.

71. The area drained by the Mississippi river is 1,235,334 sq. mi., and the number of people living in this area in 1880 was 21,984,480; in 1890, was 27,934,906, and in 1900, was 33,395,247. Find, to the nearest .1, the average population per sq. mile of this area for each census year.

72. The product of two numbers is .0963976 and one number is 37.076. Find the other number.

73. Find to the nearest .001 the length of the diagonal of a square whose perimeter is 80 rods.

74. Find, in feet, to the nearest .001, the side of a square whose area is 1 acre.

75. Find, to the nearest .001, what part of the entire population of the United States lived in the Mississippi basin in 1880. In 1890. In 1900. (Pop. U. S. in 1880, 50,155,783; in 1890, 62,622,250; in 1900, 76,303,387.)

76. How many sq. rd. does a rectangle $24\frac{3}{8}' \times 121\frac{1}{2}'$ contain? (Result to nearest .001).

77. How many grains does 1 cu. in. of water weigh if 1 cu. ft. of water weighs $62\frac{1}{2}$ pounds. (Give result to the nearest .01.)

78. The hills of a row of corn are $3\frac{2}{3}$ feet apart, and the row is 80 rods long. How many hills are in the row, if the first hill is $1\frac{5}{6}$ feet from one end and the last hill is $1\frac{5}{6}$ feet from the other end of the row?

79. If there is an average of 6 ears of corn in each hill, how many ears are there in the row? How many bushels are there in the row, containing 100 ears to the bushel?

80. How many rows $3\frac{2}{3}$ feet apart will a strip 2 rods wide contain, supposing the first row is $1\frac{5}{6}$ feet from one edge and the last row is $1\frac{5}{6}$ feet from the other edge?

81. How many bushels of corn would there be on this strip with the yield mentioned in problem 79.

82. How many square rods are there in a strip of ground 2 rods wide and 80 rods long? How many acres?

83. If the yield of a strip of ground 8×80 rd. be 4 ears of corn per hill, the hills and rows being $3\frac{2}{3}$ feet apart, how many bushels per acre does the yield amount to, counting 100 ears per bushel? Find the total yield.

84. $\frac{3}{7}$ is what percent of $\frac{8}{9}$? $54\frac{11}{16}$ is what percent of $62\frac{1}{2}$?

85. What percent profit does a grocer make who buys berries by dry measure and sells them by liquid measure, buying and selling at the same price per quart?

86. At what price must the following articles bought by the dealer, under the specified conditions, be marked to gain 35% and allow a discount of 25%.

12 Tables	@ $16.00	$ 192.00
10 Bedsteads	@ 40.00	400.00
6 Dressers	@ 20.00	120.00
6 Davenports	@ 48.00	288.00
		$1000.00
Less 10%		100.00
		$ 900.00

87. At what price should the articles bought as specified in prob. 86 be marked to make a profit of 30%, to allow discounts of 25% and 20%?

88. Mark the same articles, bought as in prob. 86, to make a profit of 25% to allow for a loss from bad debts of 20%, and to allow a discount of 20% and 10%.

89. A wholesale dry goods merchant buys in France a stock consisting of two qualities of silk, paying for one 7.38⅓ francs per meter and for the other, 22 francs per meter. Calling a yard .914 meter and a franc $.193, find to three decimal places the prices per yard of the two grades of silk in United States money.

90. If the dealer must pay an exchange rate of 20¼ cents per franc, how much do the two grades of silk of the last problem cost in United States money.?

91. Using the key-word "James Brown," how must the dealer mark the two qualities of silk that he may drop 20% from the marked price and still make 33⅓% on his investment, the goods being sold in America?

92. How must the dealer mark the two qualities of silk, if he must pay an import duty of 40% on bringing the

silks into this country, that he may drop 20% from the marked price and still make $33\frac{1}{3}\%$ on his investment?

93. A man contracts to do a piece of work for \$1000, to be paid \$200 at the end of a year; \$200 at the end of 2 years; and so on for 5 payments. Counting money worth 6%, find the value at the time of contracting, of the first payment; of the second payment.

94. An author is paid a $4\frac{1}{2}\%$ royalty on the wholesale price of a book which sells, retail, at 45 cents. The wholesale price is 90% of the retail price. What is the amount of the author's royalty on 16,400 books sold?

95. Find to the nearest .01 ft. the perimeter and the diagonal of 1 square acre.

96. Find the length of the diagonal, to the nearest .01, from one lower corner of a room 12.6 ft. $\times$ 16.4 ft. $\times$ 9.8 ft. to the opposite upper corner.

97. How many square feet of area are there in the surface of a spherical ball 2 ft. in diameter.

98. Find the radius, to the nearest .01 in., of a circle whose area is 1 sq. yd.

99. Find the radius in inches to the nearest .01, of a sphere whose surface is 1 square yard.

100. Find the radius to the nearest .01 in. of a sphere whose volume is 1 cu. yd.

101. A room, containing 6000 cubic feet, is twice as wide, and 3 times as long as high. Find to the nearest .01 the length of the diagonal from a lower corner to the opposite upper corner.

102. From a square piece of tin is cut, the largest possible circle. What percent of the square is cut away?

103. A wooden cylinder has an 8 in. radius. What is the

length of the side of the largest square prism that can be sawed out of it.

104. Calculate the area of a rectangle whose dimensions are the diameter and circumference of a sphere of 10 inches radius, and compare it with the area of the surface of the sphere.

105. A right, circular cone whose base is 2 inches in diameter is cut from the top of a similar cone, whose base is 4 inches in diameter. The altitude of the lower part is 6 inches. Find its volume.

106. The diameter of a circle is 14 inches. The width of a rectangle of equal area is 7 inches. What is its length?

107. A circular cast iron ring is $10\frac{1}{2}$ inches in diameter and 1 inch thick. The hole in its center is $3\frac{1}{2}$ inches in diameter. The specific gravity of cast iron is 7.2. Find the weight of the cast iron ring

108. The sum of the two acute angles of a right triangle is 90°. The acute angles are to each other as 1:2. Find them.

HINT: Call one angle, x, and the other $2x$. Thus $x+2x=90°$.

109. The sum of the 3 angles of any triangle is 180°. The angles are to each other as 1:3:5. Find them.

110. A regular octagonal flower-bed (having 8 equal sides) is made by cutting off the corners of a 12-ft. square. The distance from the center to a corner of the octagon is 6.494 feet. Find the area of the octagon to the nearest .01 sq. ft.

111. Find the area of a circle whose circumference is 80 rods.

112. Find the area of a rectangle whose length is 3 times its width, and whose perimeter is 80 rods.

113. The radius of a circle is $12\frac{542}{1309}$ rods long. Calling $\pi = 3.1416$, find the number of posts, set 1 rod apart, needed to put a fence on its circumference?

114. How many posts, set 1 rod apart, are needed for a straight fence 80 rods long?

115. If wrought iron is 7.68 times as heavy as water, how heavy is a solid wrought iron ball having a radius of 7.4432 inches? (Call $\pi = \frac{22}{7}$)

116. A solid block is composed of a circular cylinder surmounted by an inverted hemisphere of the dimensions shown in Fig. 1. How many cu. in. are there in the solid?

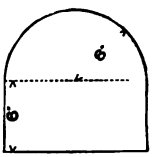


FIGURE 1

117. Suppose the moon's path to be a circle of radius 60 times the radius of the earth (= 4000 mi.) and that the moon makes a complete circuit once in $27\frac{1}{3}$ days. Find the speed of the moon's motion in miles per hour. (Take $\pi = \frac{22}{7}$)

118. Find the moon's speed under the same conditions as in problem 117 in feet per second and compare it with the speed of a rifle ball moving 1100 ft. per second.

119. Find the simple interest at $5\frac{1}{2}\%$ on \$248.60 from Feb. 16, 1904, to Apr. 28, 1907.

120 How long will it take \$120.00 to amount to \$150.00 at 4% simple interest?

121. Calculate the simple interest on this note—

\$865.00 Peoria, Ill., Oct. 12, 1899.

Five years after date, I promise to pay W. H. Upton, for value received, at the First National Bank of Peoria, (\$865.00) eight hundred sixty-five and $\frac{00}{100}$ dollars with interest at 6% from date.

Signed: ROBERT O. WALLACE.

Due Oct. 12, 1904.

122. What sum of money loaned at 6% simple interest will amount to \$324.32 from Sept. 10, 1904, to July 18, 1910?

123. How long will it take \$100 to amount to \$300 at 6% simple interest?

124. Jan. 1, a man deposited \$1200 in a savings bank which paid 3% interest. Apr. 25 he deposited \$250. He made no withdrawals. How much did his interest amount to July 1, of the same year?

125. A savings bank had on deposit \$1315 from Jan. 1 to July 1, and \$385 from Apr. 1 to July 1 of the same year. It paid \$22.61 for the use of this money. Find the rate of interest paid.

126. Prove that $87 \times 93 = 90^2 - 3^2$.

127. Prove that $84.3 \times 84.7 = 84 \times 85 + .3 \times .7$.

128. Prove 11×243 can be found by writing 3 units, then $3+4$ tens, then $4+2$ hundreds then 2 thousands.

129. Make a rule for multiplying quickly by 11.

130. Prove $98 \times 6432 = 643200 - 2 \times 6432$.

131. Make a rule for multiplying quickly by a number that is a few units less than 100; less than 1000.

132. Make a rule for multiplying quickly by a number that is a few units greater than 100; greater than 1000.

133. Show by a drawing that—

$$34^2 = 30^2 + 2 \times 4 \times 30 + 4^2$$

134. Find mentally the squares of the following numbers, using the equation of problem 133:

24 32 48 56 64 77 83 91

135. Show by a drawing that—

$$27 \times 33 = (30-3)\ (30+3) = 30^2 - 3^2 = 891$$

SUMMARY FOR WORK OF PART I

I. What the pupil should know at the end of this part.

1. In Notation and Numeration:

(1) How to read and to write all sorts of whole numbers, common fractions, and decimals to four places.
(2) How to read, write, and interpret the Roman notation.
(3) How to read, and to write all sorts of denominate numbers.
(4) How to read, write, and interpret simple literal numbers.
(5) The salient feature of the English notation.
(6) How to read and to denote percents as common and as decimal fractions, and *vice versa*.
(7) How to denote factors with exponents.

2. In the Fundamental Operations:

(1) How to add, subtract, multiply, and divide all sorts of whole numbers, common and decimal numbers (to 4 places) in separate classes or in combination, and how to point, simplify, and interpret the results intelligently.
(2) How to reduce, add, subtract, multiply, and divide compound denominate numbers, and to abbreviate the names of the units properly.
(3) How to add, subtract, multiply, and divide simple general (literal) numbers.
(4) The fundamental laws and rules of the fundamental operations, and how to check work done under them.
(5) The connections of addition and subtraction, of addition and multiplication, of multiplication and division, and of division and subtraction.
(6) The standard "terminology of the science" for all these operations.

3. In Graphing and Scale Drawing:

(1) How to graph thermometer readings and statistical data to show the laws of change embodied in such data.
(2) How to make and to interpret scale-drawings of some complexity.
(3) How to represent the fundamental operations with lines and areas, and to show operations with fractional number with divided rectangles.
(4) How to represent and to deal with such *signed* numbers as occur with thermometer readings, or opposing forces.

4. In Bills and Accounts:

(1) How to draw up lists of items into accounts, and how to foot, balance, and receipt these accounts.
(2) How to keep family expense accounts, or accounts with separate parts of a farm, or (by easy implication) with the separate branches of other businesses.

5. On the Equation:

(1) The use of the equation as a means of stating problems and the fundamental operations as $5+9=?$ $14-9=?$ $8\times12=?$ $60\div12=?$ etc.
(2) The use of the equation in stating problems containing an unknown number, or magnitude.
(3) The use of the equation-method of solving common and decimal fractions, percentage, and interest problems.
(4) The equation itself as an expression of *balance of values.*

6. In Common Fractions:

(1) "The language of the science" as applied to common fractions.
(2) How to reduce fractions to higher, lower, and lowest terms.
(3) All the fundamental operations as applied to fractions containing one-, two-, or three-digit terms.
(4) How to deal with complex fractions.
(5) How to reduce common to decimal fractions.
(6) Greatest common divisor, least common multiple, and least common denominator, and how to use them.

7. In Ratio and Proportion:

(1) The meaning and use of ratio; of proportion.
(2) Necessary language and laws of ratio and proportion.
(3) How to state problems in the form of proportion and to find the unknown numbers by dealing with the proportion.
(4) Scale-drawing sensed as an application of ratio and proportion.
(5) The fraction as a ratio and the proportion as an expression of equality of two fractions.

8. In Decimal Fractions:

(1) The necessary language of decimals and the laws for pointing results.
(2) How to reduce a decimal to a common fraction, and to express decimals as percents.
(3) How quickly to check results with decimals.
(4) Effect on quotient of moving the point in either dividend, or divisor, or in both.

9. In Compound Denominate Numbers:

(1) The standard units and tables for U. S. money; English money, and the most used equivalents of monetary units of other nations.
(2) Troy, and avoirdupois weight, common linear, surface and cubic measure, liquid and dry measure, time and counting.
(3) How to reduce, add, subtract, multiply, and divide compound numbers.
(4) The essential units and tables of the metric system.
(5) How to convert from common to metric standards, and *vice versa.*

10. In Mensuration:

(1) The necessary language and laws of the rectangle, the parallelogram, the triangle, the circle, and the square box (or prism).
(2) How to calculate and to express quantities of lumber in board feet.
(3) How to calculate cost of fencing fields and lots.
(4) How to analyze irregular figures into triangular parts.

(5) Enough land measure to enable pupils to read intelligently the descriptions of land in deeds, to locate the land and to calculate the areas, when tract is regular, from the description.

11. In Percentage:

(1) The necessary language of percentage.
(2) The connections of percentage with whole numbers, and with common and decimal fractions.
(3) The necessary laws of the subject.
(4) The equation-method of solving percentage problems.
(5) How to find the *percentage*, the *percent*, the *base*, or the *amount*, and how to show these ways in an equation with either numbers or letters.
(6) Applications to gain or loss, to other school subjects, to commission, to trade discount, and to marking goods.

12. In Interest:

(1) The necessary language of the subject.
(2) The connection of percentage with "interest for one year."
(3) How to find the *interest;* the *principal;* the *rate;* the *time*, by the common method.
(4) The six-percent method.
(5) How to compute an interest table.
(6) How to show the laws for calculating any element of the interest-problem both with arithmetical and with general numbers.
(7) Applications to promissory notes and to discounting notes.
(8) Bank discount.

13. In General Number:

(1) How to express sums, differences, products, quotients, and powers of general number.
(2) How to express common fractions in general number.
(3) How to express perimeters, and areas of squares, rectangles, parallelograms, and triangles in general number.
(4) How to indicate the unknown number of an arithmetical problem by a letter, to formulate a simple equation, using it, and to solve this equation.
(5) How to solve simple formal equations of the type $6x+8=20$, and $5y-7=13$.

14. In Form-Study and the Rudiments of Geometry:

(1) Such geometrical relations as are involved in the rational study and use of the square, the rectangle, the parallelogram, the triangle, the circle, the cube, and the square box.

II. The pupil should have considerable skill in the following:

(1) In adding long columns, vertically, and checking results.
(2) In adding lines, horizontally, and checking.
(3) In adding, subtracting, multiplying and dividing whole numbers and fractions, and checking.
(4) In the mechanics of long division.
(5) In finding perimeters and areas of the forms studied.
(6) In analyzing a problem involving 3, or 4, simple steps.
(7) In telling how a problem should be solved before actually solving it.
(8) In estimating approximate results.
(9) In mental addition, subtraction, multiplication, and division.
(10) In making problems from given conditions.
(11) In judging what is the best way to proceed with a problem.
(12) In the use of cancellation to shorten work.
(13) In expressing the solution of the problem in convenient form for the use of cancellation.
(14) In discerning factors of numbers.
(15) In exhibiting the meaning of operations with fractions, by means of divided rectangles.
(16) In solving easy percentage and interest problems.
(17) In finding circumferences and areas of circles from given dimensions, or for dimensions obtained by measurement.

III. Ideas and values in arithmetical study, advanced or begun in Part I.

(1) Arousing an intelligent interest in growth and extent of the nation, and in business arithmetic.
(2) Feeling of value of arithmetical study in affairs of life.
(3) Heightening the feeling of ability to do with some skill, things worth doing.
(4) Joy in consciousness of power over some real difficulties.
(5) Consciousness of ability to see into, and through some things.

(6) Growth in feeling of independence, developed through checking.
(7) Pleasure in the kind of definite thinking called for in arithmetic.
(8) Increased insight through doing habitually all one can with *his head*, adding to the stock of "spot cash" knowledge.
(9) Augmented ability to know (1) what to do, (2) when to do it, and (3) how to do it, without hints, or suggestions, of any sort.
(10) Feeling that a knowledge of arithmetic makes many difficulties much easier.
(11) Engendering taste for continued mathematical study and school work in general.

SUMMARY OF WORK OF PART II

I. What the pupil should know at the end of this part.

1. In Notation and Numeration, and Fundamental Operations:

(1) How to read and to write with ease, and to add, subtract, multiply, and divide with ease, certainty, and steadiness, whole numbers and any sort of fractions, in any sort of reasonable combinations.

(2) How to apply the foregoing severally and in combination, to a wide range of problems of daily life.

(3) Through reviews, a deeper and firmer hold on previous knowledge of these topics.

(4) Meaning and use of easy literal notation and of operations applied to literal numbers.

(5) Addition and division of lines and angles.

(6) How to get differences of angles, of longitudes, and of times.

(7) Most convenient practical ways of pointing quotients of decimals.

(8) Products of sums and differences of lines and of numbers.

(9) Greatest common divisor and least common multiple and how to find them by the factoring process.

2. In Form-Study, Mensuration, and the Rudiments of Geometry:

(1) Areas and perimeters of common forms.

(2) Construction of developments and models of solids, and nature of surfaces of solids.

(3) Mensuration of the trapezoid.

(4) Mensuration of cube, right and oblique prism, the pyramids, the cylinder, the cone, and the sphere.

(5) Fundamental concepts of perpendicularity, parallelism, bisection of angles, learned through paper-folding.

(6) Classifications of quadrilaterals.

(7) Angles learned through the turning of clock-hands.

(8) Measurement of angles with the protractor.
(9) Angle-sums of triangles, parallelograms, etc.
(10) Land measure, townships, sections, fractions of sections, lots, meridians, base lines, ranges, and how to interpret legal descriptions of locations of farms.
(11) That the square on the hypothenuse of a right triangle equals the sum of the squares on the other two sides.
(12) Practical uses of similar triangles.

3. In Longitude and Time:

(1) Use of longitude and latitude to locate places on the earth.
(2) Dependence of measurement of time on the rotation of the earth.
(3) Connection of longitude and time.
(4) The terminology and the table of converting time and longitude.
(5) Meaning and use of standard time, time belts, and the date line.

4. In Percentage and Interest:

(1) Applications to insurance, taxes, trade discount, stocks and bonds, compound interest, savings deposits, and partial payments.
(2) The essential terminology of all the foregoing topics.

NOTE.—These applications furnish a review, just before leaving the elementary school, of percentage and interest.

5. In Uses of Arithmetic in Other School Subjects and in Common Affairs:

(1) In elementary science, air pressure, longitude and time, standard time, laws of conversion of thermometer readings.
(2) In transportation problems, movements of trains, and pulling force of engines.
(3) In industrial studies, the ordinary commercial topics, insurance, taxes, commercial discounts, etc.
(4) In constructive art of the builder, exercises in modeling, making, and calculating properties of the standard geometrical forms.
(5) Many practical questions and problems, distributed in their appropriate places.

6. In Square and Cube Root:

(1) How to get squares and cubes of numbers.
(2) Tables of squares and cubes of the first 9 integers, and of the first 9 tens.
(3) How to find the square roots of numbers.
(4) How to find the cube roots of numbers.

7. In Generalized Number and the Concepts of Algebra:

(1) Intelligent use of letters to represent unknown numbers in arithmetic problems.
(2) The concepts and meaning of the addition, subtraction, multiplication, and division axioms, as laws for the use of the equation in solving problems.
(3) How to write some algebraic phrases, and statements in symbols.
(4) Practical uses of the equation in solving common problems.
(5) The expression of the laws for converting thermometer readings in equation form, and the use of these equations.
(6) Graphic representation of thermometer readings.
(7) Shortened methods for calculating.

II. What previous work should be thoroughly reviewed in the eighth grade.

(1) The laws of all the fundamental operations as applied to whole numbers, and to fractions, both common and decimal.
(2) How to add, subtract, multiply, and divide numbers, part in common fraction form and part in decimal form and to express results either as common or as decimal fractions.
(3) All the processes of checking the fundamental operations.
(4) Percentage and interest.
(5) All laws of mensuration previously taught.
(6) All the most common compound number tables, and operations.
(7) Greatest common divisor and least common multiple and how to find them.
(8) A general review at close, covering the entire ground of arithmetic.

III. The pupil should have considerable skill in the following:

1. In all the fundamental operations.
2. In checking and proving correctness of work and results.
3. In recognizing and using factoring, and cancellation.
4. In the mensuration of squares, rectangles, parallelograms, triangles, boxes (prisms), circles, cylinders, cones, and spheres.
5. In all the most practical phases of percentage and interest problems.
6. In finding square roots.
7. In shortening calculations.
8. In recognizing when to add, subtract, multiply, or divide.
9. In mental calculation.
10. In analyzing the conditions of problems and estimating results.
11. In the use of the equation as a tool for problem-solving.
12. In knowing how to take hold of a practical arithmetical question.

IV. Ideas and values advanced, or begun, in this grade.

(1) Consciousness of independent arithmetical power.
(2) Readiness to defend challenged conclusions and results.
(3) Feeling that there are some things that can be definitely proved to be correct.
(4) Feeling of neighborliness with truth and with ways of establishing it.
(5) Consciousness that hard work pays in more ways than one.
(6) Greater insight into and love for school work in general
(7) Belief that school work in arithmetic makes one stronger for the work of life.
(8) Greater readiness to attack real difficulties, and greater persistency in the struggle to master them.
(9) Feeling of the joy that comes from achievement.
(10) Consciousness of a goodly stock of "spot-cash" knowledge and power.
(11) Feeling of real value of arithmetical knowledge, and of a desire for more.

ANSWERS

MYERS ARITHMETIC BOOK III

Page 1

1. 20lb. 57½ lb.
2. 35$\frac{5}{32}$; 3890$\frac{5}{8}$; 11,671$\frac{7}{8}$
3. 662,500 lb.

Page 2

4. 4668$\frac{3}{4}$ lb.
5. 21,787½ lb.
6. 5446$\frac{7}{8}$lb.; 16340$\frac{5}{8}$lb.
7. 328,125 lb.
8. 1683 lb. 9. 9337½ lb.

Page 3

1. 88$\frac{7}{16}$; $8401.56
2. $50.40 3. $63.35
4. 18¾; $1406.25
5. 37½; $3281.25
6. $1500

Page 4

7. 160; 220; 495
8. $1650 9. $82.46
10. 60 11. 11¼oz.; 5⅝oz.

Page 8

6. (1) 250.7 (2) 475.352
   (3) 4016.974
   (4) 1384.097

Page 9

7. (1) 5052
   (2) 47,213 (3) 26,646
   (4) 4101.77
   (5) 11,579.727

Page 10

3. (5) $\frac{2}{15}$ (7) $\frac{11}{63}$
   (6) $\frac{37}{55}$ (8) $\frac{4}{45}$
4. (2) 15.301¼
   (3) 133.87$\frac{11}{12}$
   (4) 24.75 (5) 6444.23

Page 11

1. Week — No.
   1st, 1,798,798
   2d, 1,823,561
   3d, 1,814,340
   4th, 1,820,085
   5th, 605,388
   Total, 7,862,172
2. 148,890,876
3. 245,083,133

Page 12

4. 91,180,966
1. 168,665 sq. mi.; 282,535 sq. mi.
2. S. Atl. 113,870 sq. mi.
3. 765,855 sq. mi.; 620,290 sq. mi.
4. 1,187,535 sq. mi.
5. Wstn. 1,018,870 sq. mi.; Wstn. 905,000 sq. mi.; Wstn. 567,245 sq. mi.
6. 3,024,880 sq. mi.
7. 702,298 sq. mi.
8. 3,727,178 sq. mi.

Page 14

11. $2,050,000,000
1. (1) 3,604,508;
   (2) 3,644,122
2. 4,857,057;
   4,943,600;
   5,786,197;
   6,252,451;
   8,136,326;
   8,678,644;
   6,945,026;
   7,267,190
4. 3,623,068

Page 15

5. 747,889; increase
6. +147,269; +699; −92,020, etc.
7. 3,742,324,558 bu.; 1,048,266,971 bu.; 2,702,515,570 bu.; 10,294,274,207 bu.
8. 5,321,461,906 bu.; 1,620,968,732 bu.; 4,036,985,037 bu.; 14,393,235,693 bu.
9. (Corn) 2,058,854,841 bu.; (Wheat) 540,503,114 bu.; (Oats) 748,464,911 bu.; (Potatoes) 209,653,394 bu.

Page 16

10. (Corn) 2,398,872,615 bu.; (Wheat) 672,830,839 bu.; (Oats) 886,910,317 bu.; (Potatoes) 270,161,455 bu.
11. (Corn) 340,017,774 bu.; (Wheat) 132,327,725 bu.; (Oats) 138,445,406 bu.; (Potatoes) 60,508,061 bu.

Page 17

7. (1) $27x$ (4) $223d$
   (2) $32y$ (5) $20\frac{1}{4}z$.
   (3) $21.6c$
8. (1) $11x+13y$
   (2) $21a+21b$
   (3) $22m+17c+17d$

9. (1) $6x+2y$
(2) $4c+2d$
(3) $7c+6n+5y$
(4) $9x+3y+z$
(5) $9a+3b+3c$
(6) $8c+9d+3e$
(7) $5m+9n+6s$

Page 21

3. (1) 1,736,229
(2) 5,690,750
(3) 5,018,067
(4) 7,402,056
(5) 34,084,497
(6) 17,019,527

Page 24

2. 245 **4.** 231
3. 314 **5.** 243
1. 48 sq. in.; 336 sq. in.; 2560 sq. ft.; 182,250 sq. ft.; $9x$ sq. ft.; $9x$ sq. yd.; ab sq. mi.; xy sq. units
2. 27; 486
3. 81 cu. in.; 81 cu. ft.; 81 cu. yd.; 81 cu. rd.; 81; $9a$
4. 84 cu. in.; 96 cu. in.; 96 cu. ft.; 180 cu. ft.; 27 cu. yd.; 180 cu. units; $12x$; $3xy$; abc cu. units

Page 25

6. 30,000 lb.; 18,000 lb.
7. 10,336 lb.
8. 15,504 lb. **9.** $133.12
10. 576 sq. ft.; 108 sq. ft.
11. 10,944 lb.
12. 20,736 cu. ft.
1. 270 **2** $11.48\frac{1}{16}$

Page 26

3. 148 **5.** 16.758+bu.
4. 24 **6.** 13 bu.
7. $589.23+
8. 1880; $12,908.75
9. 269; 342; 225; 189; 293; 15; 20

Page 28

1. 68
2. 150
3. 25
4. 26.25
1. $153.64; $89.32
2. $.82; $1.61; $.51; $.19; $.72; $1.22
3. $14.89
4. $166.91; $178.63
5. $27.82; $29.77

Page 29

6. Oct. $168.37
Nov. 173.16
Dec. 155.82
Jan. 166.93
Feb. 148.94
March 147.91
7. $961.13

Page 30

8. 160; $\frac{1}{4}$; 640
9. 640

Page 31

10. 320; 320
11. 19.2
12. $67.20
13. 640
14. $160
15. $49.92
16. $71.25
17. $121.17
18. 55; 55.
19. 550
20. 110; 109; 219

Page 32

21. $50.37
22. 5; 10; 12
23. 20; 5; $7\frac{1}{2}$; 15; $\frac{5}{8}$; $6\frac{2}{3}$
24. 4; 8
25. $6\frac{2}{3}$; $10\frac{1}{2}$
26. $13\frac{1}{3}$; 16; 24
27. 6; 30
28. $59.40
29. $109.77
30. $181.29
31. $529.66

Page 33

8. $13.16; $1974
9. $8.16; $1305.60
10. $800

Page 34

11. $\frac{1}{2}$ crop ($120)
12. Corn; $174
13. Wheat $256 more Corn $448 more
14. Grain rent ($160)
15. 3264
16. $1103.64
17. $729.64; $374

Page 35

1. Receipts, $512.18; expenses, $128.23; profit, $383.95 per 40 acres; $9.60 per acre

Page 36

2. Receipts, $295; expenses, $106.80; profit per 20 acres, $188.20; profit per acre, $9.41
3. Receipts, $1803.20; exp., $704.40; profit for 30 acres, $1098.80; profit per acre, 36.62\frac{2}{3}$

Page 37

4. Receipts, $638.75; exp., $166.69; profit 50 acres, $472.06; profit per acre, 9.44\frac{8}{25}$
5. Receipts, $114; exp. $31.25; profit $82.75 for 10 acres; profit per acre, $8.275
6. Receipts, $350.40; exp., $122.25; profit $228.15 for 10 acre lot; profit $22.815 for 1 acre.

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7. Expenditures, $766.75
8. Receipts, $2453.91; exp., $766.75; profit, $1687.16, net earnings for the year

Page 39

1.(1) 325 (7) 309
(2) 321 (8) 407
(3) 413 (9) 418
(4) 341 (10) 481
(5) 406 (11) 814
(6) 508 (12) 909
2.(1) 95 (7) 183
(2) 89 (8) 205
(3) 102 (9) 309
(4) 98 (10) 402
(5) 107 (11) 1203
(6) 94 (12) 888
3.(1) 3412 (6) 5061
(2) 4859 (7) 6302
(3) 4267 (8) 7109
(4) 4653 (9) 9801
(5) 6123 (10) 13,648
1. 102,400
2. 640; 160

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3. 160; 120; 80; 80; 40; 40; 20
8. A. S. Park S. W. $\frac{1}{4}$
H. S. Barnes E. $\frac{1}{2}$ S. E. $\frac{1}{4}$ and N. W. $\frac{1}{4}$ S. E. $\frac{1}{4}$
J. Brown N. $\frac{1}{2}$ N. W. $\frac{1}{4}$
H. A. Dryer W. $\frac{1}{2}$ N. E. $\frac{1}{4}$
J. M. Smith S. W. $\frac{1}{4}$ S. E. $\frac{1}{4}$
P. S. Mosier W. $\frac{1}{2}$ E. $\frac{1}{2}$ N. E. $\frac{1}{4}$
J. S. Doe S. $\frac{1}{2}$ E. $\frac{1}{2}$ E. $\frac{1}{2}$ N. E. $\frac{1}{4}$
W. H. Dugan S. $\frac{1}{2}$ N. W. $\frac{1}{4}$
9. 2600
10. 480
11. 6 mi.; 6 mi.
12. 36; 23,040.

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14. 144
15. 160 rd. or $\frac{1}{2}$ mi.

Page 46

18. $\frac{2}{3}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{2}{3}$; $\frac{9}{17}$
19. $\frac{9}{11}$; $\frac{8}{11}$; $\frac{8}{11}$; $\frac{8}{12}$; $\frac{7}{11}$; $\frac{7}{8}$; $\frac{9}{11}$
2. 1, 2, 3, 4, 6, 12
3. 12
4. 1, 3, 7, 21; 21
5. 1, 2, 3, 6; 6

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6.(1) 1, 2, 3, 4, 6, 12; 12
(2) 1; 2, 7, 14; 14
(3) 1, 2, 3, 4, 6, 12; 12
8. Prime: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
Composite: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69, 70, 72, 74, 75, 76, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100
10. Even numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50
Odd numbers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49
11.(1) 8; 2; 26; 38; 42
(2) 5; 9; 21; 15; 19; 27; 41
(3) 5; 2; 19; 41
(4) 8; 9; 21; 15; 26; 27; 38; 42
12. 9, 21; 8, 16; 3, 7
14. 7×3; $2\times2\times2\times3$; 5×5; $3\times3\times3$; $3\times2\times5$
15. 5; $2^4\times5$

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1. $2\times2\times3\times3$; $2\times2\times2\times2\times3$
3. $2\times3\times7$; $3\times5\times7$; $2\times2\times2\times2\times2\times3$; $3\times3\times3\times2\times2\times2$; $5\times5\times3$; $5\times5\times5\times2$

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7.(1) 36 (2) 72 (3) 225
8. (1) 20 (5) 54
(2) 18 (6) 22
(3) 45 (7) 63
(4) 215 (8) 162
9.(1) $\frac{8}{9}$ (5) $\frac{9}{14}$ (9) $\frac{3}{4}$
(2) $\frac{24}{49}$ (6) $\frac{10}{11}$ (10) $\frac{8}{7}$
(3) $\frac{8}{15}$ (7) $\frac{15}{52}$ (11) $\frac{1}{11}$
(4) $\frac{6}{7}$ (8) $\frac{8}{7}$ (12) $\frac{1}{7}$
10. $\frac{4}{15}$
1. (14) $\frac{3}{a}$ (15) $\frac{3}{x}$

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6. (1) 36 (2) 180
(3) 1050 (4) 108

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8. (1) $\frac{888}{525}$ (2) $\frac{158}{165}$
(3) $\frac{59}{105}$ (4) $\frac{211}{224}$

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10. $\frac{47}{77}$
11. $8\frac{49}{60}$
12. $25\frac{51}{120}$
13. $110\frac{11}{49}$
14. $105\frac{2}{3}$
1.(1) $\frac{13}{15}$ (2) $\frac{8}{88}$ (3) $\frac{5}{84}$
(4) $\frac{a+b}{ab}$ (7) $\frac{13}{77}$
(5) $\frac{b-a}{ab}$ (8) $1\frac{49}{145}$
(6) $\frac{24}{85}$ (9) $\frac{ab+ac}{bc}$
(10) $\frac{my-mx}{xy}$
(11) $1\frac{24}{65}$ (13) $1\frac{38}{170}$
(12) $\frac{39}{185}$ (14) $\frac{ad+bc}{bd}$
(15) $\frac{my-nx}{ny}$

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3. $1\frac{1}{10}$ hr.
4. $\frac{33}{40}$; $\frac{1}{40}$
5. $55\frac{3}{5}$; $16.70
6. $\frac{3}{25}$
7. $\frac{7}{10}$; $\frac{3}{10}$
8. $\frac{11}{24}$
9. $\frac{7}{25}$
10. $21.01\frac{2}{3}$

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11. 300; $\frac{3}{11}$

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4. (1) $7\frac{7}{8}$ (6) 7 (11) 21
(2) 10 (7) 4 (12) a
(3) 16 (8) $7\frac{1}{2}$ (13) a
(4) $3\frac{11}{18}$ (9) 35 (14) 4
(5) $10\frac{1}{2}$ (10) $3\frac{1}{2}$ (15) a

1. (1) 20 (10) $33\frac{1}{4}$
(2) 12 (11) 21
(3) 25 (12) 24
(4) 10 (13) $\frac{3}{5}a$
(5) $11\frac{1}{7}$ (14) $\frac{1}{2}c$
(6) $11\frac{2}{3}$ (15) $\frac{4}{5}x$
(7) 24 (16) ac
(8) 33 (17) 56
(9) $8\frac{3}{4}$ (18) mx

2. (1) $4\frac{2}{3}$ (4) $3\frac{1}{2}$ (7) $5\frac{5}{8}$
(2) $7\frac{1}{2}$ (5) 27 (8) $\frac{3}{5}b$
(3) $4\frac{1}{2}$ (6) $24\frac{3}{4}$ (9) 6

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2. (1) $45\frac{5}{8}$ sq. in.
(2) $94\frac{3}{10}$ sq. in.
(3) $266\frac{1}{2}$ sq. in.
(4) $540\frac{3}{8}$ sq. in.
(5) $314\frac{3}{10}$ sq. in.
(6) $312\frac{1}{2}$ sq. in.

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4. $1277\frac{9}{25}$
5. $999\frac{19}{56}$
6. 462
7. $1390\frac{5}{8}$
8. $4.12\frac{1}{2}$
9. $5.84\frac{3}{8}$
10. $10.22\frac{3}{4}$
11. $2.93\frac{1}{8}$
12. $9354\frac{15}{32}$

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4. $\frac{3}{5}$ ft.
5. $\frac{1}{10}$

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6. $\frac{5}{8}$; $5000
7. $2\frac{3}{56}$
2. (1) $1\frac{37}{52}$ bu. (6) $2\frac{21}{40}$
(2) $1\frac{43}{72}$ yd. (7) $9\frac{9}{80}$
(3) $1\frac{7}{18}$ hr. (8) $11\frac{1}{5}$
(4) $4\frac{137}{144}$ lb. (9) $2\frac{1}{21}$
(5) $5\frac{37}{64}$ mi. (10) $8\frac{2}{3}$

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3. (1) $12\frac{1}{2}$ ft.
(2) $25\frac{1}{40}$ ft.
(3) $18\frac{19}{28}$ ft.
(4) $11\frac{4}{15}$ ft.
(5) $16\frac{1}{4}$ ft.
(6) $12\frac{15}{32}$ ft.
4. $51\frac{3}{20}''$
5. $6\frac{1}{10}''$
6. $6\frac{1}{2}''$
7. $1\frac{3}{8}''$; $\frac{1}{40}''$

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1. $3\frac{3}{8}$
2. $\frac{3}{8}$ yd.
3. $\frac{15}{68}$ cwt.
4. 28
5. 66
6. $4\frac{4}{7}$
7. $6\frac{3}{8}$; $9\frac{3}{7}$; $3\frac{2}{3}$; $8\frac{4}{5}$
8. $21\frac{1}{3}$; $14\frac{4}{9}$; $9\frac{3}{8}$; $8\frac{8}{15}$
9. 9; $9\frac{1}{5}$; $7\frac{1}{2}$; $9\frac{9}{10}$

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11. $3100
12. 9675
13. 105

Page 68

1. $1\frac{1}{2}$
2. $\frac{5}{18}$
3. $2\frac{1}{12}$
4. $\frac{15}{64}$
5. $1\frac{5}{9}$
6. $\frac{8}{5}$
7. $\frac{2}{21}$
8. $1\frac{19}{75}$
9. $3\frac{37}{97}$
10. $1\frac{3}{5}$
11. $4\frac{37}{48}$
12. $1\frac{371}{720}$
13. $1\frac{576}{1000}$
14. $1\frac{21}{525}$
15. $\frac{ad}{bc}$

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1. 150 lb.
2. Toward the right; 11 lb.
3. $3x$ lb.
4. $4m$ lb.
5. R $6\frac{3}{4}$ lb.

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6. (1) R $35\frac{7}{8}$ lb.
(2) R $5\frac{7}{8}$ lb.
(3) R $51\frac{17}{40}$ lb.
7. F $4\frac{4}{7}$ T.
8. (1) B $44\frac{7}{24}$ T.
(2) F $213\frac{5}{24}$ T.
(3) B $7\frac{7}{120}$ T
10. (1) D $4\frac{11}{24}$ lb.
(2) U $14\frac{3}{8}$ lb.
(3) D $78\frac{1}{8}$ oz.
(4) D $6\frac{3}{40}$ oz.

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12. (1) $+3\frac{3}{5}$ lb.
(2) $+\frac{7}{8}$ oz.
(3) $+33\frac{2}{7}$ T.
(4) $+205\frac{7}{8}$ lb.

1. (1) $1\frac{137}{185}$ (6) $12\frac{236}{297}$
(2) $\frac{6}{164}$ (7) $5\frac{33}{45}$
(3) $\frac{19}{60}$ (8) $5\frac{31}{46}$
(4) $1\frac{583}{864}$ (9) $260\frac{19}{35}$
(5) $\frac{113}{800}$ (10) $19\frac{11}{45}$

2. (1) $\frac{2}{3}$ (7) $1\frac{67}{500}$
(2) $\frac{4}{9}$ (8) $1\frac{113}{229}$
(3) $\frac{2}{3}$ (9) $\frac{8}{9}$
(4) 2 (10) $\frac{333}{856}$
(5) 4 (11) $4\frac{809}{917}$
(6) $2\frac{48}{85}$ (12) $\frac{333}{668}$

3. (1) $\frac{2}{28}$ (6) $16\frac{2}{3}$
(2) $\frac{11}{82}$ (7) $10\frac{766}{1085}$
(3) $2\frac{1}{20}$ (8) 0
(4) $\frac{2}{21}$ (9) $\frac{9}{40}$
(5) $\frac{149}{168}$ (10) $\frac{33}{1495}$

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1. $4\frac{1}{2}$ ft.; $3\frac{3}{8}$ ft.; 6′; $2\frac{5}{8}$′; $2\frac{5}{8}$′; $11\frac{1}{8}$′; $17\frac{1}{8}$′
3. 20′; $12\frac{1}{2}$′; 10′; $3\frac{3}{4}$′; $7\frac{1}{2}$′; $3\frac{3}{4}$′; $6\frac{1}{4}$′
4. $298\frac{7}{18}$ sq. ft.
6. 75″; 33″; 60″; 21″

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7. 33″×75″; 30″×$7\frac{1}{2}$″ 6″ above center and at sides; 9″ below

9. Cross pieces: 32″; 48″; 48″
Binding string: 138″
11. 9.11$\frac{5}{8}$ sq. ft.
12. 24″ each; 28″; 20″
14. 84″; 60″; 114″; 84″; 9″ and 6″

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16. 11″; 10″; 4$\frac{1}{2}$″
1. 320′; 400′; 470.-156 sq. rd.
2. 200′; 80′; 58$\frac{888}{1089}$

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3. 200′; 160′; 117$\frac{587}{1089}$
4. 40′; 60′; 266$\frac{2}{3}$
5. 55′; 80′; 40′; 20′
6. 50′; 85′; 80′; 90′; 110′; 120′; 120′; 170′; 210′
7. 60′; 30′; 15′
8. 70$\frac{120}{288}$; 13$\frac{87}{121}$
9. 61$\frac{887}{968}$; 20$\frac{620}{1089}$; 107$\frac{277}{1089}$
10. 45$\frac{595}{1089}$
11. 130$\frac{10}{11}$

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6. (1)32 (6)24 (11)25
(2)15 (7)3$\frac{5}{6}$ (12)2
(3)72 (8)2 (13)2
(4)88 (9)2 (14)$\frac{1}{8}$
(5)4$\frac{2}{3}$ (10)15 (15)$\frac{1}{2}$
2. 432; 360; 720; 396; 36
3. 6.9; 27.6; 82.8; .38$\frac{1}{3}$; 38$\frac{1}{3}$; 115; 1.15
4. 5; 15; 25; 100

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5. 32$\frac{1}{2}$ ft.; 97$\frac{1}{2}$ ft.; 130 ft.; 14$\frac{1}{12}$ ft; 5$\frac{5}{12}$ ft.; 27$\frac{1}{12}$ ft.
6. 272,160
7. 49
9. 240; 50; 40
10. 30ft.
11. 40ft.
12. 75ft.

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13. 36
14. 9″; 6$\frac{3}{4}$″; 13$\frac{1}{2}$″; 22$\frac{1}{2}$″
15. 9″ to mile
16. 1″ to 8$\frac{1}{3}$ ft., or $\frac{1}{100}$; 1″ to 25′, or $\frac{1}{800}$; 1′ to 8′, or $\frac{1}{8}$
17. Water, 25$\frac{1}{8}$ oz.; F., 8$\frac{3}{8}$ oz.; P., 7$\frac{1}{8}$ oz.; W., 6$\frac{3}{8}$ oz.
Water, 46$\frac{1}{8}$ oz.; F., 15$\frac{3}{8}$ oz.; P., 14$\frac{8}{10}$ oz.; W., 12$\frac{1}{10}$ oz.
Water, 100$\frac{3}{8}$ oz.; F., 33$\frac{3}{8}$ oz.; P., 31$\frac{1}{8}$ oz.; W., 26$\frac{3}{8}$ oz.
18. Water, 68$\frac{1}{4}$ oz.; F., 20$\frac{1}{8}$ oz.; P., 21$\frac{1}{3}$ oz.; W., 17$\frac{1}{15}$ oz.
Water, 206$\frac{3}{8}$ oz.; F., 62$\frac{3}{8}$ oz.; P., 64 oz.; W., 51$\frac{1}{8}$ oz.
Water, 258 oz.; F., 78 oz.; P., 80 oz.; W., 64 oz.
19. Water, 15$\frac{3}{4}$ lbs.; F., 5$\frac{1}{4}$ lb.; P., 5 lb.; W., 4 lb.
Water, 7$\frac{7}{8}$ lb.; F., 2$\frac{5}{8}$ lb.; P., 2$\frac{1}{2}$lb. W., 2 lb.
Water, 9$\frac{9}{20}$ lb.; F., 3$\frac{3}{20}$ lb.; P., 3 lb.; W., 2$\frac{3}{5}$ lb.
Water, 25$\frac{1}{8}$ lb.; F., 8$\frac{3}{8}$ lb.; P., 8 lb.; W., 6$\frac{3}{8}$ lb.
20. Water, 3$\frac{3}{5}$ lb.; F., 2$\frac{9}{16}$ lb.; P., 1$\frac{7}{10}$ lb.; W., 2$\frac{1}{2}$ lb.
Water, 9$\frac{1}{2}$ lb.; F., 5$\frac{1}{2}$ lb.; P., 4$\frac{1}{4}$ lb.; W., 6$\frac{1}{4}$ lb.
Water, 5$\frac{1}{8}$ lb.; F., 3$\frac{3}{16}$ lb.; P., 2$\frac{1}{16}$ lb.; W., 4 lb.
Water, 27$\frac{1}{2}$ lb.; F., 16 lb.; P., 12$\frac{3}{4}$lb. W., 18$\frac{3}{4}$ lb.

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1. 36.04 in.

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2.
1891	25.54 in.
1892	36.56 in.
1893	27.47 in.
1894	27.47 in.
1895	32.89 in.
1896	33.14 in.
1897	25.85 in.
1898	33.77 in.
1899	31.49 in.
1900	28.65 in.
1901	24.52 in.
Total	327.35 in.

3. Footings. Av. in in.

Jan.	21.89 in.	1.99
Feb.	25.15 in.	2.29
Mar.	26.50 in.	2.41
Apr.	18.75 in.	1.70
May	33.48 in.	3.04
June	39.25 in.	3.57
July	33.38 in.	3.03
Aug.	29.04 in.	2.64
Sept.	31.38 in.	2.85
Oct.	14.54 in.	1.32
Nov.	28.50 in.	2.59
Dec.	25.49 in.	2.32

1. Al. 35.2
Fil. 47.9
Pecans 46.8
Wal. 42.
Shred. Cocon. 100.

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	Waste	Not Waste	Diff.
2. Al.	64.8	35.2	29.6
Fil.	52.1	47.9	4.2
Pea.	24.5	75.5	-51.0
Wal.	58.0	42.0	16.0
But.	86.4	13.6	72.8

3. Fat—H_2O

1st. 6	50.1
	61.6
	68.2
	61.6
	53.8
	36.5
last four	61.5
	63.7
	-39.6
	56.7

4. Al.—Fil. = 5.4
Al.—Wal. = 4.3
Pea.-But. and raw pea. = 3.5
Raw—roast pea. = 4.7
5. Ash—Protein.
Roasted pea. 28
Hickory nuts 13.3
Pecans 9.5

6. Carboh. and fat.
Al.—Pecans 12.3
Shred.—unshred.
Cocoan. 10.4

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3. (1) 13,972.8
(4) 15,436.76
4. (1) 242.7 lb.
(2) 253.335 lb.
(3) 394.226¼ lb.
5. (1) 280 lb.
(2) 213.69⅜ lb.
(3) 631.856¼ lb.
(4) 716.96⅞ lb.

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3. (1) 92.8
(2) 20.34
(3) 12.68
(4) 16.64
4. 18,740.556 sq. mi.
31,392.778 sq. mi.
63,821.25 sq. mi.
77,536.25 sq. mi.
107,957.727 sq. mi.
61,732.245 sq. mi.

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5. (1.) 14.29 (5.) 14.35
(2.) 14.43 (6.) 14.32
(3.) 14.48 (7.) 14.46
(4.) 14.43
1. 3.144 **2.** 3.138

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3. 3.143; 3.151;
3.143; 3.160;
3.141; 3.194;
Av. 3.155
4. 3.119; 3.143; 3.142;
3.142; 3.142;
Av. 3.137.
6. 3.1435; Diff., +.0006; +.0013
1. Alum. 2.664
Zinc 6.984
Cast Iron 7.200
Tin 7.333
Wrt. Iron 7.680
Steel 7.840
Brass 8.381
Copper 8.832
Silver 10.481
Lead 11.350
Gold 19.214
Plat. 21.552

Wood

Cork .240
Spruce .500
Pine, Yel. .554
Cedar .561
Pine, White .448
Walnut .670
Maple .750
Ash .845
Beech .852
Oak 1.040
Ebony 1.216
Lignum V. 1.333

Liquid

Alcohol .800
Turpen. .870
Petr. .891
Olive Oil .912
Lin. Oil .952
S. Water 1.026
Milk 1.032
Acetic ac. 1.064
Mur. acid 1.200
Nitric ac. 1.520
Sulph. ac. 1.842
Mercury 13.60

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2. Glass(av.) 2.81
Chalk 2.79
Marble 2.71
Granite 2.66
Stone(com.) 2.53
Salt (com.) 2.13
Soil (com.) 1.99
Clay 1.95
Brick 1.89
Sand 1.82

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5. (1) 491.06¼ sq. ft.; $491\frac{1}{14}$ sq. ft.
(2) 122.78⅛ sq. ft.; $122\frac{1}{8}\frac{1}{8}$ sq. ft.
(3) 1257.14 sq. in.; $1257\frac{1}{7}$ sq. in.
(4) 11,314.29 sq. in; 11,314⅐ sq. in.
6. 1414.26 sq. in.; $1414\frac{2}{7}$ sq. in.
7. 268.911⅜ sq. in.; $268\frac{1}{8}\frac{1}{8}$ sq. in.

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8. 21,480.9578 lb.
9. 1.7506 in.;
2.407 sq. in.
10. 7976.719 lb.
11. (1) 7951.495 lb.
(2) 47,078.966 lb.
(3) 31,175.707 lb.
(4) 37,192.9606 lb.
(5) 42,270.473 lb.
(6) 31,569.624 lb.
12. $\frac{cr}{2}$ sq. ft., or πr^2 sq. ft.
13. $c = 2\pi r$ rd.; $\frac{cr}{2}$ sq. rd. or πr^2 sq. rd.
14. 763.4088

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1. (1) 99.89
(2) 317.702
(3) 1081.4
(4) 366.731
(5) 2917.7183
(6) 9473.19495
2. (1) 78.58
(2) 96.04
(3) 12.93
(4) 36.159
(5) 37.896
(6) 701.32
(7) 723.037
(8) 2.797
(9) 25.252
(10) .017
(11) 58.1211
(12) .1003
3. (1) 3759.53
(2) 667.8582
(3) 756.0743
(4) 8680.0868
(5) 71.517
(6) .71517
(7) .06464
(8) 1.2096
(9) 16.210801
(10) 12.096
(11) 12.096
(12) 79.105488
(13) 79.105488
(14) 79,105.488
(15) 2184.1974
(16) .21841974

(17) 52.904544
(18) 52.904544

4. (1) 4.501 (6) 38.6
(2) 4.592 (7) 4620
(3) 88 (8) .0465
(4) 32.015 (9) 462
(5) 28.4 (10) 462
(11) 462
(12) 231,000

5. (1) 26.666
(2) 8763.888
(3) 1285.714
(4) 12,222.222
(5) .333
(6) .333
(7) .666
(8) .444
(9) 488.888
(10) 4.333
(11) .855
(12) 12.312
(13) 363.636
(14) 75.757
(15) 78.787
(16) .078
(17) 3.453
(18) 11.441
(19) .191
(20) .636
(21) 11.222
(22) 33.333
(23) .186
(24) 1.111

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3. 49; 4.8

4. 10 pounds; 200 marks; 250 francs; 66⅔ rubles; 250 crowns; 250 lire

5. 20.55 pounds; 419.29 marks; 518.13 francs; 129.53 rubles; 492.61 crowns; 518.13 lire

6. $7.9625

7. $323.08

8. (1) 288 (4) 4080
(2) 1440 (5) 4608
(3) 4080 (6) 26,880

9. 10

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10. 30 **11.** 1240; $2\frac{7}{11}$

12. 128 oz.; 96 oz.

13. 384; 774

14. 4800; 2240; 8008

15. (1) 56 oz. (2) 300 oz. (3) 1391 oz. (4) 1728 oz. (5) 13,008 oz. (6) 15,200 oz.

16. (1) 2 T. (4) $4\frac{1}{2}$ T.
(2) $1\frac{1}{2}$ T. (5) $6\frac{1}{2}$ T.
(3) $8\frac{1}{4}$ T. (6) $6\frac{3}{4}$ T.

17. (1) 3 L. T. (2) 8.5 L.T. (3) 5 L. T. (4) 42.5 L. T. (5) 50 L. T. (6) 1.75 L. T.

18. 500 lb.; 1120 lb.

19. 198 inches; 792 inches

20. 66 ft.

21. 678 inches; 456 inches

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22. 32 rods, 2 yd.

23. 60 ft.

24. 237.6 bds.

25. 2840.2 bds.

26. 90 **27.** $1676\frac{4}{11}$

28. (1) 432 sq. in.
(2) 440 sq. in.
(3) 1296 sq. in.
(4) 3024 sq. in.
(5) 7824 sq. in.
(6) 4536 sq. in.
(7) 6516 sq. in.
(8) 39,204 sq. in.

29. (1) 4 sq. rd. (2) 4 A. (3) 6400 sq. rd. (4) $\frac{1}{2}$ A. (5) $272\frac{1}{4}$ sq. ft. (6) 4840 sq. yd.

30. (2) 200 sq. ch. (3) 6400 sq. ch. (4) 23,040 A. (6) 102,400 sq. rd.

31. 2 rd.; 10 rd.; 40 rd.; 160 rd.

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32. 10 rd.; 36

33. 60 **35.** 43,560

34. 79 **36.** 201

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39. $57\frac{3}{4}$; $28\frac{7}{8}$

40. 9.45

42. 1 oz.

43. 28; 25

44. $806\frac{2}{3}$; 32.256

45. $\frac{4}{5}$

46. 224 pt.; $2\frac{1}{2}$ bu.

47. $\frac{4}{5}$ oz.

48. (1) 3600 sec.
(2) 86,400 sec.
(3) 604,800
(4) $\frac{1}{110}$ hr.
(5) 2 hr.
(6) 7 da.
(7) 2 wk.
(8) 1 wk., 4 da., $13\frac{7}{8}$ hr.
(9) 10,080 min.

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49. $52\frac{1}{4}$; Wed.

52. $1800

53. $2190

55. 12 hr., 37 min.; 12 hr., 56 min.; 13 hr., 13 min.; 13 hr., 29 min.; 13 hr., 45 min.; 19 min.; 17 min.; 16 min.; 16 min.

56. 6 hr., 14 min.; 6 hr., 23 min.; 9 min.

57. 6 hr., 26 min.; 6 hr 30 min.; 4 min.; 6 hr., 36 min.; 6 hr., 37 min.; 1 min.; 6 hr., 46 min.; 6 hr., 43 min.; 3 min. shorter; 6 hr., 55 min.; 6 hr., 50 min.; 5 min. shorter.

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59. Jan. 1, 1 hr., 39 min.; Apr. 1, 1 hr., 35 min.; July 1, 2 hr., 5 min.; Oct. 1, 1 hr., 30 min.

60. Jan. 1, 1 hr. 39 min.; Apr. 1, 1 hr., 34 min.; July 1, 2 hr., 5 min.; Oct. 1, 1 hr., 30 min.

61. $4.20

62. 70; 2000; 24,000

63. 1776

64. $24
65. $129.60
66. $432
1. 15,552
2. $46\frac{37}{47}$

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3. $25
4. $37.56
5. 1,244,160
6. 2804
7. 7 reams, 7 quires; 51 reams, 9 quires
8. 560; 6720; 26,880
9. 216; 864
10. 27,783
11. 82,266
12. £119, 6s., 11d.
13. 234; 2808
14. 2838
16. 19,200; 160
17. 19,371

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19. 5781 in.
20. 160 yd., 1 ft., 9 in.
21. 277 qt.
22. 55,815 lb.
23. 635 cu. ft.
24. $1925.92
25. $371\frac{1}{4}$ gr.
26. 116.1
27. 54.87 gr.; 18.29 gr.
28. 232.2 gr.
29. 20
30. $\frac{4}{5}$, or .772; 1, or .95$\frac{5}{8}$; $\frac{5}{4}$, or 1.3–; 1, or 1.05–

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31. 8.616 oz.
32. $1.32+
33. 35 pounds; 700 M.
34. 283.8
35. $\frac{10}{11}$
36. 54 ft., 10 in.
37. 2 mi. 44 ch. 1rd. 4 li.
38. 112 rd., 4 yd., $10\frac{4}{5}$ in.
39. 24.18+ mi.
40. 660
41. 198
42. $975
43. $5\frac{8}{40}$A.

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44. 10,000
45. $109.20
46. $14,400
47. 6392.208
48. 27,720; $16\frac{1}{24}$
50. $0.37\frac{1}{2}$
51. $8.04
52. $45
53. 31,500 lb.
54. $79.20

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55. $122,828.28
56. $530.30
57. 24,589+ lb.
58. $22.75
59. $1.80
60. $132\frac{1}{2}$ cu. in.
61. $727.27
62. $1188
63. $4235
64. $646.70
66. 1 pt., 1.376 gill
67. $13.86

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68. $15\frac{15}{16}$
69. Gain, $3.24
71. .921875 bu.
72. 478.125
73. $76.65
74. $\frac{1}{24}$
75. $13\frac{1}{3}$c.
76. 2150.22 cu. in.
77. 10.8
78. $116.40
79. 32.4
80. 40; 56

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6. 2.112 Km.
7. $0.711
8. 48
9. 51.52 m.

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10. 4035
11. 10,000 Dm^2; 1,000,000 m^2
17. 215.28
20. 42 m^2
21. 252.12 m^2
22. $1664
23. 372
24. 35

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25. 258.7846
26. 2000; 2; .02; 12 dry qt. 1.1+ dry qt. greater than 12 l.
31. 1000
32. $13\frac{1}{8}$ lb.; 6000 g.
33. 39.81456 gr.
34. 220.46; 1102.3
35. 38
36. 32 cents
37. $39.41
38. $17
39. 56
40. 31.25

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41. 138.88+ lb.

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24. .$83\frac{1}{3}$; .$62\frac{1}{2}$; .$91\frac{2}{3}$; .$14\frac{2}{7}$; .$71\frac{3}{7}$; .$44\frac{4}{9}$; .$63\frac{7}{11}$; .$46\frac{8}{13}$; .$47\frac{7}{15}$; .$48\frac{12}{17}$
25. $48\frac{3}{4}$
26. $10\frac{19}{19}$

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27. $33\frac{1}{3}\%$; 25%; $41\frac{2}{3}\%$; $77\frac{7}{9}\%$; $97\frac{1}{2}\%$; $2\frac{1}{9}\%$
28. (1) $5\frac{1}{3}\%$ (5) $4\frac{8}{9}$ (2) $1\frac{4}{7}$ (6) 5 (3) $62\frac{1}{2}$ (7) $12\frac{1}{2}$ (4) $7\frac{17}{24}$ (8) 50 (9) $4\frac{4}{5}$
29. $\frac{1}{10}$; $\frac{3}{25}$; $\frac{1}{12}$; $\frac{1}{8}$; $\frac{9}{25}$; $\frac{4}{25}$; $\frac{1}{6}$; $\frac{3}{40}$; $\frac{27}{40}$, $\frac{7}{8}$
30. .07; .0625; .$83\frac{1}{3}$; .875; .015; .0275; .00875; .001; .0001
31. 11.27
32. 4%
33. 25%; .$12\frac{1}{2}$; .$06\frac{1}{4}$; .$18\frac{3}{4}$; .$18\frac{3}{4}$; .$06\frac{1}{4}$
34. $9000

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35. 59.3%; 58.1%; 54.3%; 50.0%; 38.1%; 38.0%; 37.4%

36. $2000 37. $2100
38. 70; 68; 60; 81; 80; 70; 70

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6. $4.00; $7.50
8. $.05n$
9. $.125a$
11. $\frac{100a}{100}$ or $.01a$; $\frac{100x}{100}$ or $.01x$; $\frac{900x}{100}$ or $.09x$
12. $\frac{1600}{100}$ or .16; $\frac{2900}{100}$ or .29; $\frac{100a}{100}$ or $.01a$; $\frac{100m}{100}$ or $.01m$; $\frac{1600x}{100}$ or $.16x$
13. $1.6r$; $3.5r$; $.01ra$ or $\frac{ra}{100}$; $.01r$R or $\frac{r\text{R}}{100}$
14. $p = .01rb$ or $\frac{br}{100}$

Page 124

1. $20
2. 25c.

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3. 13c.
4. $0.80
5. 160
6. $128; $179.20; $224; $40; $80; $560.
7. $5192
9. $80\frac{5}{14}$ ft.
10. $177\frac{7}{9}\%$
11. $16\frac{2}{3}$
12. $27\frac{3}{11}\%$

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13. $66\frac{2}{3}\%$
14. 11,200
15. $52.80
16. $14.00
17. 20%
18. 32,000
19. 576
20. 2000
21. $9.775
22. $750
23. $250
24. $29.75
25. $1.16\frac{2}{3}$

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26. $5000
27. 15%; $4.50
28. $925
29. 20%
1. $29\frac{1}{11}$; $48\frac{4}{11}$; $22\frac{8}{11}$; $29\frac{1}{11}$
2. Feb. $35\frac{5}{7}$; $39\frac{3}{7}$; 25; $32\frac{1}{7}$
Mar. $38\frac{2}{11}$; $45\frac{5}{11}$; $16\frac{4}{11}$; $35\frac{1}{11}$
Apr. $33\frac{1}{3}$; $43\frac{1}{3}$; $23\frac{1}{3}$; $43\frac{1}{3}$
3. $21\frac{10}{11}$; $24\frac{6}{11}$; $29\frac{1}{11}$; $24\frac{6}{11}$

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4. Cloudy, $28\frac{1}{18}$; $20\frac{2}{18}$; $26\frac{2}{18}$; $24\frac{3}{18}$
P. Cloudy, $26\frac{1}{18}$; $26\frac{1}{18}$; $19\frac{3}{18}$; $26\frac{1}{18}$
Rainy or S. $21\frac{3}{7}$; $21\frac{3}{7}$; $26\frac{4}{7}$; $30\frac{2}{11}$
5. $71\frac{3}{7}\%$; $68\frac{4}{7}\%$; $81\frac{1}{7}\%$; 100%; $77\frac{1}{7}\%$; $64\frac{2}{7}\%$; 80%; $67\frac{1}{7}\%$; $74\frac{2}{7}\%$; $82\frac{2}{7}\%$; $74\frac{2}{7}\%$; $82\frac{2}{7}\%$;
6. Approx., 82%, 66%, 90%, 83%, 80%, 70%, 73%, 64%, 76%, 85%, 88%.
7. 39.5%; 2.4%
8. 23g.; 1.6g.
9. 120
10. Approx. def. 46.8%
" exc. 43.6%
" def. 33.7%
" exc. 38.1%

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1. Approx., 60.3%
2. 9 hr., 29 min., 52 sec.; 8 hr., 49 min., 14 sec.
3. 14 hr., 51 min., 52 sec.; 15 hr., 31 min., 3 sec.

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	hr.	min.	sec.
5. Feb.,	9	52	55.20
Mar.,	11	11	25.62
Apr.,	12	36	31.32
May,	13	57	46.44
June,	14	55	58.08
July,	14	57	36.90
Aug.,	14	26	52.26
Sept.,	13	13	51.24
Oct.,	11	49	18.48
Nov.,	12	7	25.50
Dec.,	9	23	16.44

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1. 142.9%; 17.9%
3. 73.4%; 26.6%
36.2 + %
4. 30,800 mi. 5. 7600 mi.
6. N.A. 24,043.9 + mi.; S.A. 13,606.02 + mi.; Af. 14,080.14 + mi.; Eur. 17,200 mi.
7. 4200 mi.
8. 3896.10 + mi.

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9. 3300 mi.
10. 31,223.92 + sq. mi.
11. 23,799.65 + sq. mi.
12. 9950.54 + sq. mi.
13. 875 ft.
14. 16,233,766.23 + sq. mi.
1. $4.00 2. $62.50

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3. $1.50
4. (1) $300
(2) $382.50
(3) $3.00
(4) $14.25
(5) $3
(6) $112.50
(7) $75
(8) $240
5. $46.25; $64.75;
6. $1200
7. $13,684
8. $53.61
9. $17.52

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2. $60 3. $2992.50

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4. $5.30

5.(1)$30.38 (2)$128.25
(3) $1054.62
(4) $41.30 (6) $684
(5)$233.29 (7)$17.86

6. 24% **1.**(1) 42½ (4) 46
(2) 41⅛ (5) 26⅛
(3) 28⅞

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2.(1) $\frac{ro}{ao}$ (2) $\frac{on}{ms}$
(3) $\frac{hso}{io}$ (4) $\frac{hha}{zs}$
(5) $\frac{hio}{hao}$ (6) $\frac{moo}{rao}$
(7) $\frac{nrs}{mos}$

3.(1) $\frac{ms}{rs}$ (2) $\frac{zs}{ns}$
(3) $\frac{mas}{rho}$ (4) $\frac{has}{es}$
(5) $\frac{rns}{ais}$ (6) $\frac{zns}{nmo}$

4.(1) $\frac{oo}{ms}$ (2) $\frac{ee}{ia}$
(3) $\frac{arh}{hnz}$ (4) $\frac{ren}{azz}$
(5) $\frac{zzs}{nms}$ (6) $\frac{hhrr}{zam}$

5.(1) $\frac{es}{ns}$ (2) $\frac{hos}{hss}$
(3) $\frac{aos}{hni}$ (4) $\frac{rnn}{amm}$
(5) $\frac{ios}{oss}$ (6) $\frac{zhs}{oms}$

6.(1) $\frac{kk}{ce}$ (2) $\frac{ss}{ol}$
(3) $\frac{lab}{bhr}$ (4) $\frac{ash}{lrr}$
(5) $\frac{rre}{hce}$ (6) $\frac{bbaa}{rlc}$

8.(1) $\frac{ni}{os}$ (2) $\frac{zs}{ns}$
(3) $\frac{hns}{has}$ (4) $\frac{rrr}{aos}$
(5) $\frac{zor}{nms}$

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11. $31.50; $31.50; $63; $110.25; $t\times$ $31.50

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12. $100; $100; $300; $583⅓; $x\times$$100

13. $128

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15.(1)$62.30 (3)$101.54
(2)$66.97 (4)$179.35

16.(1)$332.44 (2)$197.84
(3)$999.86

17.(1) $34.13; $734.13
(2) $33.90; $433.90
(3) $31.43; $241.43
(4) $20.50; $170.50
(5) $19.56; $299.56
(6) $19.40; $379.40
(7) $20.75; $280.75
(8) $39.25; $339.25

18. $100 **20.** $102.75

19. $150 **21.** $250

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22. $850

25.(1)$200.00; $219.50
(2)$640.23; $809.25
(3)$720.00; $278.50
(4)$135.00; $63.42
(5) $1200.98; $1323.48
(6) $3424.86; $3600.86
(7) $2888.89; $2921.39

26. 4½% **28.** 12%

27. 11.6% **29.** 9$\frac{8}{14}$%

Page 142

31. (1) 5.6 (2) 12
(3) 10.4

32. 4 yr., 4 mo., 15 da.

33. 5 yr., 9 mo.

34. 1 yr., 6 mo. 15 da.

35. 12 yr., 6 mo.

36. 16⅔ yr.

37. 5 yr., 1mo., 16 da.

40.(1) 6 yr., 2 mo., 9 da.
(2) 1 yr., 1mo., 15da.
(3) 3 yr., 6mo., 27da.

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41. (1) $11.78; $71.78
(2) $41.93, $216.93
(3) 2 yr., 3 mo.; $126.00
(4) 3 yr., 9 mo.; $617.50
(5) 1 yr., 7 mo., 26 da.; $83.85
(6)$1500; $2009.25
(7) 4; $87.16⅔
(8) 3 yr., 7 mo., 6 da.; $937.986

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47. 4yr. 8 mo. **53.** $350

48. 20 yr. **54.** $500

50. 4% **55.** $250

51. 20%

1.(1)$37.50 (2)$75.00
(3) $243.75
(4) $137.50
(5) $968.75
(6) $x\times$$37.50
(7) $t\times$$37.50

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2.(1) $62.50
(2) $206.25
(3) $525.00
(4) $250.00
(5) $368.75
(6) $t\times$$37.50
(7) $r\times t\times$$12.50
(8) $r\times x\times$$12.50
(9) $r\times n\times$$12.50

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11.(1) $11.52 (3) 4$\frac{7}{17}$%
(2)$266.67 (4) 2 yr.
(5) 8⅓%

Page 147

2. Int. $3.75; Amt. $253.75

3. $384.85

4.(1) $292.04
(2) $797.37
(3) $2805.53
(4) $4195.46
(5) $4411.81

Page 148

1. $109.09
2. $108; $1.09
3. $65.76

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4. $63.54
5. $4.55; $86.40
6. (1) 5.99; $59.41
   (2) $31.21; $284.01
   (3) $46.70; $362.36
   (4) $123.12; $730.88
   (5) $229.76; $810.42
   (6) $376.48; $1167.54
   (7) $350.31; $1862.19
   (8) $193.99 $2646.65
   (9) $157.44; $3920.14
   (10) $1101.88; $8755.87

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3. Floor, 1,080,000 lb.; end wall, 432,000 lb.; side wall, 540,000 lb.
4. 44,496 lb.

Page 152

1. 39.6 mi. per hr.
2. 3:10 P.M.
3. 378 mi.
4. 1 P.M.
5. 4301 mi.
6. 38 mi. per hr.
7. 23½ mi. per hr.; 18½ mi. per hr.
8. 34½ mi.; 41½ mi.
9. 7 mi.

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10. 50½ mi. per hr.
11. 13½ mi. per hr.
12. 5 sec.
13. ½ hr.
14. 679 ft.
15. 510.39 T.
16. 2.55195 T.

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17. 2712 ft.; 1329.45 T.
18. 6381.36; 3.2-
19. 3.192 T.
20. 1.666 T.
21. 2.5134 T.
22. $156,464.40
23. 26⅔ T.

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24. 11¼ T.
25. 18,750 lb.
26. 19,200 lb.

1. 45¼ min.; 50¼ min.; 47 min.; 46 min.; 49¾ min.; 47¼ min. 45¾ min.
2. 1¾ min.; 1½ min.; 2½ min.; 1½ min.; 1½ min.

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3. A 2½ min.; B ½ min.; A ½ min.; A 2 min.; B 2 min.; on time; A 1 min.
5. 5.5 mi.; 24.5 mi.; 30.8 mi.; 50 mi.; 35.6 mi.
6. 50 min.; 20 min.; 25 min.; 43 min.

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7. 1.11; 1.225; 1.232; 1.163−
8. 1.11+; 1.07+; 1.17+; 1.20+

Page 159

6. $.07
7. $.18

Page 160

11. $904.56
12. $\frac{1}{10}$th part
13. $62.40

Page 161

14. 1897—$79.86
    1898—$80.40
    1899—$95.28
    1900—$95.67
    1901—$101.59
15. $21.73
16. No Breadstuffs, $.31; $.57; $1.24; $5.51 Meat, $.18; $.27; $1.16; $1.26 Dairy and Garden, $.91; $2.24; $1.86; $.31
17. $\frac{1851}{1886}$, $\frac{1885}{1915}$, $\frac{9000}{10150}$
18. $\frac{1445}{7986}$; $\frac{427}{1882}$; $\frac{1555}{10150}$
19. $90.56
20. Bread $15.01
    Meat $8.04
    Dairy $13.67
    Other Food $9.01
    Clothing $15.57
    Metals $14.54
    Misc. $14.73
21. 1897—98—$.54
    1898—99—$14.88
    1899—1900—.39
    1900—01—$5.92

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1. Jan. 1833
   Feb. 1867
   Mar. 1884
   Apr. 1848
   May 1832
   June 1821
   July 1810
   Aug. 1870
   Sept. 1903
   Oct. 1895
   Nov. 1892
   Dec. 1889
   Total 22,344
2. 40; 75; 189; 1233; 325
3. 5%; 5%; 6%; 6%
4. Jan. 1837
   Feb. 1872
   Mar. 1887
   Apr. 1876
   May 1860
   June 1855
   July 1852
   Aug. 1823
   Sept. 1871
   Oct. 1870
   Nov. 1868
   Dec. 1849
   Total 22,320
6. 188; 185; 159; 965; 363

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1. (1) 1⅛ (8) $\frac{11}{16}$

(2) $\frac{17}{20}$ (9) $\frac{45}{61}$
(3) $1\frac{6}{13}$ (10) $\frac{11}{15}$
(4) $1\frac{23}{35}$ (11) $\frac{11}{24}$
(5) $1\frac{1}{14}$ (12) $\frac{13}{24}$
(6) $\frac{1}{3}$ (13) $\frac{197}{204}$
(7) $\frac{8}{15}$ (14) $4\frac{23}{24}$
(15) $19\frac{25}{36}$

4. (1) $\frac{3}{5}$ (8) $10\frac{3}{7}$
(2) $1\frac{1}{7}$ (9) $5\frac{1}{4}$
(3) $\frac{11}{21}$ (10) $\frac{2}{3}$
(4) $\frac{9}{14}$ (11) $\frac{173}{264}$
(5) $1\frac{14}{27}$ (12) $1\frac{1}{18}$
(6) 33 (13) 10
(7) $11\frac{9}{91}$ (14) $21\frac{133}{135}$
(15) $5\frac{47}{48}$

5. (1) 6.48 (5) 71.24
(2) 32.6 (6) 7.124
(3) .652 (7) 71.24
(4) 3260 (8) 24.831

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6. (1) 38.65 (4) 32.87
(2) 386.5 (5) 56.38
(3) 40.65 (6) 561.2
8. (1) 2.56 (4) 328.7
(2) 4.72 (5) 2.751
(3) 83.6 (6) .6032
9. (1) 38.62 (4) 42.23
(2) 20.52 (5) 3291.
(3) 71.85 (6) 4223.

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6. (7) $1260\frac{4}{11}$ cu. ft.
(8) *abc* cu. in.

Page 172

2. 7
4. $x+y+x+y$; $2x+2y$; $2(x+y)$

Page 173

7. $\frac{2x+2y}{2}=x+y$
9. 1650; 5

Page 174

1. Form 9. $p=a+z+2x+2y$.
5. (2+a+b) in.; etc.

Page 175

8. Area $=xy-2ab$

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36. 26; $2(n-2)$
37. 120

Page 188

17. $6677\frac{1}{2}$ **19.** 723
18. 741 **20.** $172\frac{53}{121}$

Page 189

21. $168\frac{33}{121}$; $163\frac{117}{121}$
22. 4941 **24.** $2\frac{1}{2}$; $4\frac{1}{2}$
23. $1\frac{9}{40}$ **25.** $8\frac{1}{8}$
26. $8\frac{221}{440}$; $9\frac{1}{176}$

Page 195

2. 70'c.; 120'c.
3. 60° 45′
4. 7 hr., 3 min.
5. 1st is east, 45°30′
6. 1st from E., 2d from W.
7. 20° 15′; 92°

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10. 64° 33′ 54″
11. 166° 23′ 24″; 11 hr., 5 min., 33.6 sec.
12. 3° 46′ 48″; 15 min., $7\frac{1}{2}$ sec.
13. 17° 5′; 1 hr., 8 min., 20 sec.
14. 2° 15′ 33.3″; 9 min., 2.22 sec.

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17. 73° 44′ 59.85″;
3° 2′ 8.25″;
71° 7′ 45.75″;
77° 3′ 56.7″
18. Albany, 25 min., 2.94 sec.
Ann Arbor, 2 hr., 34 min., 7.53 sec.
Bombay, 10 hr., 26 min., 10.93 sec.
Cambridge, 1 hr., 5 min., 55.79 sec.
Paris, 5 hr., 59 min., 47.81 sec.
Edinburgh, 5 hr., 44 min., 53.73 sec.
Bombay, 3 hr., 37 min., 20.98 sec.
Bombay, 4 hr., 41 min., 54.77 sec.

19. 6° 15′ 44.10″
38° 31′ 52.95″
156° 32′ 43.95″
16° 28′ 56.85″
89° 56′ 57.15″
86° 13′ 25.95″
54° 20′ 14.70″
70° 28′ 41.55″

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1. 9 A.M.; 8 A.M.;
8 A.M.; 8 A.M.;
7 A.M.; 6 A.M.;
6 A.M.

2. New Or., 9 hr., 10 min., 45 sec., P.M.
Phila., 10 hr., 10 min., 45 sec., P.M.
Buffalo, either of the above
Washington, 10 hr. 10 min., 45 sec., P.M.
Austin, 9 hr., 10 min., 45 sec., P.M.
Boise City, 8 hr., 10 min., 45 sec., P.M.
Los Angeles, 7 hr., 10 min., 45 sec., P.M.
El Paso, 7, 8, 9 hr., 10 min., 45 sec., P.M.
Salt Lake City, 7 hr., 10 min., 45 sec. P.M.

3. New Or., 2:15 P.M.
Phila., 3:15 "
Buffalo, 3:15 "
or 2:15 "
Wash., 3:15 "
Austin, 2:15 "
BoiseCity, 1:15 "
LosAng., 12:15 "
El Paso, 2:15 "
or 1:15 "
or 12:15 "
S.L.City, 12:15 "
Dodge C., 3:25 P.M.
New Or., 4:25 "
Phila., 5:25 "
Buffalo, 4:25 "
or 5:25 "
Wash., 5:25 "
Austin, 4:25 "
BoiseCity, 3:25 "
LosAng., 2:25 "
El Paso, 3:25 "

or 2:25 "
or 1:25 "
S. L. City, 2:25 "

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5. 12 o'c. noon; 1 P.M.; 4 P.M.; 9 P.M.; 11 P.M

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6. 180°
7. 1 hr.
8. 12 o'c. noon; Sat. noon.
9. Oct. 15, 4 P.M.; Oct. 15, 3 P.M.; Oct. 15, 5 P.M.; Oct. 15, 6 P.M.; The answers above are for just East of the 180th Mer. For just West of the 180th Mer., the date is Oct. 16.
10. Yes; Because it burned after 12 o'c. local time.

Page 201

11. (1) 10:25 A.M.
(2) 10:25 A.M.
(3) 9:25 A.M.
(4) 8:25 A.M.
(5) 12:25 P.M.
(6) 10:25 A.M.
(7) 12:25 A.M.
(8) 11:25 P.M.

Page 202

2. $1.60; $14.40
4. 17,625; 16,800; 8000; 26,250; 5200 25,312.5; 32,500

Page 203

8. 90,000; 68,060; 75,000; 90,000; 126,000
10. 13,333⅓; 13,333⅓; 10,000; 11,544⅔; 10,736⅔
12. 800; 889; 1334; 1143
13. $38.72; $9.30

Page 204

14. 30,200 16. $302.00
15. $108.72 17. 18,192

Page 205

19. $222.77 20. $515.35
21. 1728; $47.52

Page 206

23. 1920; $168
24. 22,186; $310.60

Page 209

5. 80; 80; 80; 40; 38; 39; 40
8. J. Hay, Lot 50, No. A., 82
S. Perry, Lot 51, No. A., 81
J. Hay, E. ½ N. W. ¼ Sec. 7, No. A., 80
J. Ochiltree, Lot 49, No. A., 41
J. Ochil., S. E. ¼, N. W. ¼, Sec. 6, No. A., 40
J. Ochil., E. ½, S. W. ¼, Sec. 6, No. A., 80
J. Church, Lot 48, No. A., 38
J. Church, Lot 47, No. A., 41

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3. 5⅓ cu. ft.; 4 cu. ft.; 4⅔ cu. ft.
4. ⅔ l cu. ft.; ⅓ l cu.ft.; $\frac{10}{27}$ l cu. ft.
5. 32
7. (1) 75.3984 cu. in.
(2) 269.3922 cu. in.
(3) 667.5900 cu. in.
(4) 2150.4252 cu. in.
(5) 296.8812 cu. ft.
(6) 3.1416 ar^2 cu units

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8. 3.1416 in.; 50.2656 sq. in.; 56.5488 sq. in.
9. 761.426
10. 9.953 min.
11. 14.72625 lb.
12. 211.58
13. 552.7 14. ¼

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15. ⅓; ⅛
18. (1) 16.5 cu. in.
(2) 188.5 cu. in.
(3) 326.85 cu. in.
(4) 41.87 cu. in.
19. 21.2 20. 169.6

Page 213

21. 148.4
22. 1.807238+
23. 27.2397+
24. 5901.56+
26. 7¼; 28⅞
28. 1257⅐; 314⅛

Page 214

29. 197,040,561½
30. $S = 4\pi R^2$
31. 360 cu.in.
32. $V = \frac{B \times a}{3}$
33. 64 cu. in.
34. 91,636,272

Page 215

35. 7⅛
36. 2,234,261,760
37. 12,330,981,000

Page 216

39. 33⅓
40. 33⅓; 2.806; 12.77; 541.51+
41. Circum. mi.
Moon, 6788⅛
Merc., 9522⅛
Venus, 24,200
Earth, 24,885½
Mars, 13,294⅛
Jup., 271,857½
Sat., 229,428⅛
Uran., 100,257½
Nept., 109,371⅜
Sun., 2,722,971⅜

Surface
Moon, 14,663,314$\frac{8}{7}$
Merc., 28,854,257$\frac{1}{7}$
Venus, 186,340,000
Earth, 197,040,561$\frac{1}{7}$
Mars, 56,234,828$\frac{2}{7}$
Jupiter, 23,515,642,857$\frac{1}{7}$
Saturn, 16,748,285,714$\frac{2}{7}$
Uranus, 3,198,202,857$\frac{1}{7}$
Neptune, 3,806,125,714$\frac{2}{7}$
Sun., 2,359,182,445, 714$\frac{2}{7}$

Volume mi.
Moon, 5,278,793,142$\frac{8}{7}$
Merc. 14,571,399,857$\frac{1}{7}$
Venus, 239,136,333$\frac{1}{3}$
Earth, 260,027,860,521$\frac{11}{21}$
Mars, 39,645,554,142$\frac{8}{7}$
Jup., 339,017,184,523,809$\frac{11}{21}$
Sat., 203,770,809,523,809$\frac{11}{21}$
Uranus, 17,003,778,523,809$\frac{11}{21}$
Nept., 22,075,529,142,857$\frac{1}{7}$
Sun., 340, 665, 945,161, 142, 857$\frac{1}{7}$

42. $\frac{3}{2}$; 1

43. $\frac{\sqrt{5}}{4}$ or, .559+

44. $\frac{2}{3}$ **45.** $\frac{1}{2}$ **46.** $\frac{1}{3}$

Page 219

7. 8″

8. (1) 15 (3) 20 (5) 28 (2) 16 (4) 40 (6) 36

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11. (1) 22 (2) 27 (3) 34 (4) 35 (5) 36 (6) 75 (7) 87 (8) 97 (9) 98 (10) 332

Page 223

14. (1) 43.6 (5) 8.71 (2) 68.7 (6) 8.94 (3) 71.1 (7) .74 (4) 7.82 (8) .677 (9) 1.103

15. (1) 2.236+ (2) 2.645+ (3) 3.872+ (4) .921+ (5) .5 (6) 11.180+ (7) 41.-024+ (8) 82.915+ (9) 85.603+ (10) 1.360+ (11) 5.175+ (12) 8.055+

16. (1) $\frac{1}{4}$ (2) $\frac{4}{5}$ (3) $\frac{12}{25}$ (4) $\frac{144}{169}$ (5) $\frac{64}{225}$ (6) $\frac{441}{1024}$ (7) $\frac{a^2}{b^2}$ (8) $\frac{x^2}{y^2}$

18. (1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{4}{5}$ (4) $\frac{12}{13}$ (5) $\frac{8}{11}$ (6) $\frac{15}{17}$ (7) $\frac{8}{17}$ (8) $\frac{40}{27}$ (9) $\frac{a}{b}$

19. 42.8+″

20. 70.037+ft.

21. 90′

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22. 4950 sq. ft.

23. $8.78

24. 11,250 sq. ft.

25. 80′ **26.** 2200 **27.** 660

28. 173.2 sq. ft.

29. 3367+ sq. ft.

Page 225

30. 804.96 sq. in.

1. 125 cu. in.; 216 cu. in.; 343; 1331; 5832; 9261; 13,824

2. 2 and 3; 3 and 4; 4 and 5; 7 and 8; 8 and 9; 9 and 10; 38 and 39; 78 and 79

Page 226

3. 15.625; 300.763; 1953.125; $\frac{1}{8}$; $\frac{8}{27}$; 37,037$\frac{1}{27}$; 6591$\frac{8}{13}$

7. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{7}$; $\frac{9}{20}$; $\frac{7}{10}$; $\frac{9}{50}$

9. (1) 124 (2) 232 (3) 43.2 (4) $\frac{13}{17}$

10. 12.5 ft. **11.** $\frac{11}{16}$ in.

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1. 62° **2.** 518 ft. **3.** Yes

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4. 49$\frac{1}{2}$ ft. **7.** 128 rd.

5. 72 ft. **9.** $\frac{1}{4}$″

6. 23$\frac{1}{7}$ ft. **10.** 3′

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11. 46.98 lb.; 13.02 lb.; 11.745 lb.; 3.255lb.

12. 10.8 lb.; 1.32 lb. .6 lb.; .24 lb.; .06 lb.; 2.7 lb.; .33 lb.; .15 lb.; .06 lb.; .015 lb.

13. 18%; 2$\frac{1}{2}$%; 1%; $\frac{3}{5}$%; $\frac{1}{10}$%.

14. 82$\frac{210}{217}$%; 10$\frac{210}{217}$%; 4$\frac{168}{217}$%; 1$\frac{168}{217}$%; $\frac{100}{217}$%.

15. 45$\frac{3}{4}$ oz.; 55$\frac{1}{3}$ oz.; 287$\frac{7}{8}$ oz.; 7$\frac{2}{3}$ oz.

16. 100 oz.; 73.6 oz.; 603.2 oz.; 15.2 oz.; 8 oz.

17. 60$\frac{10}{11}$ c.; 1.06\frac{3}{55}$; 1$\frac{7}{55}$ lb.; 1$\frac{17}{55}$ lb.

Page 233

1. $11.90

Page 234

2. $6250 **3.** $14,000
4. $2800 **5.** $16,000
6. 1$\frac{188}{231}$% **7.** 2$\frac{8}{21}$%
8. 2.6% **9.** $700
10. $4500 **11.** 2.4%
12. 3% **13.** $72,640
14. $1800 **15.** 1$\frac{1}{4}$%

Page 235

16. $18.6624
17. 1500; $84 **18.** 4%
19. $350; $70; $77
20. 75%; $67.50

Page 236

1. $54 **3.** 2$\frac{1}{2}$%
2. $2880 **4.** $150
5. $24,102.56

6. $33,874.35½
7. $7390 512

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8. $684,042.55
9. $3,312,133⅓
10. 18 mills on a dollar
11. $3001.47 13. $171.00
12. 15 14. $21.44
15. $89.60 16. $100
17. .013; $13,639.11

Page 238

18. $12,767.48
19. 19
20. $21,354.17
1. $578.85; $572.76

Page 239

2. $124.64 4. $834.74
3. $125.97

Page 240

5. $825.95 6. $285.22

Page 242

1. 2.1%; $420
2. $26,350; $26,550; $26,200; $26,400
3. $35,000; $35,050; $34,950; $34,950
4. Nothing
5. Loss, $500; Loss, $500
6. Gain, $937.50
7. 8.75%
8. 9.77%

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9. 1.84%; 1.83%
10. (1) 2.77%; 2.75%
(2) 2.76%; 2.75%
(3) 3.55%; 3.54%
(4) 2.87%; 2.86%
11. 3.7%
12. 4.57%; 4.55%
13. Wab. R. R., 5s., .43%; .42%
14. Straight loan, .24%
15. 14.8−%; $8250

Page 244

2. $225.53 4. $614.43
3. $632.60

Page 246

1. $463.59
2. (1) $77.27
(2) $156.85
(3) $387.34
(4) $1326.51
(5) $2696.74
(6) $4245.45
(7) $6840.63
(8) $5873.19
3. $365.88
4.

	Tot. Dep.	Tot. W'd'ls.	Bal.
Feb. 26	$280	$135	$145
May 2	$355	$225	$130
Jun. 15	$505	$305	$200

$355−$305=$50
Withd'l value July 1 =$200.37, Ans.

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5. $1292.73

Page 249

2. $410.57 2. $x+y$
3. $1920.64 3. $x+y$
4. 80c.−45c.=35c.
5. $m−$s

Page 250

6. 45c. 10. $\frac{x}{y}$ in.
7. xy
8. $5A^6$
9. 7 rd. 11. $\frac{a}{b}$
12. 30c.; 40c.
13. $8x$; $3x^2$ 15. 28°

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16. $12°+x°$; $12°-y°$
17. 2°, 6°, 12°
18. $1.50, hat; $3.00, coat
19. $\$x-\$30$ 21. $x-y$
20. $\frac{\$x-\$30}{2}$ 22. $3a$
23. $x-y$
24. $(a+b)$ mi.
25. $(3+a)$ mi. per hr.
26. $(a-3)$ mi. per hr.

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27. $5a$ cents
28. ax cents
29. $40
30. $\frac{\$x}{15}$ 31. $\frac{m}{l}$ rd.
32. 6mi.; The length of a side of the township.
33. $(10x+8y)$ cents
34. $\frac{m+c}{3}$
35. $\frac{5(m+c)}{3}$ cents·
$\frac{2(m+c)}{3}$ cents.
36. $72xy$ sq. in.
37. $\frac{x}{3}$ ft.
38. $6x+6y$, or $6(x+y)$; $6x+6y=6(x+y)$
39. 4 in.; 8 in.

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1. 75 lb. 2. $p=75$
3. $F=300$ 4. $F=156$
5. $F=78$

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1. (1) 5 lb. (2) 3 ft. (3) 2¼ mi. (4) 4c. (5) $4 (6) 21 sq. in. (7) 5 (8) 24 (9) 3
2. 6; 21; 1½
3. 15; 2; 19
4. 5; 15; 25; 25
5. 4; 12; 28; 16; 2
6. 9; 27; 81; 3
7. 3 8. 21⅓

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9. 4 10. 2; 2 11. 7; 7
12. 10; 5; 3½; 2½; 1; ½
14. (1) 13 (2) 3 (3) 80 (4) 169 (5) 9 (6) 89 (7) 9 (8) 169 (9) 39 (10) 39 (11) 712 (12) 15

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5. (2) 2 (3) 3 (4) 5

(5) 4 (6) 2 (7) $6\frac{1}{16}$
6. (8) 7 (9) 2 (10) 4

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4. Ash, 12; oak, 4; hickory, 16
5. 15 5c. stamps; 60 2c. stamps; 30 10c. stamps
6. 20
7. 6; 24; 3
8. 9; 18; 21

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9. 56,650 sq. mi.
10. 16 **13.** 58 mi.
11. $126\frac{2}{3}$ mi. **14.** 248 mi.
12. 285 mi. **15.** 369 mi.
16. Son, $15; Father, $60
17. Joseph 6; William 18
18. 12 in.; 20 in.

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19. 30 ft.; 45 ft.
20. 40 rd. **23.** 40 rd.
21. 440 rd. **24.** $37.50
22. $2.50 **25.** 350 rd.
26. 8m.; $50\frac{4}{7}$ m.
27. $R = \sqrt{\frac{7}{22}}$ in.; $C = 6\frac{2}{7}\sqrt{\frac{7}{22}}$ in.
1. 3 **2.** 8 **3.** 7
4. 2 **5.** 24 **6.** 36
7. 10 **8.** 3 **9.** 20
10. 12 **11.** 2

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12. 1 **13.** 3 **14.** $2\frac{1}{2}$
15. 8 **16.** 15 **17.** 12
18. 8 **19.** 9 **20.** 3
21. 3
22. Hemp. cable, 1″ diam.
W., .577 lb.; L., .109 T.; S., .654 T.
Hemp cable, $2\frac{1}{2}$″ diam.
W., 3.61 − lb.; L., .68 + T.; S., 4.09 − T.
23. Tarred hemp rope, 1″ diam.
W., 1.036 lb.; L., .247 T.; S., 1.480 T.
Tarred hemp, $2\frac{1}{2}$″ diam.
W., 6.475 lb.; L., 1.544 − T.; S., 9.25 T.
Manila rope, 1″ diam.
W., .765 lb.; L., .329 T.; S., 1.877 T.
Manila rope, $2\frac{1}{2}$″ diam.
W., 4.78 + lb.; L., 2.056 + T.; S., 11.731 + T.
24. Iron wire, $1\frac{1}{4}$″ diam.
W., 6.011 − lb.; L., 4.472 − T.; S., 26.581 + T.
Iron wire, $2\frac{1}{8}$″ diam.
W., 17.372 − lb.; L., 12.924 − T.; S., 76.82 − T.
25. Steel wire, $1\frac{1}{4}$″ diam.
W., 6.166 − lb.; L., 6.939 + T.; S., 43.172 − T.
Steel wire, $2\frac{1}{8}$″ diam.
W., 17.819 − lb.; L., 20.054 − T.; S., 124.767 − T.

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2. 12; 6
3. $5\frac{1}{4}$ ft.; $2\frac{1}{4}$ ft.
4. Water, $12\frac{1}{2}$ oz.; acid, $3\frac{1}{2}$ oz.
5. 26; 14

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6. 125 ft.; 50 ft.; 6250 sq. ft.
7. 24 lb.; 16 lb.
8. $15\frac{1}{2}$; $7\frac{1}{2}$
9. $12\frac{1}{2}$ ft.; $7\frac{1}{2}$ ft.
11. Acid, 7; water, 21
12. Silver, 371.25 gr.; alloy 41.25 gr.
13. Tin, 3; copper, 12
14. Copper, $22\frac{1}{2}$ T.; tin, $2\frac{1}{2}$ T.

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15. Water, $202\frac{1}{8}$; acid, $28\frac{1}{8}$
16. 7; 2 **20.** 24; 8
17. 8; 5 **21.** 16; 12
18. 7; 3 **22.** 24; 40
19. 15 in. 9 in. **23.** $\frac{3}{8}$

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25. 2; 12
26. −10; −35
27. 5; 7
28. $x=12$; $y=5$
29. $x=14$; $y=3$
30. $x=10$; $y=5$
31. $x=7\frac{1}{2}$; $y=5$
32. $x=16$; $y=14$
33. $x=1\frac{1}{4}$; $y=-1\frac{3}{4}$
34. $a=10$; $b=-21\frac{1}{2}$
35. $c=6\frac{2}{7}$; $d=2\frac{1}{7}$

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6. (1) $F=+113°$; $C=+45°$
(2) $F=+194°$; $C=+90°$
(3) $F=+131°$; $C=+55°$

Page 277

1. Ice, 32°F.; 0°R.
Benzol, 39.92°F.; 3.52°R.
Tallow, 109.4°F.; 34.4°R.
Paraffin, 114.8°F.; 36.8°R.
Wax, 143.6°F.; 49.6°R.
Sulphur, 239°F.; 92°R.
Tin, 446°F.; 184°R.
Bismuth, 482°F.; 200°R.
Cadmium, 608°F.; 256°R.
Lead, 618.8°F.; 260.8°R.
Zinc, 773.6°F.; 329.6°R.
Antimony, 809.6° F.; 345.6°R.
Silver, 1832°F.; 800°R.

Copper, 2012°F.; 880°R.
Gold, 2192°F.; 960°R.
Cast-Iron, 2192°F.; 960°R.
Cast-Steel, 2507°F; 1100°R.
Wrought-Iron, 2912°F.; 1280°R.
Platinum, 3227°F.; 1420°R.
Iridium, 3542°F.; 1560°R.

2. Ether, 35°C.; 28°R.
Carbon Dis., 46°C.; 36.8°R.
Sulph. Acid,—10° C.; —8°R.
Chloroform, 61°C.; 48.8°R.
Alcohol, 78°C.; 62.4°R.
Water, 100°C.; 80° R.
Mercury, 357°C.; 285.6°R.
Zinc, 1,040°C.; 832°R.

4. 98.96°F.; 29.76°R.

6. Ether va., 384.8°F.
Carb. acid, 87.8°F.
Etheline, 48.2°F.
Oxygen, −180.4°F.
Nitrogen, −229°F.
Hydrogen, −281.2° F.

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2. $114.5625
3. $33,812.50
4. $5\frac{1}{2}$ in.; $2\frac{13}{32}$ sq. in.

Page 281

5. 254.47
6. 48.4375 ft.

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2. $156.45

Page 283

2. (1) 884 (4) 2436
(2) 396 (5) 3575
(3) 1591 (6) 8036
(7) 4899 (8) 8019
(9) 8075

4. (1) 1296 (5) 361
(2) 2209 (6) 9025
(3) 4096 (7) 15,625
(4) 3364 (8) 21,904

6. (1) $30\frac{1}{8}$ (5) 930.21
(2) $272\frac{1}{2}$ (6) 2550.16
(3) $420\frac{3}{16}$ (7) 110.1875
(4) $1122\frac{3}{8}$ (8) $4970\frac{3}{81}$
(9) $8190\frac{5}{16}$

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8. (1) 114.44
(2) 20,159.81
(3) 8.56
(4) 4780.32

9. (1) 248.729
(2) 496.885
(3) 1977.280
(4) 4035.960

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2. (1) 568.75 (3) 42.51
(2) 73.28 (4) 2.39

2. (1) 80.20 (2) 196.54
(3) 93.62

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4. (1) 34 (5) 222
(2) 212 (6) 279
(3) 76 (7) 234
(4) 13.2 (8) 538
(9) 328

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1. $74\frac{11}{77}$ **3.** 2516
2. 2837 **4.** 321
5. 91,258,318 bu.; 107,705,575 bu.; 198,963,893 bu.
6. 16,447,257 bu.
7. 7,350,241 tons; 12,164,065 tons; 15,867,615 tons; 7,480,400 tons; 6,466,134 tons; 56,649,473 tons

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8. 69,489,932½ tons.
9. 75,807,199$\frac{1}{11}$ tons

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10. Queen, 216.5 A.M.; 224.7 P.M.
11. Beauty, 364.2 A.M.; 330.6 P.M.
Beechwood, 395.0 A.M.; 359.2 P.M.
Myrtle, 207.7 A.M.; 164.5 P.M.
Spot, 361.7 A.M.; 343.8 P.M.
Rose, 349.6 A.M.; 304.9 P.M.

12.

114.2	107.9
112.3	117.8
115.1	114.0
125.1	120.6
116.4	114.2
116.7	125.5
127.1	125.9
126.8	128.5
119.0	121.3
118.0	117.4
117.3	122.1
123.9	121.9
124.1	121.3
128.0	127.1
129.7	123.2

13. Queen, 441.2
Beauty 694.8
Beechwood, 754.2
Myrtle, 372.2
Spot, 705.5
Rose, 654.5

14. 3622.4
15. 42,659,947
16. 67,296,477 tons; 26,637,855 tons
17. (1) 4263
(2) 29,163
(3) 68,076
(4) 48,883
(5) 28,773
(6) 41,870
(7) 118,037
(8) 854,106
(9) 100,919

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18. 7.84; 2.50 **20.** 18¾ T.
19. $83,700 **21.** $\frac{25}{1250}$
22. 2925; $148.83
23. 146,200 lb.; $8819.70

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24. $1\frac{17}{20}$; $73\frac{1}{10}$
25. $301.44
26. 5280; 88
27. $59.22
28. $122,843,954.50
29. $128.25
30. 54 bu.
31. 95 ft.
32. $4740
33. 93 sec.

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34. $6
35. 33.105+ ft.
36. 17−; 21+; 26−
37. 69,564
38. $1043.46
39. $\frac{1}{2}$; 30
40. $9\frac{1}{2}$
41. 7227
42. 11; 7
43. 168

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44. 13
45. $\frac{1}{512}$
46. $\frac{243}{512}$
47. 39,578,910
48. $1\frac{7888}{128157}$
49. $\frac{171}{999}$
50. $31\frac{3}{4}$
51. $\frac{3}{88}$
52. $\frac{3}{56}$
53. $1\frac{1}{132}$
54. $\frac{5}{512}$
59. 11; $\frac{D}{d}$

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60. 8
61. $\frac{8}{10}$ in.
62. 11 in.
63. .75; .68; .65$\frac{1}{3}$; .25$\frac{5}{7}$; .0008$\frac{1}{3}$
64. $\frac{1}{3}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{4}{9}$; $\frac{5}{9}$; $\frac{1}{9}$; $\frac{8}{9}$
65. $\frac{10444}{12878}$; 3.25
66. $3\frac{807}{8481}$
67. 5215.183
68. 521.2162
69. 129.7025; 13,040.61$\frac{13}{32}$
70. .81621
71. 17.8; 22.6; 27.0
72. .0026
73. 28.284 rd.
74. 208.710 ft.

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75. .438; .446; .438
76. 10.878
77. 253.18
78. 360
79. 2160; 21.6
80. 9
81. 194.4
82. 160; 1
83. 129.6; 518.4 bu.
84. $48\frac{3}{14}$; $87\frac{1}{3}$
85. $16\frac{4}{11}$

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86. Table, $25.92
Bedstead, $64.80
Dresser, $32.40
Davenport, $77.76
87. Table, $31.20
Bedstead, $78.00
Dresser, $39.00
Davenport, $93.60
88. Table, $31.25
Bedstead, 78.12\frac{1}{2}$
Dresser, 39.06\frac{1}{4}$
Davenport, $93.75
89. $1.302; $3.881
90. $1.367; $4.072
91. $\frac{aao}{jmr}$; $\frac{brw}{enr}$
92. $\frac{mjw}{jmr}$; $\frac{wsn}{enr}$

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93. $188.68; $178.57
94. $298.89
95. 834.84 ft.; 295.16 ft.
96. 22.89 ft.
97. 12.5664
98. 20.31 in.
99. 10.16 in.
100. 22.33 in.
101. 37.42 ft.
102. $21\frac{33}{50}$
103. 11.3136+ in.

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104. 1256.64 sq. in.; Equals area of surface of sphere.
105. 43.9824+ cu. in.
106. 21.9912+ in.
107. 20.044+ lb.
108. 30°; 60°
109. 20°; 60°; 100°
110. 119.23+ sq. ft.
111. 509.29+ sq. rd.
112. $1\frac{7}{8}$A

Page 299

113. 78
114. 81
115. 480 lb.
116. $1131\frac{3}{7}$
117. $2299\frac{137}{287}$ mi. per hr.
118. $3372\frac{249}{287}$ ft. per sec.
119. $43.75
120. 6 yr. 3 mo.
121. $259.50

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122. $240
124. $19.25
123. 33 yr. 4 mo.
125. 3%